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Hybrid GA and OPF evaluation of network capacity for distributed generation connections

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ABSTRACT

Many methods have been applied to examining the capacity of existing distribution networks to accept distributed generation (DG). One aspect missing from existing approaches is the capability to efficiently site and size a predefined number of DGs. Here, a hybrid method employing genetic algorithms and optimal power flow aims to overcome this shortcoming. It could be applied by Distribution Network Operators to search a network for the best sites and capacities available to strategically connect a defined number of DGs among a large number of potential combinations. Some applications of the proposed methodology in the UK under current Ofgem financial incentives for DNOs confirmed its effectiveness in siting and sizing an assigned number of DG units.

Keywords: **— distributed generation, power flow analysis, optimization methods, power generation planning.**

1 INTRODUCTION

Distributed generation (DG) creates a variety of well-documented impacts on distribution network operation and implies significant changes to planning and design practices [1]-[4]. One area of interest is in providing Distribution Network Operators (DNOs) with the means to make best use of the existing network with DNOs encouraging development at the most suitable locations by issuing information to developers regarding the existence of spare connection capacity or from locational signals created by connection pricing. As such, DNOs require methods of quantifying the capacity of new DG that may be connected to distribution networks without the need for reinforcement.

This task has attracted significant research interest with a wide range of methods, objectives and constraints being applied within two broad approaches. The first approach aims to site DG of discrete, pre-specified, capacities at the best sites, requiring the use of methods like genetic algorithms (GAs) able to handle discrete formulations [5], [9]–[13]. The discrete formulation of DG capacity will not provide a truly optimal solution, while the use of multiple capacities extends the search space significantly. The second approach requires network locations of interest to be pre-specified with algorithms guiding capacity growth within network constraints. The methods tend to use continuous functions of capacity solved using methods like optimal power flow [4]–[6], linear programming [7] or gradient search [14] which are robust and repeatable. A downside is that where a large number of locations are searched the perceived optimal solution may contain a number of sites with very small available capacities. Although mathematically correct, the upfront costs of connection indicate that the very small plant would not be economic. However, the requirement to pre-specify locations is the major issue with this approach as the determination of the overall combination of locations is defined by ${}^{n}C_{r}$. With r DG to be located in a network of n buses, finding the best set is a significant effort beyond the feasibility of manual searches for even a small distribution network.

Both approaches require capacity or location to be pre-specified. Here, a method is presented that overcomes these limitations. It is a hybrid method that uses a GA to search a large range of combinations of locations, employing OPF to define available capacity for each combination. Although this is achieved at the expense of requiring the number of DG units to be pre-specified this opens up the potential to examine the benefits of strategic placement of small numbers of DG.

The paper is set out as follows: Section 2 sets out the basis for the hybrid DG capacity evaluation approach; Section 3 presents a case study using the tool which is discussed further in Section 4.

2 CAPACITY EVALUATION

The capacity of the existing network to accept DG is defined by a range of constraints imposed by statute (e.g., voltage limits), equipment specification (e.g., thermal limits on lines and transformers) or other operational or planning limits. In line with existing DNO practice in the UK these assessments are made assuming the traditional worst case situation of maximum DG output at minimum load which provides the largest reverse power flows and voltage rise [1], [15]–[18].

The hybrid method requires the user to define the number of DG units to be connected. The Genetic Algorithm generates combinations of locations from those available in the network. For each combination of locations, an optimal power flow is performed to define the capacity available; this information is fed back to the GA which searches for the ‘DNO optimal’ connectable capacity. In doing this the method should deliver the best locations as well as the capacities available for a user-specified number of DG. The methodology is shown in Fig. 1 and explained in more detail as follows.

2.1 UK Context

The optimal DG capacity is deemed to be from the point of view of the DNO. Clearly, the attitude of the DNO is dependent on the actual or perceived benefits or costs associated with DG connection, and these will vary between systems. A significant driver of the costs and benefits will be the regulatory rules or incentives applicable to DG. Here, we are using the current arrangements in force in the UK as the basis for decision-making.

In the 2005 distribution price control [19], [20], the UK regulator, Ofgem set out the regulatory environment for DNOs for a five year period. An incentive scheme for connecting DG was introduced wherein DG developers pay annual distribution use of system charges to the DNO (rather than the full upfront cost of connection). These consist of a charge based on 80% of the total cost of the reinforcement works (if any) required to connect the DG, a capacity charge of £1.50/kW of DG capacity installed (in lieu of direct recovery of the remaining 20% of the reinforcement assets) and an operations and maintenance (O&M) charge of £1/kW of DG capacity installed to recover operational costs relating to the reinforcement. As such, where DG is connected such that no reinforcement is required the DNO directly benefits by £2.50/kW of capacity installed.

