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An optimisation framework for thermal energy storage integration in a residential heat pump heating system

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Abstract

Domestic heating has a large share in the UK total energy consumption and significant contribution to the greenhouse gas emissions since it is mainly fulfilled by fossil fuels. Therefore, decarbonising the heating system is essential and an option to achieve this is by heating system electrification through heat pumps (HP) installation in combination with renewable power generation. Potential increase in performance and flexibility can be achieved by pairing HP with thermal energy storage (TES), which allows the shifting of heat demand to off peak periods or periods with surplus renewable electricity. We present a design and operational optimisation model which is able to assess the performance of HP-TES relative to conventional heating system. The optimisation is performed on a synthetic heat demand model which requires only the annual heat demand, temperature and occupancy profiles. The results show that the equipment and operational cost of a HP system without TES are significantly higher than for a conventional system. However, the integration of TES and time-of-use tariffs reduce the operational cost of the HP systems and in combination with the Renewable Heating Incentive make the HP systems cost competitive with conventional systems. The presented demand model and optimisation procedure will enable the design of low carbon district heating systems which integrate the heating system with the variable renewable electricity supply.

Keywords: thermal energy storage, heat pump, optimisation, heat demand model, mixed integer linear programming

1. Introduction

Nearly half of the UK total energy consumption is for heating purposes and this proportion is even higher in Scotland [1, 2]. The domestic heat demand is responsible for the majority of this end use and supplied mostly by natural gas. Therefore, there is a large potential and need to reduce the environmental impact of domestic heating by decarbonising the heating systems.

Reducing the overall heat demand by increasing energy efficiency of the built environment has been acknowledged as one of the most viable solutions. The potential savings of this effort on primary energy consumption have been shown for different countries [3, 4]. However, the heat demand is unlikely to decrease significantly in the near future due to the low construction rates of more energy efficient new buildings and the gap between planned and actual energy savings from retrofitting activities [5].

The utilisation of heat pumps (HP) to fulfil heat demand is another potential solution towards heating system decarbonisation. The main premise of this effort is to use electricity generated by renewable sources to provide heating and thus the decarbonisation effect will not fully materialise as long as the electricity generation relies heavily on fossil fuels. However, the further integration of heating and electricity networks also expands the opportunities for demand side management and will thus enable the integration of more variable renewable generation into the energy system. One of these opportunities is the combination of HPs with thermal energy storage (TES) to shift electrical load from on-peak to cheaper off-peak hours [6], and in the future to times with surplus renewable electricity. An early example is the study by Tassou et al. which explores the implementation of heat pumps in the UK and compares its economic performance with typical heating systems in the late 1980s [7]. Technological improvements and supporting policies have promoted heat pumps beyond the early stage limitations [8]. Results from a recent field trial in the UK illustrate the real performance of the technology [9]. The effects of off-peak tariff periods and building fabric characteristics on heat pump annual performance are investigated by Cabrol and Rowley [10].

The simultaneous design and operational optimisation of HP-TES systems is essential to ensure that the installations of new energy systems lead to improvements, both financially and environmentally, compared to conventional heating systems. Furthermore, this optimisation has to be performed for every installation due to the vastly different local weather conditions, occupancy profiles, energy tariffs, government subsidies and building types. For example, the optimal sizing of the HP is crucial due to the

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C	cost, £		*Subscripts*	
COP	coefficient of performance		cap	thermal capacity
$E7$	economy 7		ch	charge
$E10$	economy 10		dch	discharge
HH	heating hours		dem	demand
HP	heat pump		el	electricity
LF	load factor, -		ext	external
$MKEP$	milton keynes energy park		i	heat pump type
P	electrical power, kW		in	input
Q	thermal energy, kWh		$loss$	standing loss
$\dot{Q}$	thermal power, kW		min	minimum
R	revenue, £		nom	nominal
RHI	renewable heat incentive		out	output
SPF	seasonal performance factor		rh	resistive heater
T	temperature, K		sto	store
TES	thermal energy storage		t	time step
ΔT	temperature difference, K		$thld$	threshold
c_p	heat capacity of water, kJ/kgK			
r	discount rate, -		*Superscripts*	
y	binary selection status of heat pump unit, -		ac	active
η	efficiency, -		ic	inactive
δ	binary operational status of heat pump unit, -		inv	investment
ρ_w	density of water, kg/m^3		opr	operational
τ	time horizon, year		tot	total

large variations in heat demand throughout the year [8, 9]: an undersized HP might worsen the overall economic and environmental performance by increased utilisation of electric resistive heating to cover the heat demand while an oversized HP would increase the capital costs. Additionally, the operational optimisation is particularly relevant when different energy vectors are intertwined in the future smart energy system, e.g. widespread installation of HP-TES systems. However, such an integration brings new challenges in the control and operation of the energy system. For example, Kelly et al. [11] showed that synchronised load-shifting with HP-TES systems can lead to a significantly increased peak load in the electricity system. Thus it is essential to be able to assess the performance of a HP-TES system before the installation and also during the operation of the system. As Kelly et al. showed, the latter point is particularly important to enable a concerted operation of multiple HP-TES systems in either an integrated energy system or a district heating network.

