THE UNIVERSITY *of* EDINBURGH

Edinburgh Research Explorer

Achieving Arbitrary Signals Transmission Using a Single Radio Frequency Chain

Citation for published version:

Zhou, L, Khan, FA, Ratnarajah, T & Papadias, CB 2015, 'Achieving Arbitrary Signals Transmission Using a Single Radio Frequency Chain', *IEEE Transactions on Communications*, vol. 63, no. 12, pp. 4865-4878.
<https://doi.org/10.1109/TCOMM.2015.2493140>

Digital Object Identifier (DOI):

[10.1109/TCOMM.2015.2493140](https://doi.org/10.1109/TCOMM.2015.2493140)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Early version, also known as pre-print

Published In:

IEEE Transactions on Communications

General rights

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact openaccess@ed.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.

Achieving Arbitrary Signals Transmission Using a Single Radio Frequency Chain

Lin Zhou, *Student Member, IEEE*, Fahd Ahmed Khan, *Member, IEEE*, Tharmalingam Ratnarajah, *Senior Member, IEEE*, and Constantinos B. Papadias, *Fellow, IEEE*

Abstract—Electronically steerable parasitic array radiator (ESPAR) antenna has been proposed to obtain multiple antenna functionality with only a single radio frequency (RF) chain. However, for some signals, the input resistance of the ESPAR antenna (EA) becomes negative. A negative input resistance would lead to oscillatory/unstable behaviour of an EA to transmit signals. In this work, we address this issue of an EA by obtaining approximate signals for transmission, which are close to the ideal signals in the mean squared error (MSE) sense, and satisfy the requirement that the input resistance is positive. Specifically, we formulate and solve an optimization problem to minimize the MSE between the ideal and the approximate signals taking into account the constraint on the input resistance. Closed-form expressions to calculate the approximate signal vector from ideal signal vector are also obtained, resulting in simplified processing at the transmitter. We denote the EA utilizing the novel closed-form expressions as EA with preprocessing (EA-P) and the EA without any preprocessing as the standard EA (EA-S). The performance of EA-P is compared with the performance of EA-S and the standard multiple antenna system, with multiple RF chains, under various communication scenarios. Our simulations show that the performance of EA-S significantly degrades compared with the standard multiple antenna system and an error floor is obtained. However, using the proposed EA-P scheme, the error floor is avoided and nearly the same performance as that of the standard multiple antenna system is achieved.

Index Terms—Reconfigurable antenna, ESPAR, MIMO transmission, a single RF chain, optimization.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication has attracted great interest in recent years due to its spatial multiplexing and diversity benefits. These benefits increase as the number of antennas increases. However, each additional antenna element requires an

additional radio frequency (RF) chain resulting in increased cost and complexity. In addition, in order to maximize the benefits offered by multiple antennas, the antennas need to be placed sufficiently apart to minimize the coupling. This results in an increase in the size of the device and reduced mobility [1]. Electronically steerable parasitic array radiator (ESPAR), a pattern reconfiguration antenna with only a single active element [2], has been proposed to reduce the cost of multiple antenna devices. Reconfigurable antennas can achieve dynamically changeable antenna properties in frequency, radiation pattern and polarization [3], [4], [5]. The studies in [4], [6], [7], [8] have shown that reconfigurable antennas can improve the performance of MIMO wireless communication systems. For ESPAR antenna (EA), the antenna elements in ESPAR antenna (EA) are placed close together, resulting in strong coupling among antenna elements. This coupling is exploited to control the currents at the antenna elements and the overall radiation pattern is shaped by varying the feeding voltage at the active element and adjusting the impedance of the parasitic elements. Therefore, ESPAR gives multi-antenna functionality utilizing a single RF chain and alleviates the complexity and implementation issues associated with standard multiple antenna systems.

Recently, there has been a lot of research related to different aspects of EA design and to achieve its low-cost implementation in real-world systems. EA based multiple antenna system designs utilizing the beamspace model were proposed in [1], [9] [10], [11]. However, there are several weaknesses in the beamspace MIMO model. In the beamspace model, the receiver cannot guarantee the orthogonality of basic beam patterns and the radiating modes cannot be computed accurately for an arbitrary EA. Moreover, arbitrary channel-dependent precoding cannot be realised for an arbitrary antenna array [12]. Therefore, an alternate model based on the currents at the ports of the transmit antenna was introduced in [13]. The authors in [14] provided the design conditions that an EA has to satisfy in order to support an arbitrary precoding scheme.

The main design condition considered in [14] was that the real part of the input impedance, also labelled

Lin Zhou, Fahd Ahmed Khan and Tharmalingam Ratnarajah are with the Institute for Digital Communication (IDCOM), School of Engineering, The University of Edinburgh, Edinburgh, UK, e-mails: l.zhou@ed.ac.uk; fahd.khan@kaust.edu.sa; t.ratnarajah@ed.ac.uk.

Constantinos B. Papadias is with Athens Information Technology (AIT), Athens, Greece, e-mail: cpap@ait.gr.

This work was supported by the Seventh Framework Programme for Research of the European Commission under grant number HARP-318489.

as *input resistance* [15], should be positive. Satisfying this condition is essential because a negative input resistance results in a positive reflection coefficient (dB), which implies that the EA is reflecting power back and exhibiting oscillatory/unstable behavior [16]. In a standard multiple antenna system, the input resistance is designed to be positive and therefore, this issue does not arise [15]. However, in the case of the EA system, the value of the input resistance depends on the currents at the antenna elements (which depend on the signal to be transmitted) and the mutual coupling (MC) matrix (which depends on the antenna geometry). If the MC matrix is fixed, which is typically the case for an EA, a change in the antenna signal/current vectors can result in a negative input resistance. This implies that for such signals/current vectors, the EA will reflect back power and exhibit unstable behaviour. In order to transmit such signals, a new MC matrix, implying a new EA, needs to be designed. Therefore, one issue with the approach in [14] is that, once an EA is designed, there are still some signals for which the EA will be unstable, as such signals result in a negative input resistance. In order to transmit such signals a new EA needs to be designed.