The price control also introduced an incentive scheme to reduce losses. The regulator sets a target loss level for each DNO and DNOs are rewarded if losses are below this and penalised if they are above: each unit of loss is valued at £48/MWh (in 2004 values). While the impact of DG on losses is site and time specific, depends on the DG technology and control of reactive power, there is a tendency for losses to follow the U-shape trajectory, as Fig. 2 illustrates [21]. Specifically, losses begin to decrease when connecting small amounts of DG capacity until they achieve their minimum level. If DG capacity still increases, then losses begin to rise. It is worth pointing out that at high DG capacities, losses can become larger than those without DG connected. As such, the DNO is financially exposed to the impact of significant volumes of DG connections and while there is some protection offered by limits on loss adjustment factors, this does not apply to 11 kV systems.

2.2 Capacity evaluation with OPF

For a given set of locations, the network capacity available for new DG can be found using OPF following the approach of [4] and [6]. The maximum DG capacity can be determined by modelling DG as generators with negative cost coefficients. By minimizing the (negative) cost of all these generators, the DG capacity and benefit resulting from it are maximized. The available capacity is dictated by a range of network planning and DG control constraints.

Although there is great interest in active management of distribution networks (e.g., [15], [22]), there are difficulties associated with coordinating control of DG and other network elements. Here, the traditional approach of operating DG in power factor control mode is assumed, necessitating the power factor constraint [22]:

$$\cos\phi_g = P_g \Big/ \sqrt{P_g^2 + Q_g^2} = const. \tag{1}$$

The safe operation of power system equipment and quality of supply requires voltages to be maintained close to nominal:

$$V_b^{\min} \le V_b \le V_b^{\max} \tag{2}$$

where $V_b^{\min}$ and $V_b^{\max}$ are the lower and upper bounds of the bus voltage V_b. The thermal capacity, $S_t^{\max}$, of each line or transformer, t, also sets a limit to the maximum apparent power, S_t, transfer:

$$\left|S_t\right| \le S_t^{\max} \tag{3}$$

For simplicity, other constraints on DG penetration such as fault levels are not considered here but could be included within the methodology as illustrated in [6] or [8].

Earlier versions of the OPF approach [4], [6], [8] provided an objective function that was dependent only on the capacity of the DG connected. It can be adapted, instead, for the specific DNO requirements in force in the UK specifying incentives for DG connection and losses:

$$f_{OPF} = \sum_{g=1}^{n} C_g\left(P_g\right) - C_L\left(P_L^{BM} - P_L^{ACT}\right) \tag{4}$$

Here $C_g(P_g)$ is the benefit or incentive (£/kW per year) of connecting a generator g of capacity P_g, C_L represents the value of the loss incentive as applied to the difference between the actual level of losses P_L^{ACT} and the target losses, P_L^{BM} . As the target losses are specified by the regulator, it has been assumed that the DNO's target loss level is the same as those in the absence of new DG. The addition of loss incentives results in a more sophisticated problem than loss minimisation or capacity maximisation alone as there is a trade-off between extra DG capacity and loss reduction.

2.3 Genetic Algorithm

The genetic algorithm searches the combinations of DG locations arising from the specified number of DG. The GA randomly generates a population of solutions by defining a set of bus combinations with each combination represented by a vector of integers identifying individual buses. For each combination of locations in the population the nested OPF calculates the optimal capacities and losses considering the worst case of minimum load [15]-[18], according to the objective function defined in (4).

For each generation, a new set of improved individuals is created by selecting individuals according to their fitness according to a normalized geometric ranking scheme. After the new population is selected genetic operators are applied to selected individuals: simple crossover (randomly selected cut-point dividing each parent into two) and binary mutation (changes each of the bits of the parent based on the probability of mutation). An elitism mechanism ensures the survival of the best performing combination. The iteration process continues until the process reaches the maximum number of iterations or the best individual fails to improve beyond a specified amount over a specified number of generations.

Further, an alternative chromosome representation of individuals, considering both the number and location of DG units, without the number constraint [23], suggests that the methodology should be entirely flexible.