The modelling and simulation of energy systems is usually performed with one of two approaches. In the first approach, the system is modelled using a specific energy simulation software, such as TRNSYS [10] and ESP-r [11]. The main benefit of this approach is the detailed simulation of the physical characteristics of the energy system, including non-linear behaviours, and the utilisation of validated equipment models. However, these tools can be computationally expensive and, more importantly, system optimisation is not their main purpose. Furthermore, they can be difficult to set up which prevents their widespread use. On the other hand, the second approach involves modelling reduced complexity models of the energy systems using mathematical programming methods, for example evolutionary algorithms [12, 13] or mixed integer linear programming (MILP) [14, 15, 16]. These studies typically include sizing and operational optimisation of
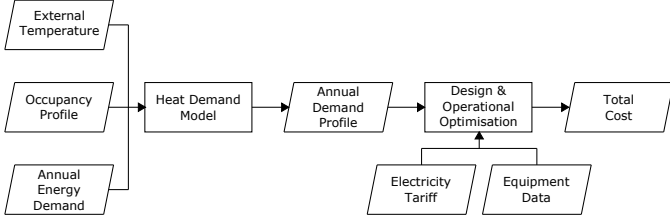
Figure 1: Optimisation framework.

Figure 2: Heating system consisting of an air source heat pump and a hot water storage tank with resistive back-up heater.

various potential generation and storage equipment to fulfil the given energy demand. This approach is very flexible and the MILP methods can be computationally efficient.

While the MILP methods offer efficient optimisation, they either rely on external tools to provide the demand profile or include simple correlations which might not capture the essential characteristics of the heat demand. However, the heat demand is the most important input parameter for any heating system optimisation and it is essential to get a good representation of the real system. Unfortunately, measured demand profiles are usually not available and thus synthetic demand profiles are generated and utilised as an input. Some studies use building simulation software [11] or multi-purpose simulation tools [16] which require a large number of input parameters and are not very accessible. Other studies use relatively simple correlations between demand and external temperature but neglect the occupancy profile [17]. For example, Tassou et al. [7] used a heat demand model which depends only on the outside temperature and neglects further effects such as occupancy. This can lead to lower peaks in the heat demand which will affect the sizing of the HP. In addition, the comparison of the results with a dedicated energy simulation software, e.g. TRNSYS, or experimental data is usually missing. Thus there is a need for a simple, yet sufficiently accurate framework for the optimisation of domestic heating systems.

This work presents a complete framework for the simultaneous design and operational optimisation of different heating system configurations in terms of their annual operational and total cost. The framework is self-contained and requires for the calculation of the heat demand only the external temperature, annual heat demand and occupancy profile. The effects of different HP and resistive heating power ratings, TES sizes and required heating temperatures are investigated in the tariff and government incentive context of the UK. The results are comparable to a similar study which utilised iTRNSYS simulation model.

2. Model description

The main parts of the optimisation framework are the heat demand model and the design and operational optimisation as shown in Fig. 1.

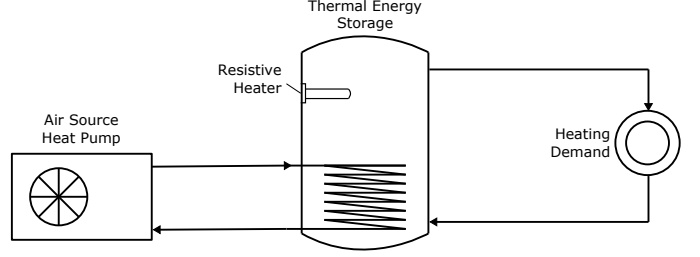The heat demand model includes both space heating and domestic hot water demand. The modelled heating system consists of a monovalent air-source heat pump system with thermal energy storage, as illustrated in Fig. 2. The heat pump supplies energy to the storage tank through a coil heat exchanger located at the bottom of the tank. A detailed description of each part of the model is given in the following paragraphs.

2.1. Optimisation problem

The main optimisation problem of this study is to find the optimal sizing and operational profile in order to minimize the total cost C^{tot}, which includes investment cost C^{inv}, operational cost C^{opr}, and revenue from subsidy R:

$$min \quad C^{tot} = min \left\{ C^{inv} + \sum_{\tau=1}^{20} \frac{C^{opr} - R}{(1+r)^\tau} \right\} \qquad (1)$$

The investment cost includes equipment price and installation cost and is assumed to be £1500 and £500 for HP and TES, respectively (Eq. 2). The annual operational cost consists of the total amount of electricity input to the heat pump and resistive heater multiplied by the appropriate electricity tariff (Eq. 3). In Eq. 1, the net present value is utilised to consider the future operating cost and revenue, with 5.5% interest rate, r and 20 years time horizon τ [18].

$$C^{inv} = C_{HP}^{inv} + C_{TES}^{inv} \qquad (2)$$

$$C^{opr} = \sum_{t=0}^{8759} (P_{HP,in,t} + P_{rh,t}) \cdot \Delta t \cdot C_{el} \qquad (3)$$

The type of subsidy considered in this study is the Domestic Renewable Heat Incentive (RHI) [19]. The UK government launched this policy in order to foster the implementation of non-fossil fuel domestic heating systems. It is a financial incentive policy which offers payments to the consumers for the amount of heat their system produces for 7 years. Eligible heating systems are biomass boilers, heat pumps (both air and ground source), and solar thermal collectors. The current tariff, i.e. September 2015, for air source heat pumps is 0.0742/kWh. The annual revenue is calculated based on the annual heat demand (e.g. based on Energy Performance Certificate) and

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the average seasonal performance factor (SPF) of the heat pump (Eq. 4)

$$R = Q_{dem} \cdot \left(1 - \frac{1}{SPF}\right) \cdot R_{RHI} \qquad (4)$$

An overview of the optimisation framework is depicted in Fig. 1. The MILP design and operational optimisation model receives inputs in the form of equipment data, electricity tariff, and heat demand data. The latter is synthetically generated using a heat demand model. During the optimisation the system sizes, e.g. HP rating and TES size, and the operational state of the units, e.g. HP output, are modified to find the lowest total cost for the given conditions, e.g. tariff and resistive heater rating.