The entire method has been implemented in the Matlab®environment, incorporating some features of the MATPOWER suite [24]. Its use is illustrated in the following case study to identify the best combinations of a specified number of DG within the network.

3 CASE STUDY

The method was applied to an 11-kV radial distribution system comprising two substations, four feeders, 69 nodes and 78 branches (including normally open tie lines) [25]. The network is shown in Fig. 3 while detailed network data is given in the Table 1. The system was assumed to be operated under UK regulation voltage limits of ±6% of nominal with thermal limits of 3 MVA for all lines.

All DG units were assumed to have fixed 0.9 lagging power factors. The total active and reactive power are 4.47 MW and 3.06 MVAr respectively, while the loss target, evaluated for conditions without any DG unit installed on the network, is 0.228 MW. The objective functions use the incentives currently in force in the UK: a benefit of £2.50 per year for every kW of new DG connected as well as a penalty/reward of 4.8p/kWh for losses increase/reduction relative to the target.

The GA has a population size of 30 and the algorithm stops if improvement of the best objective function value is below a threshold of £0.01/hour over 50 generations or the number of generations exceeds 300. These values were found to guarantee the convergence of the algorithm to a satisfactory solution.

3.1 Analyses

In demonstrating the approach, several analyses explore a series of issues: identification of spare capacity in the network foe accommodating DG; the benefit of larger numbers of DG; and the impact of the loss incentive on optimal capacity.

A series of simulations were run to define the optimal connection points and capacities for sets of 3, 5, 7 and 9 potential DG units. Given the 67 possible sites these represent search spaces of 4.79×10^4, 9.66×10^6, 8.70×10^8 and 4.28×10^{10} combinations, respectively. Each simulation is a fairly lengthy process, but the duration is reasonable given the strategic nature of the process.

3.2 Results

The optimal locations and capacities for the four simulations are shown in Table 2 and illustrated in Fig. 4. Bus 35 appears in all four cases, while buses 40 (e.g., for 5, 7 and 9 DG search) and 62 (e.g., for 3, 5 and 9 DG search) appear in three assessments. Five other locations appear in two of the assessments.

It appears that many of the optimal locations are near to the centre of the feeders, often close to branch points. There is logic to this as they will tend to supply load at the end of the feeders whilst exporting modest amounts to the loads nearer to the sub-station. It is also apparent that the law of diminishing returns applies as, although overall connectable capacity increases with the number of DGs, the rate of increase falls. This will mainly be due to the opportunity to ‘spread’ capacity by connecting more but smaller DG and, to a large extent, will be influenced by the loss incentive which will tend to promote a more even spread of capacity. This finding supports the logic of promoting micro-generation within domestic and commercial properties, at the extreme end of the distributed generation spectrum.

The change in DNO incentive resulting from the connection and loss incentives is shown in Table 3. It is clear the overall incentive payments increase with the number of DGs increase, albeit by just over £2.79/hour. The interesting issue is the much larger contribution from the loss improvement incentive relative to that for connection. In saying that, the imbalance reduces as the number of DGs goes up: the ratio of the loss and connection incentives drops from 10.48 to 7.34 when DG numbers increase from 3 to 9. This occurs as the connection incentive rises by nearly 81% over this range while the loss incentive rises by only 27%. This would appear to suggest a law of diminishing returns as far as loss improvement goes.

The impact of the loss incentive can be explored by contrasting the results from the GA-OPF procedure with the optimal capacities suggested by an OPF applied to the same nodes with only the connection incentive as an objective. While this may not represent the best combinations of 3, 5, 7 and 9 DGs anywhere in the system, Table 4 does show some significant differences with the earlier results. Firstly, the connectable capacities are universally bigger than before while the benefit from the loss incentive becomes a significant penalty in the four cases so that the total incentive becomes a penalty for the cases with 5, 7 and 9 DGs because the loss penalty exceeds the connection incentive. The improvement in DG capacity from considering more locations seems less clear, particularly as the maximum capacity across the four cases occurs when considering 5 locations. This is as a direct result of these locations not being the optimal ones under the altered objective function. In the 5 DG case, the connection of a large DG unit of 3.35 MW at bus 35, and of three medium DG units, namely a 1.13 MW unit at bus 4, a 1.32 MW unit at bus 62 and a 1.26 MW unit at bus 26, creates significant losses due to the reverse power flows and this explains the larger decrease of loss incentive for the DNO in this case.