2.2. Heat demand model

One important input to a heating system optimisation is heat demand data. A real measurement-based demand profile with complete supporting information is hard to obtain and rarely available in the literature. For example, hourly gas and electricity consumption for several houses in the Milton Keynes Energy Park (MKEP) project are available, but details on housing characteristics and social information are missing [20]. Thus, this study employs a heat demand model to generate synthetic heat demand profiles.

Heat demand depends on numerous factors, such as weather conditions, building characteristics, occupancy profile, installed heating system and occupants behaviour. A heat demand model typically reduces this complexity by various simplifications depending on the modelling approach. Residential energy demand can be modelled by two modelling approaches: top-down and bottom-up [21]. The top-down approaches rely on highly aggregated historical energy consumption data and are relatively straightforward to develop. On the other hand, the bottom-up approaches, which can be further categorised into bottom-up statistical and bottom-up engineering approach, require more detailed input information (e.g. building characteristics and billing data) and can be computationally intensive.

In this study, a synthetic heat demand model is developed by combining different aspects of the aforementioned modelling approaches: aggregated consumption data from the top-down approach and occupancy data from the bottom-up approach. The model requires the total annual heat demand, external temperature data and occupancy profile as inputs. The latter two inputs are selected over other influencing factors, e.g. solar gain, due to their relative importance as reported by various studies (e.g. [22, 23]). On the other hand, it has been shown for the low-voltage electricity network that the inclusion of the user occupancy and activity profile in the load model leads to more realistic load profiles [24]. The model is based on the energy signature method, where the heat demand is assumed to be a linear function of external temperature [25, 17], with the inclusion of occupancy profiles.

Table 1: DHW probability distribution

Time period	Ratio of daily DHW-volume
Weekdays / Weekends	
07.00 - 09.00 / 08.00 - 10.00	50 %
09.00 - 18.00 / 10.00 - 16.00	10 %
18.00 - 23.00 / 16.00 - 00.00	30 %
23.00 - 07.00 / 00.00 - 08.00	10 %

The working status of the heating system is dependent on external temperature and occupancy profile (Eq. 5). During the occupied period the heating system is operational if the external temperature is below the threshold temperature. Here, the night-time (23.00 - 07.00) counts as inactive period with the inactive threshold temperature $T_{thld}^{ic} = 0$ °C while the rest of the day uses the active threshold temperature $T_{thld}^{ac} = 14$ °C. The occupancy profile of 2 adults working full-time is assumed in this publication but other occupancy profiles are straightforward to implement. This corresponds to a scenario which has an unoccupied period from 09.00 to 18.00 during weekdays [26]. The signature variables k_1 and k_2 are computed to match the annual demand with the heating hours, as shown in Eq. 7 - 8.

$$\dot{q}(t) = \begin{cases} k_1 \cdot T_{ext}(t) + k_2^{ac} & \text{if } T_{ext} < T_{thld}^{ac} \wedge t \in t^{ac} \\ k_1 \cdot T_{ext}(t) + k_2^{ic} & \text{if } T_{ext} < T_{thld}^{ic} \wedge t \in t^{ic} \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

$$k_1 = \frac{Q_{dem}^Y}{HH^{ac} + HH^{ic}} \qquad (6)$$

$$HH^{ac} = \int\limits_{t \in t^{ac}: T_{ext} < T_{thld}^{ac}} T_{ext}(t)\,\mathrm{d}t - \int\limits_{t \in t^{ac}: T_{ext} < T_{thld}^{ac}} T_{thld}^{ac}(t)\,\mathrm{d}t$$

$$HH^{ic} = \int\limits_{t \in t^{ic}: T_{ext} < T_{thld}^{ic}} T_{ext}(t)\,\mathrm{d}t - \int\limits_{t \in t^{ic}: T_{ext} < T_{thld}^{ic}} T_{thld}^{ic}(t)\,\mathrm{d}t$$

$$(7)$$

$$\begin{aligned} k_2^{ac} &= -k_1 \cdot T_{thld}^{ac} \\ k_2^{ic} &= -k_1 \cdot T_{thld}^{ic} \end{aligned} \qquad (8)$$

Domestic hot water (DHW) demand is included by calculating the draw profile with the *DHWcalc* software [27]. In estimating the DHW draw profile, *DHWcalc* requires a number of inputs, such as house type, mean daily draw-off volume and probability distributions of the draws. Table 1 shows the distribution used in this study. The 10% daily draw assumption during the unoccupied hours (e.g. weekdays, 09.00-18.00) is chosen to consider the small irregularity in occupancy profiles and possible demand from appliances.

Fig. 3(a) illustrates an example synthetic heat demand profile generated by the model. The annual energy consumption of the modelled dwelling is calculated by multiplying the average natural gas consumption for space and water heating in a Scottish dwelling (i.e. approximately 15000 kWh/year [28]) with assumed boiler efficiency of 90%. The external temperature data are gathered from Met Office data for an Edinburgh weather station in 2013 [29].

In order to qualitatively evaluate the resulting synthetic demand profile, the heat demand and external temperature data of a house in Milton Keynes Energy Park (MK0805, year 1990) are shown in Fig. 3(c). In this case, the heat demand is derived from the gas consumption, which is available as hourly measured data in the MKEP datasets. Annual gas consumption of the MKEP house is 13456 kWh. The general trend of the synthetic heat demand profile is comparable to the demand profile of the selected house.