4 DISCUSSION

The method presented here attempts to overcome limitations in evaluating network capacity to absorb DG by avoiding the pre-specification of unit size or location within approaches described in the literature. This hybrid of OPF and GA techniques provides a means of finding the best combination of sites within a distribution network for connecting a predefined number of DGs. As such, it would allow DNOs to search a given network for the best sites to strategically connect a small number of DG among a large number of potential combinations. While current UK incentives have been used as a basis for guiding the search, alternative objective functions would make its application in other systems feasible [21].

The simulation results showed the method allowing the DNO to improving earnings, primarily by reducing network losses. On the basis of the simulation results DNOs could guide DG connections during time to achieve better network exploitation and avoiding network ‘sterilisation’ [4].

It is clear that the use of both connection and loss-reduction incentives alters the ‘optimal’ capacities and locations. Given the much larger incentive offered for loss reduction it is apparent that this tends to limit DG penetration relative to that allowed by the network constraints. It would be natural therefore for the DNO to seek to guide development at locations most favourable to it. These sites, however, may not be optimal from the environmental point of view which is likely to benefit from the connection of greater amounts of renewable energy sources. This issue clearly warrants further investigation and the authors are currently investigating appropriate means of exploring this issue [26]. A key limitation of the approach is that it does not consider the benefit of the DG in deferring network upgrades, nor consider the benefits associated with network upgrading in terms of enhanced DG capacity.

The method developed to analyse the optimal connection of essentially deterministic energy sources (e.g., CHP) within the deterministic network constraints applicable in the UK. However, it could be adapted to cope with variable energy sources and probabilistic network constraints in order to develop a cost-benefit model for network capacity.

5 CONCLUSIONS

A method combining optimal power flow and genetic algorithms aims to provide a means of finding the best combination of sites within a distribution network for connecting a predefined number of DGs. In doing so it overcomes known limitations inherent in current available techniques to optimize DG capacity. Its use would be to enable DNOs to search a network for the best sites to strategically connect a small number of DGs among a large number of potential combinations.

6 ACKNOWLEDGEMENTS

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Figure 2: Schematic of loss dependence on DG capacity

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Figure 4: Optimal location of DG across all 4 cases (one circle per case)