The inclusion of occupancy profile has a profound impact on the synthetic demand profile, as can be seen in the comparison between Fig. 3(a) and 3(b). Without occupancy profile, the resulting demand profile does not show sharp peaks and no demand period, which is not realistic in a domestic house case. The demand peaks are crucial for the sizing of HP and TES.
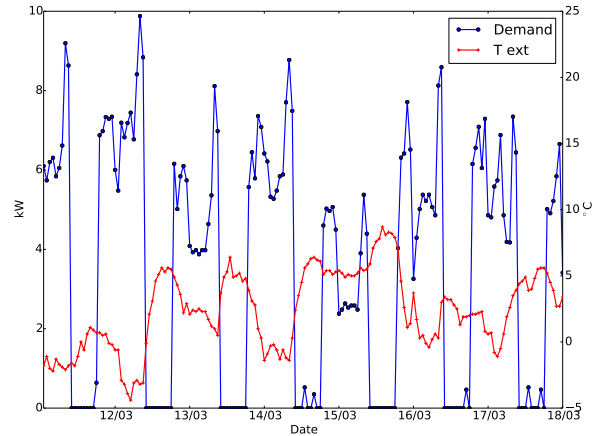
2.3. Heat pump

The performance of a heat pump can be quantified by the coefficient of performance (COP), which is defined as the ratio between the thermal power output and the electrical power input (Eq. 9). The COP is affected by different variables, such as external temperature, supply water temperature, inlet water temperature and load factor. Simplifications can be made in order to reduce this complexity, but this should be done with care as it can affect the optimal control result. For example, it has been shown that a simplified model which neglects the dependency of the COP on the external temperature can produce higher electricity consumption, relative to the more complex model [30].

$$COP = \frac{\dot{Q}_{HP,out}}{P_{HP,in}} \qquad (9)$$

The COP of the heat pump in this study is modelled as a function of temperature lift which is the difference between the supply water temperature and the external air temperature (Eq. 10). Required data to produce the linear regression fits are derived from manufacturer's data [31]. Relevant heat pump data can be found in Table 2.

$$COP_i = a \cdot T_{lift} + b \qquad (10)$$

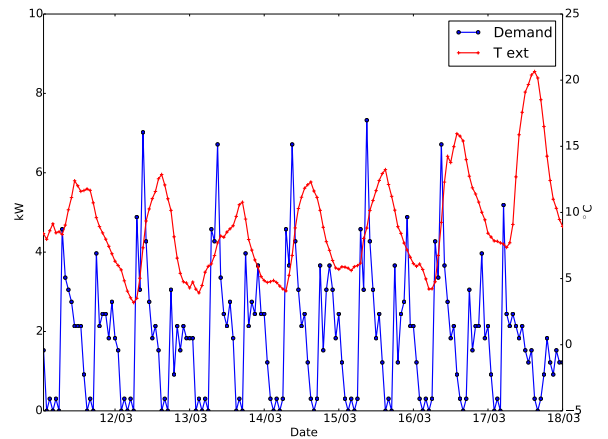Equipment modelling in the MILP model is performed by prescribing constraints which reflect the characteristics of the equipment. The current study considers a discrete set of heat pumps, from which only one must be selected.

(a)
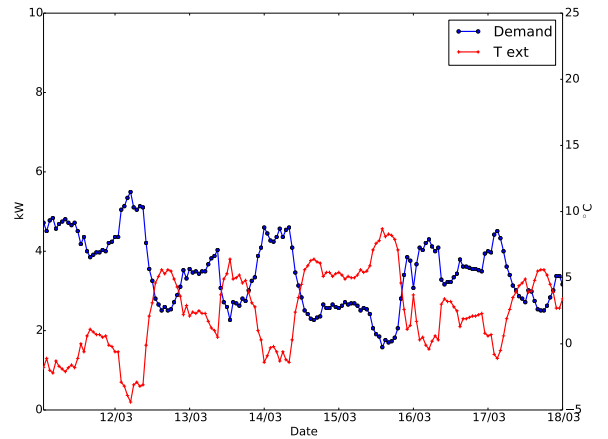
(b)

(c)

Figure 3: Example of weekly synthetic demand profile with occupancy profile (a), without occupancy profile (b) and weekly demand of an MKEP house (c).

Table 2: Air source heat pump data

Type	Capacity (kW_{th})	C_{HP}^{inv} (£)	a (K^{-1})	b
1	5.0	2778	-0.066	5.7
2	8.5	3784	-0.087	6.8
3	11.2	4506	-0.072	5.6
4	14.0	5701	-0.077	6.1

This is ensured by Eq. 11 and 12, where y_i is a binary variable describing whether heat pump model i is installed or not.

$$\sum_{i=1}^{4} y_i = 1 \qquad (11)$$

$$\dot{Q}_{HP,nom} = \sum_{i=1}^{4} y_i \cdot \dot{Q}_{i,nom} \qquad (12)$$

The thermal power output of the HP is limited by a minimum load factor and maximum capacity, as shown in Eq. 13. The binary variable δ_t describes the operational status of the heat pump at time step t. Eq. 13 includes non-linearity in the form of a bilinear term $\delta_t \cdot \dot{Q}_{HP,nom}$. This non-linearity is reformulated into linear equations [32]. The minimum load factor LF_{min} is set at 35%.

$$LF_{min} \cdot \delta_t \cdot \dot{Q}_{HP,nom} \leq \dot{Q}_{HP,out} \leq \delta_t \cdot \dot{Q}_{HP,nom} \qquad (13)$$

2.4. Thermal energy storage

The thermal energy storage included in this study is a typical domestic hot water tank with 120 - 300 L volume range. The energy content of the ideally stratified TES is calculated by Eq. 14. For a heat pump heating system, the temperature increase in the storage tank, ΔT_{TES} is set to 10 K [16].

$$Q_{cap} = \frac{V_{TES} \cdot \rho_w \cdot c_p \cdot \Delta T_{TES}}{3600} \qquad (14)$$

TES related constraints in the MILP formulation are shown in Eq. 15 - 18. The energy stored in the TES at time t, $Q_{sto,t}$ is calculated according to Eq. 15, and limited by the maximum energy content (Eq. 16). Standing losses $\dot{Q}_{loss}$ are gathered from the manufacturer's datasheet [33]. Furthermore, the TES charging rate is limited by the heat pump thermal power output (Eq. 17)

$$Q_{sto,t} = Q_{sto,t-1} + (\dot{Q}_{ch,t} - \dot{Q}_{dch,t} - \dot{Q}_{loss}) \cdot \Delta t \qquad (15)$$

$$Q_{sto,t} \leq Q_{cap} \qquad (16)$$

$$\dot{Q}_{ch} \leq \eta_{ch} \cdot \dot{Q}_{HP,out} \qquad (17)$$

The house heat demand is fulfilled by discharging energy from the TES, along with additional back-up resistive

Table 3: Thermal energy storage data

Type	Capacity (L)	C_{HP}^{inv} (£)	$\dot{Q}_{loss}$ (kWh/24h)
1	120	1290	1.0
2	150	1375	1.38
3	180	1425	1.63
4	210	1453	1.9
5	250	1575	2.21
6	300	1700	2.43

Table 4: Electricity tariffs

Tariff	On-peak (£/kWh)	Off-peak (£/kWh)
Standard	0.144	0.144
Economy 7	0.1747	0.0765
Economy 10	0.1744	0.071

heater, as stated in Eq. 18. The ON/OFF status of the resistive heater is represented by the binary variable $\delta_{im,t}$, while its thermal power output $\dot{Q}_{im}$ is fixed at 3 kW for the initial optimisation.

$$\eta_{dch} \cdot \dot{Q}_{dch,t} + \delta_{im,t} \cdot \dot{Q}_{rh} \geq \dot{Q}_{dem,t} \qquad (18)$$

Table 3 contains information on the included range of TES tanks. Charge η_{ch} and discharge efficiency η_{dch} are assumed constant at 98% [34].

2.5. Electricity tariff

Three types of electricity tariffs are considered: Standard, Economy 7 (E7), and Economy 10 (E10). Both E7 and E10 are two rate tariffs with off-peak duration of 7 and 10 hours, respectively. The off peak hours for E7 are from 00.00 to 07.00, while E10 off peak hours are between $00.00 - 05.00$, $13.00 - 16.00$, and $20.00 - 22.00$. Table 4 shows the summary of the electricity tariffs [35].

2.6. Modelling tools

The linear programming problem is formulated in Pyomo 4.0 [36] and solved with CPLEX 12.6.2 [37] on a Windows computer with 3.4GHz i7 Intel processor and 16 GB of RAM.

3. Results and discussion

3.1. Effect of electricity tariff

The results of optimisation runs with different electricity tariffs are shown in Table 5. The optimised heat pump size of 8.5 kW is identical for all tariffs. On the other hand, the optimal TES size is 300 L for E10, and 210 L for E7 and the Standard tariff. The lowest total cost is achieved for the E10 tariff. The equivalent CO_2 emission is calculated by using the average carbon dioxide intensity of the electricity grid of 0.49 kgCO_2/kWh [38]. The case for the Standard tariff has no incentive for shifting heat demand with the TES which incurs charge/discharge and

Table 5: Results for different electricity tariffs

Variables	E7	E10	Standard
HP size (kW)	8.5	8.5	8.5
TES size (l)	210	300	210
HP electricity input (kWh/year)	5449	5483	5420
Equivalent CO_2 emission (kg/year)	2670	2687	2656
Annual operational cost (£)	711	634	781
Total cost for 20 years (£)	12052	11378	12898

Table 6: Influence of heating flow temperature on operational cost and emission

Variables	$T_{out} = 50$ °C	$T_{out} = 35$ °C
HP size (kW)	8.5	8.5
TES size (l)	300	300
Electricity input (kWh/year)	5483	3678
Operational cost (£/year)	634	425
Cost reduction relative to gas boiler (%)	6%	37%
Equivalent CO_2 emission (kg/year)	2687	1802
Emission reduction relative to gas boiler (%)	5.7%	37%

self-discharge losses and thus has the lowest electricity input. Consequently, this case has also the lowest equivalent CO_2 emission. However, this neglects the generally lower CO_2 intensity during off-peak hours.

It is also interesting to compare the HP-TES heating systems with a conventional gas boiler. Assuming a gas price of £0.045/kWh, the modelled heat demand will have an operational cost of approximately £675/year. The optimal solution in Table 5 has a 6% lower operational cost. Furthermore, using a CO_2 factor of 0.185 kgCO_2/kWh for natural gas, the boiler scenario will have equivalent emission of around 2850 kgCO_2. This is 6-7% higher than the HP-TES systems.

The lower operational cost and emission of heat pump systems relative to a gas boiler system are also observed in the simulation study of Cabrol and Rowley [10], although in significantly higher values. For example, it is concluded that 45% operational cost reduction and CO_2 reduction of up to 26% are achievable for the case with E10 tariff in a similar location and number of degree days as the modelled demand in this study. The discrepancy with results of the present study can be attributed to the difference in the modelled heating system. The previous study implemented an under-floor heating system, which requires lower flow temperature than regular radiators. The current study assumes 50 °C heating flow temperature, which is within the required range for systems with regular wall radiators. This difference can have a large influence on the COP since the temperature lift is higher in the latter, thus reducing the COP.

In order to investigate the resulting effect on operational cost and emission, an optimisation problem with heating flow temperature of 35 °C is solved. The chosen temperature is typical for an under-floor heating system. The value of linear regression coefficients in Eq. 10 are modified accordingly, taking into account the lower heating flow temperature. The resulting values for the two heating flow temperature for the E10 tariff are given in Table 6. A significant increase in cost and emission reduction can be observed in the scenario with lower flow temperature. This illustrates that the combination of HP with regular radiators can limit the overall benefits of HP based systems and that a careful design of these systems is required.