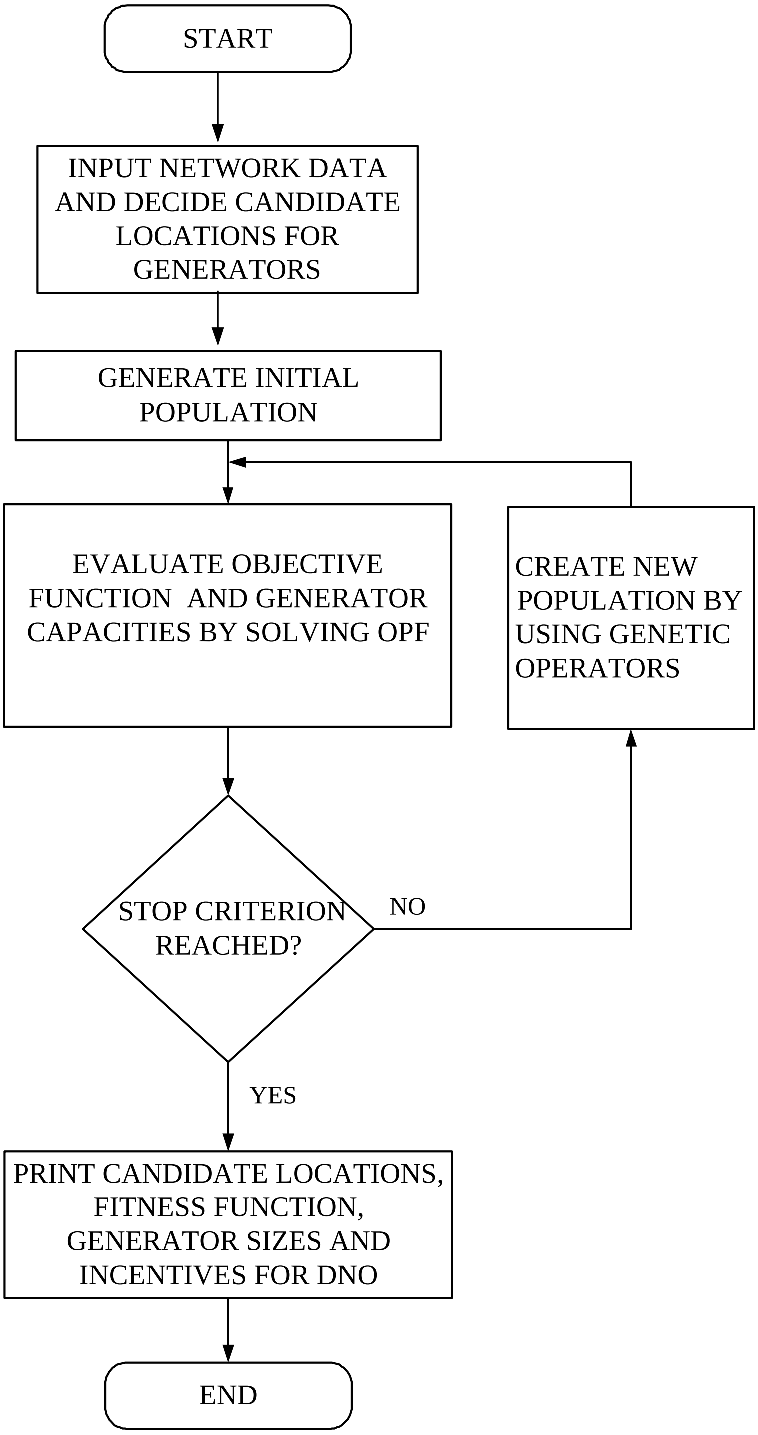

Fig. 1. Schematic of the GA-OPF methodology

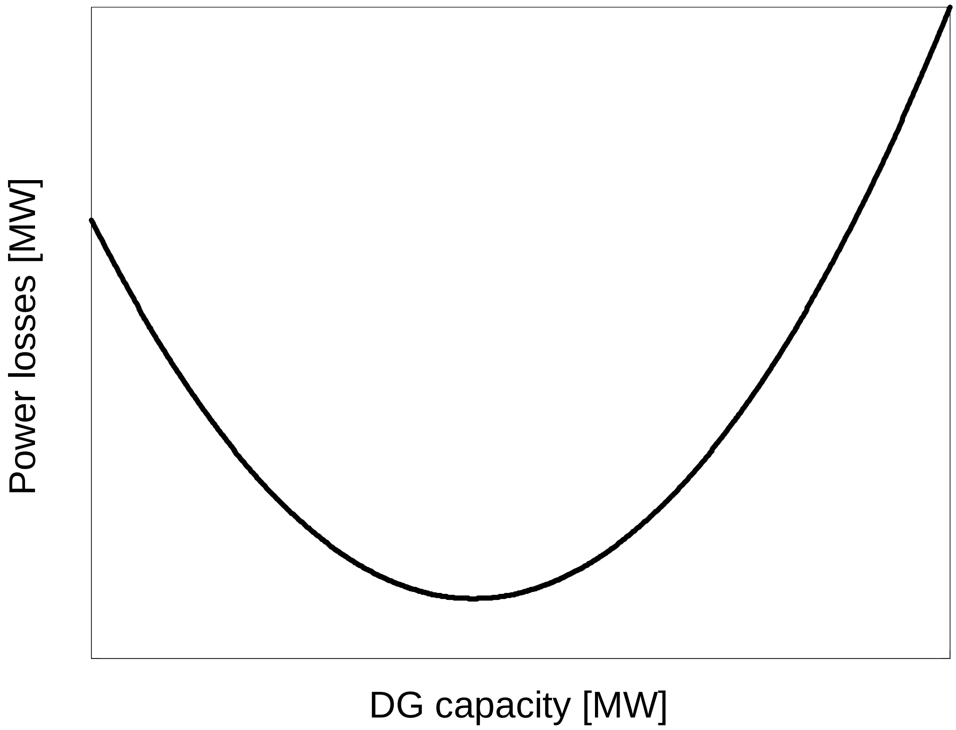