Fig. 4 and 5 illustrate operational profiles over two winter days for heating systems with 210 L and 300 L TES size, respectively. It should be noted that the operational profiles are generated by the solver and there is no load shifting prescribed in the optimisation model. In Fig. 4, it can be observed that the resistive heater is working during the hour with peak demand on every electricity tariff. This is not the case for a system with 300 L TES due to the larger storage capacity. A small energy discharge occurs during periods without demand at time steps 15 and 39 for the case of 210 L TES on the E10 tariff. This can be explained by looking at the adjacent charging graph where the HP charges the TES with energy larger than its capacity during those time periods. Therefore, the extra energy needs to be removed from the system, i.e. heat dumping through a radiator.

The main difference between tariffs is on the TES charging profile, as shown in the right graphs in Fig. 4 and 5. For a system with 210 L TES, the system tries to maintain 100% state-of-charge during the off-peak period. An extra charging period occurs in E10 due to the extra off-peak time slot in the afternoon. However, the charging during periods without demand can be observed in all tariffs for systems with 300 L TES. This can be attributed to a higher standing loss as the TES size increases. Thus, given the draw-off profile in this study, a system with a larger TES size can have lower efficiency. Furthermore, this also contributes to increasing operational costs when a TES size larger than 210 L is utilised on the Standard tariff.
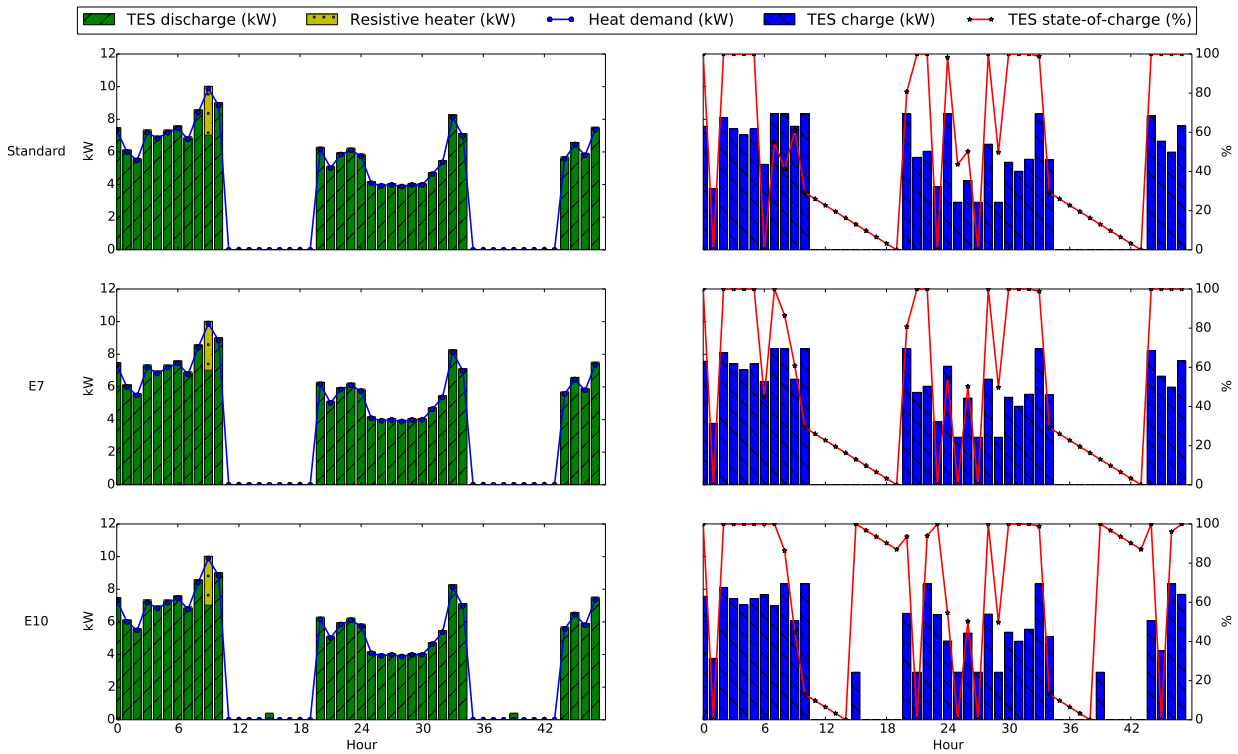
Figure 4: Operational profiles over two winter days of system with 210 L TES on different tariffs.
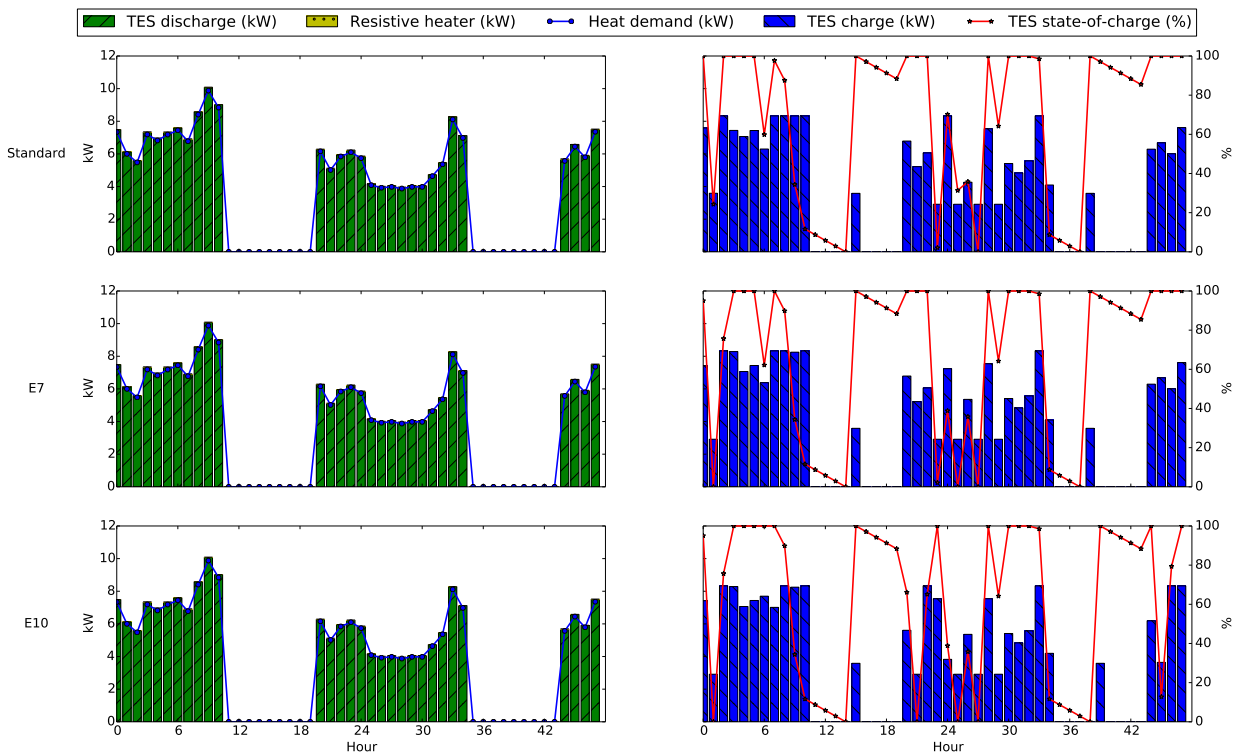