Fig. 2. Schematic of loss dependence on DG capacity

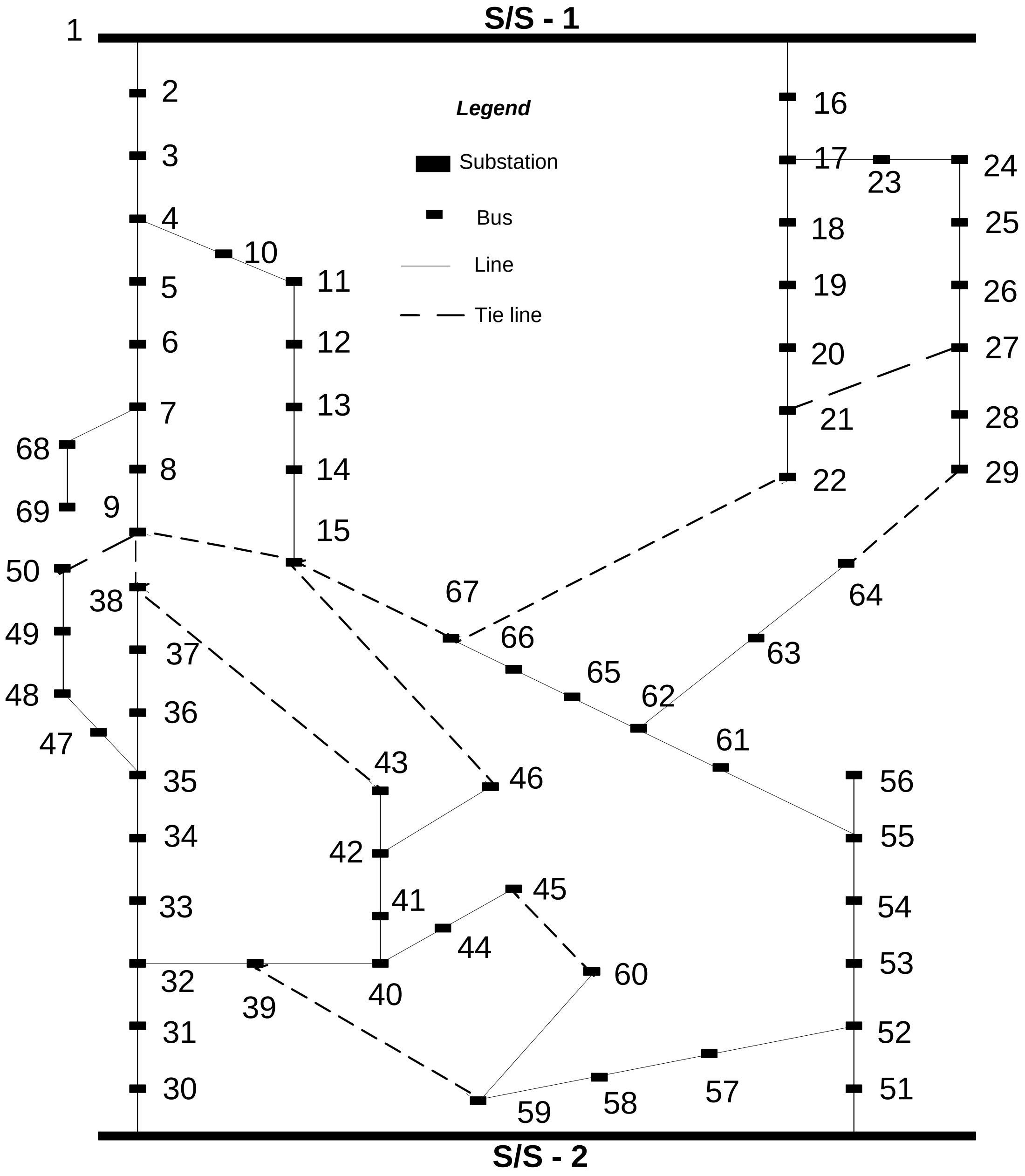

Fig. 3. Radial distribution system