Figure 5: Operational profiles over two winter days of system with 300 L TES on different tariffs.

3.2. Effect of resistive heater

The influence of the resistive heater in the optimisation results (Table 5) is negligible since the heat demand can be covered by the heat pump through energy storage for most

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Table 7: Impact of resistive heater on operational cost and emission

Variables	$\dot{Q}_{im} = 3$ kW	$\dot{Q}_{im} = 6$ kW
HP size (kW)	8.5	5
TES size (l)	300	300
Electricity input (kWh/year)	5483	6675
Operational cost (£/year)	634	784
Equivalent CO_2 emission (kg/year)	2687	3270
Total cost for 20 years (£)	11378	12160

(a)

(b)

(c)

Figure 6: Total cost of different heating systems on Standard (a), E7 (b) and E10 electricity tariff (c).

of the time. Thus the installation of a smaller heat pump, i.e. 5 kW, with lower investment cost might be beneficial. However, this HP requires a resistive heater with a higher rating to fulfil the demand during the coldest winter days. Such a resistive heater can be installed, especially in the larger tanks. The optimisation of an E10 scenario with lower HP rating (i.e. 5 kW) and 300 L TES with 6 kW resistive heater is performed. As shown in Table 7, the higher resistive heater rating inside the TES makes it possible to install a lower capacity HP. However, the financial saving in initial equipment cost is negated by the negative impact of increased electricity input due to the increased heat demand covered by the resistive heater.

3.3. Total cost

The results of total cost calculation for an 8.5 kW HP with different TES sizes on all tariffs are illustrated in Fig. 6, along with the total cost of the HP-only and the gas boiler scenario. It is clear that the utilisation of TES can decrease the operational cost relative to the HP-only scenario. Without RHI, all heat pump scenarios have a significantly higher total cost than the gas boiler scenario. This can be attributed to the significantly higher equipment cost for the heat pump with TES system. For E10 scenarios with TES (Fig. 6(c)), a higher storage volume produces lower total cost, albeit relatively small. Furthermore, the total cost is lower for heat pump with TES than heat pump only. This is because the operational cost savings from TES compensate for its capital cost. It is interesting to note that a similar trend is not found for E7 (Fig. 6(b)). Increasing the TES size beyond 210 L on the E7 tariff will increase the total cost. This is also observed on scenarios with the Standard tariff. One possible explanation for this is that the increased operational cost due to a larger proportion of heat loss in the larger tank can only be compensated in E10 scenarios. This drop in operational efficiency of cases with the same consumption pattern and increasing storage size is in line with a reported experimental observation [39]. It should be noted that the difference in total cost between the cheapest and
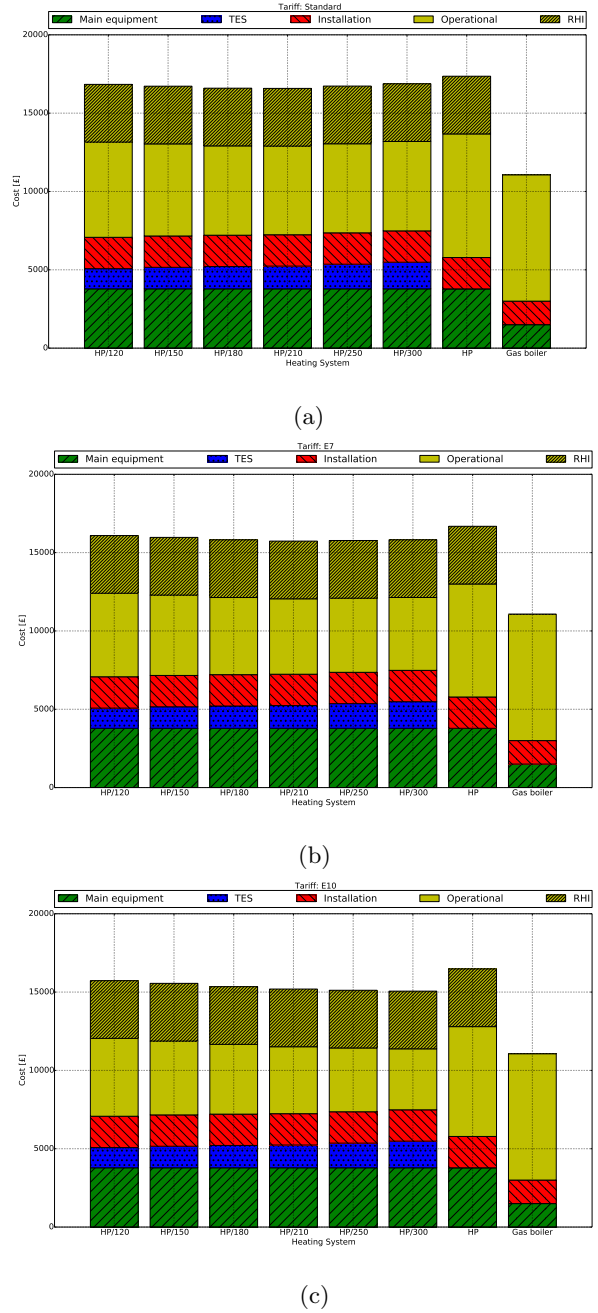
most expensive HP-TES solution in a specific tariff is relatively small (i.e. 2-7%).

The inclusion of RHI has a significant impact on reducing the total cost of HP-based systems, as shown in Fig. 6. It is clear that RHI reduces the operational cost by a large margin, and can make the heat pump scenarios cost competitive with the gas boiler option.

3.4. Study limitations

As in any modelling-based study, the results of this study have to be considered along with the model assump-

tions and limitations. Briefly described below are examples of these limitations in the present study.

While the use of the synthetic heat demand model instead of a detailed building thermal model is central to the complete optimisation framework, it leads to some limitations in the component models. For example, the HP model assumes a constant supply water temperature while it has been shown in [30] that a dynamic supply water temperature can increase the COP. On the other hand, the reduced computational complexity of the presented framework makes comprehensive optimisations of district level heating systems tractable. Furthermore, the synthetic heat demand model can be integrated with user occupancy and activity profiles generated from Time Use Surveys [21] to generate district level heat demand profiles which retain the stochastic variations inherent in these systems. Thus it is believed that this simplified model is of sufficient accuracy to produce viable results.

Another modelling decision that can influence the results is the capacity model implemented for the thermal energy storage. It has been shown that the capacity model can contribute to approximately 7% underestimation of operational cost relative to the stratified model for a case study of monovalent residential heating system with combined heat and power (CHP) [40]. The same study also describes that the increase in accuracy with the stratified model comes with higher computational cost. The lower computational cost of the capacity model employed in the current study has made it possible to solve the optimisation problem with hourly time step for the whole year, as opposed to using typical days to represent the yearly profile. The framework will be developed further to include a synthesis optimisation step [41]; thus, it is important to balance the computational cost and accuracy. Furthermore, the optimisation framework is designed to complement rather than replace dedicated energy systems simulation tools, such as TRNSYS or EnergyPlus, which are capable to model equipment with greater detail.

4. Conclusions

An integrated framework for the design and operational optimisation of residential heating systems has been presented. The optimisation is based on the MILP technique, with discrete equipment sizing. Included in the optimisation framework is a heat demand model which is capable of producing heat demand profiles based on cumulative heat demand, ambient temperature, and occupancy profile. The design and operational optimisation of a residential heating system were then performed using the output of the heat demand model and manufacturers equipment data as inputs. The framework has been successfully applied to optimise a HP-TES heating system for different UK electricity tariffs (2013), TES sizes and resistive heater capacities. These tests showed that the optimisation framework generated comparable results

to a TRNSYS simulation study while requiring only generally available inputs. Thus the framework could find widespread use in the evaluation of low carbon heating systems.

For the investigated tariffs, the heating system with 8.5 kW HP and 300 L TES operating on E10 has the lowest operational and total cost. It also has slightly lower operational cost and equivalent CO_2 emission than gas boiler system. These are achieved for a heating system with conventional radiators and the current high CO_2 grid intensity. By moving to underfloor heating which requires lower heating flow temperatures the cost and emission savings of the HP system are 37% relative to a conventional gas boiler system. The emission savings will increase with the continuing reduction in CO_2 grid intensity.

The undesired effects of undersized heat pump have also been described in this study. Despite its lower investment cost, a system with an undersized heat pump suffers from higher operational cost due to increased utilisation of electric resistive heating to cover part of the demand.

The total cost of the studied heating systems have dissimilar trends for different tariffs. The total cost of HP-TES systems on E10 decreases with increasing storage capacity, while systems on E7 and Standard tariff show an increasing trend as the storage goes beyond 210 L. In general, it can be concluded that HP-based heating systems, with or without TES, have significantly higher cost than natural gas boiler heating system. However, for cases with TES, the operational cost are lower than the HP-only scenario. Moreover, it has been shown that the recently introduced RHI can reduce the operational cost and make heat pumps a more attractive option for end users in the UK.

The MILP-based optimisation framework which employs low complexity models can be solved relatively fast compared to dedicated software tools, such as TRNSYS. This enables the operational optimisation over a whole year instead of reducing the time horizon through the use of typical days. The benefits of such an approach increase as the investigated system grows in terms of complexity, e.g. community-level energy system with various energy vectors and consumer types. This enables the design and operational optimisation of district heating system which will be reported in a future publication.

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