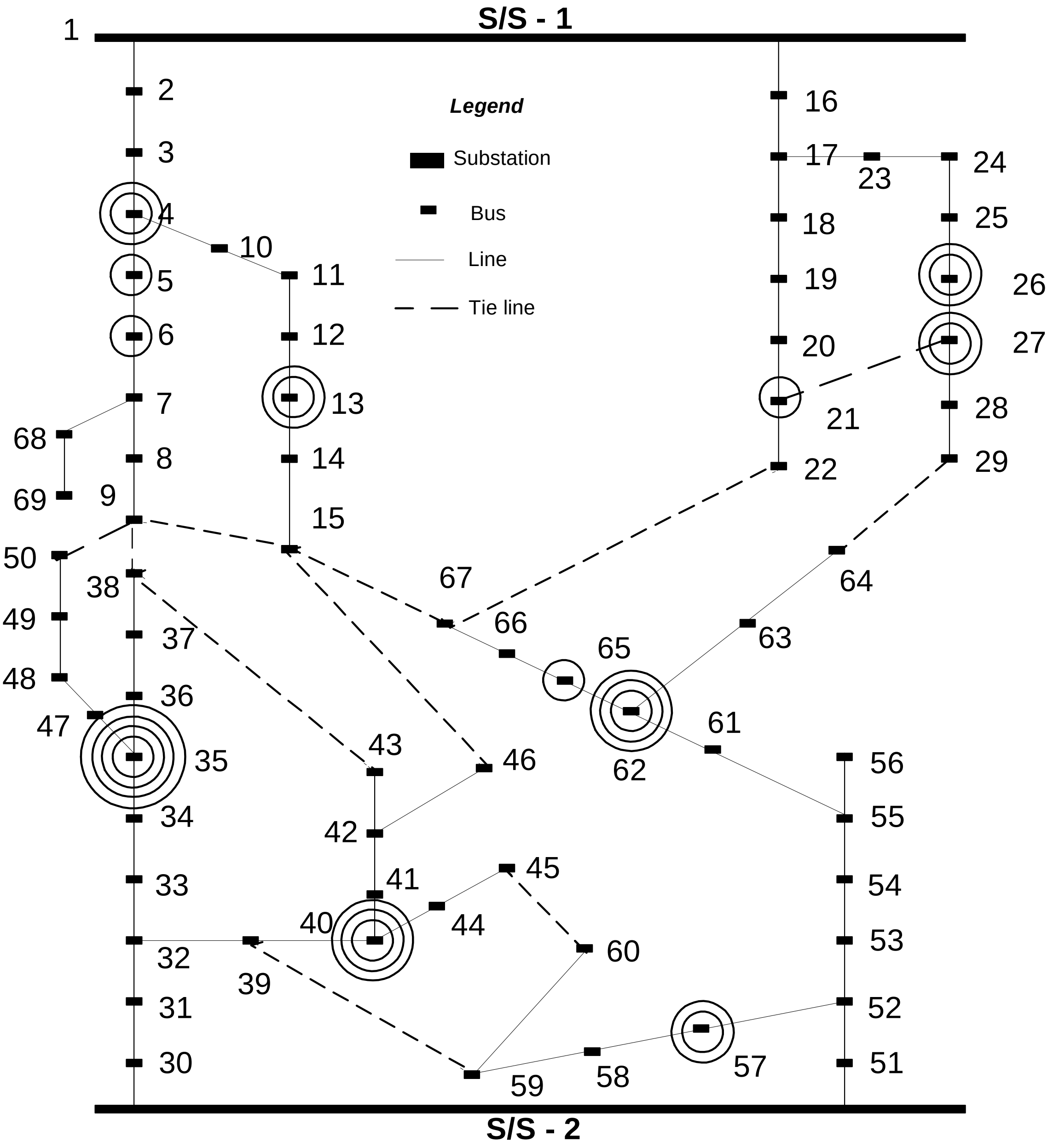

Fig. 4. Optimal location of DG across all 4 cases (one circle per case)

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Table 1: 69-bus system data

n.line	s-bus	r-bus	R (Ω)	X (Ω)	P (r-bus) (kW)	Q (r-bus) (kVAr)
1	1	2	1.0970	1.0740	100	90
2	2	3	1.4630	1.4320	60	40
3	3	4	0.7310	0.7160	150	130
4	4	5	0.3660	0.3580	75	50
5	5	6	1.8280	1.7900	15	9
6	6	7	1.0970	1.0740	18	14
7	7	8	0.7310	0.7160	13	10
8	8	9	0.7310	0.7160	16	11
9	4	10	1.0800	0.7340	20	10
10	10	11	1.6200	1.1010	16	9
11	11	12	1.0800	0.7340	50	40
12	12	13	1.3500	0.9170	105	90
13	13	14	0.8100	0.5500	25	15
14	14	15	1.9440	1.3210	40	25
15	7	68	1.0800	0.7340	100	60
16	68	69	1.6200	1.1010	40	30
17	1	16	1.0970	1.0740	60	30
18	16	17	0.3660	0.3580	40	25
19	17	18	1.4630	1.4320	15	9
20	18	19	0.9140	0.8950	13	7
21	19	20	0.8040	0.7870	30	20
22	20	21	1.1330	1.1100	90	50
23	21	22	0.4750	0.4650	50	30
24	17	23	2.2140	1.5050	60	40
25	23	24	1.6200	1.1100	100	80
26	24	25	1.0800	0.7340	80	65
27	25	26	0.5400	0.3670	100	60
28	26	27	0.5400	0.3670	100	55
29	27	28	1.0800	0.7340	120	70
30	28	29	1.0800	0.7340	105	70
31	1	30	0.3660	0.3580	80	50
32	30	31	0.7310	0.7160	60	40
33	31	32	0.7310	0.7160	13	8
34	32	33	0.8040	0.7870	16	9
35	33	34	1.1700	1.1450	50	30
36	34	35	0.7680	0.7520	40	28
37	35	36	0.7310	0.7160	60	40
38	36	37	1.0970	1.0740	40	30
39	37	38	1.4630	1.4320	30	25
40	32	39	1.0800	0.7340	150	100
41	39	40	0.5400	0.3670	60	35
42	40	41	1.0800	0.7340	120	70
43	41	42	1.8360	1.2480	90	60
44	42	43	1.2960	0.8810	18	10
45	40	44	1.1880	0.8070	16	10
46	44	45	0.5400	0.3670	100	50
47	42	46	1.0800	0.7340	60	40
48	35	47	0.5400	0.3670	90	70
49	47	48	1.0800	0.7340	85	55
50	48	49	1.0800	0.7340	100	70
51	49	50	1.0800	0.7340	140	90
52	1	51	0.3660	0.3580	60	40
53	51	52	1.4630	1.4320	20	11
54	52	53	1.4630	1.4320	40	30
55	53	54	0.9140	0.8950	36	24
56	54	55	1.0970	1.0740	30	20
57	55	56	1.0970	1.0740	43	30
58	52	57	0.2700	0.1830	80	50
59	57	58	0.2700	0.1830	240	120
60	58	59	0.8100	0.5500	125	110
61	59	60	1.2960	0.8810	25	10
62	55	61	1.1880	0.8070	10	5
63	61	62	1.1880	0.8070	150	130
64	62	63	0.8100	0.5500	50	30
65	63	64	1.6200	1.1010	30	20
66	62	65	1.0800	0.7340	130	120
67	65	66	0.5400	0.3670	150	130
68	66	67	1.0800	0.7340	25	15
69	9	50	0.9080	0.7260	-	-

70	9	38	0.3810	0.2440	-	-
71	15	46	0.6810	0.5440	-	-
72	22	67	0.2540	0.2030	-	-
73	29	64	0.2540	0.2030	-	-
74	45	60	0.2540	0.2030	-	-
75	43	38	0.4540	0.3630	-	-
76	39	59	0.4540	0.3630	-	-
77	21	27	0.4540	0.3630	-	-
78	15	9	0.6810	0.5440	-	-
79	67	15	0.4540	0.3630	-	-

TABLE 2 OPTIMAL DG LOCATION/CAPACITIES

BUS	**CAPACITY ADDED [MW]**			
	3 DG	**5 DG**	**7 DG**	**9 DG**
4		0.9421		0.4686
5			0.6404	
6				0.2315
13			0.2679	0.2436
21				0.2647
26	0.7395	0.7606		
27			0.7337	0.6775
35	1.0314	0.7638	0.7639	0.7641
40		0.7106	0.7178	0.7243
57			0.7892	0.7519
62	0.8904	0.8903		0.7072
65			0.6529	
Total	**2.6614**	**4.0674**	**4.5658**	**4.8334**

TABLE 3 Optimal capacity and incentives

	3 DG	**5 DG**	**7 DG**	**9 DG**
TOTAL CAPACITY [MW]	2.6614	4.0674	4.5658	4.8334
CAPACITY/MINIMUM LOAD RATIO (%)	59.53%	90.99%	102.14%	108.13%
DG INCENTIVE [£/HOUR]	0.7595	1.1607	1.3030	1.3795
LOSS INCENTIVE [£/HOUR]	7.9607	9.5705	9.9700	10.1293
TOTAL INCENTIVE [£/HOUR]	8.7202	10.7314	11.2729	11.5991

TABLE 4 Capacities and incentives with losses omitted from objective function

	3 DG	**5 DG**	**7 DG**	**9 DG**
TOTAL CAPACITY [MW]	5.647	6.987	6.9763	6.9572
DG INCENTIVE [£/HOUR]	1.6118	1.9939	1.9910	1.9855
LOSS INCENTIVE [£/HOUR]	-0.7156	-3.0800	-3.9612	-3.7217
TOTAL INCENTIVE [£/HOUR]	0.8960	-1.1011	-1.9703	-1.7362