In this work, a new approach is adopted, i.e. instead of trying to transmit the actual/ideal signals, which give a negative input resistance at the EA; Signals closely approximating the actual/ideal signals which do not lead to a negative input resistance are transmitted. In this way, the EA does not need to be redesigned. To the authors best knowledge, this approach is new and has not been proposed in literature previously. In order to obtain the approximate signal, an optimization problem is proposed to minimize the mean squared error (MSE) between the ideal and approximate signals considering the implementation constraint of the EA. The underlying problem is originally not convex. We recast it to convex form via equivalent reformulation and Lagrange duality¹. Finally, as practical transmitters require real-time operation, the optimization problem is solved analytically and novel closed-form expressions are derived to easily calculate the approximate current signals for the EA system.

In this paper, we denote the EA utilizing the novel closed-form expressions as EA with preprocessing (EA-P) and the EA without any preprocessing as the standard EA (EA-S). The performance of EA-P is compared with the performance of the EA-S and the standard multiple antenna transmitter in various communication scenarios.

¹Note that recasting a non-convex optimization to a convex optimization problem using Lagrange duality is a well known approach in optimization theory [17]. However, to the best of authors knowledge, none of the previous works related to EA have formulated an optimization problem to obtain the current/signal vectors taking into account the input resistance constraint.

Fig. 1: Circuit comparison of a) the EA transmitter and b) standard multiple antenna transmitter

Specifically, when the transmitter does not possess the channel state information (CSI), the spatial multiplexing scenario and space-time block precoding are considered. Whereas, when the transmitter possesses the CSI, the performance of EA-P, EA-S and standard multiple antenna with maximum ratio transmission (MRT) are compared. The performance metric compared is the symbol error rate (SER). Our results show that the EA-P performs significantly better compared with the EA-S and gives almost the same performance as the standard multiple antenna transmitter. However, replacing the ideal signal with approximate signal leads to some performance loss. If in addition, when one takes into account the cost and complexity of the EA, the EA-P scheme is well performing and practical.

The rest of the paper is organized as follows. The system model along with the working of an EA is explained in section II. The processing required for the EA-P is derived by solving an optimization problem in Section III. The performance of the EA-P is compared with the performance of the EA-S and standard multiple antenna transmitters, in different communication scenarios, in section IV. Finally, the main results are summarized in the concluding Section V.

II. SYSTEM MODEL FOR ESPAR MIMO

A. System Model with ESPAR Transmitter

The circuit diagrams of a $(M + 1)$ -element EA and a standard multiple antenna are shown in Fig. 1 [18]. The EA consists of a single active element with a RF unit and M parasitic elements without any RF units. On the other hand, the standard multiple antenna consists of $(M + 1)$ elements each connected to a RF chain.

As depicted in Fig. 1, $\mathbf{i} = [i_0, i_1, \dots, i_M]^T$ is the vector of currents at the antenna elements, where T denotes the transpose operation. The matrix $\mathbf{Z} \in \mathbb{C}^{(M+1) \times (M+1)}$ denotes the MC matrix and it depends on the antenna geometry [19]. $\mathbf{Z}_L = \text{diag}(z_0, z_1, z_2, \dots, z_M)$ is the source impedance matrix where z_0 is the source resistance which typically equals 50Ω [15] and $z_1, z_2, \dots, z_M$ are the loads at the parasitic elements. For the standard multiple antenna system, the vector of source voltages $\mathbf{v}$ is $[v_0, v_1, \dots, v_M]^T$. The currents are driven by the RF

voltage supply of each antenna element through fixed impedances $z_0, z_1, z_2, \dots, z_M$ [20], [21].

In the case of the EA system, $\mathbf{v}$ is $[v_0, 0, \dots, 0]^T$. The currents at the elements are varied by varying the input voltage v_0 at the active element and the loads $z_1, z_2, \dots, z_M$ at the parasitic elements. When feeding the active element, the currents are induced on the parasitic elements due to the mutual coupling between antenna elements. The MC matrix $\mathbf{Z}$ depends on the array's geometry and is given as

$$\mathbf{Z} = \begin{bmatrix} Z_{00} & Z_{01} & \dots & Z_{0M} \\ Z_{10} & Z_{11} & \dots & Z_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{M0} & Z_{M2} & \dots & Z_{MM} \end{bmatrix}. \quad (1)$$

According to Ohm's law, the port current vectors for both circuits can be expressed as

$$\mathbf{i} = (\mathbf{Z} + \mathbf{Z}_L)^{-1} \mathbf{v}. \quad (2)$$

If current $\mathbf{i}$, which is the function of $\mathbf{Z}$ and $\mathbf{Z}_L$ in (2), flows through the antenna elements, then the input impedance, Z_{in} , is given by [14]

$$Z_{in} = Z_{00} + \frac{\sum_{m=1}^M Z_{0m} i_m}{i_0}. \quad (3)$$

Note that Z_{in} is a function of $\mathbf{Z}$ and current vector $\mathbf{i}$ as shown in (3) and $\mathbf{i}$ varies with voltage feeding and loads value $\mathbf{Z}_L$ in (2). Therefore, Z_{in} depends on the MC matrix $\mathbf{Z}$ as well as the loads $\mathbf{Z}_L$. After transmission from an EA, the received signal at the receiver, with n_r antennas, is given as [13], [22], [23]

$$\mathbf{y} = \mathbf{H}\mathbf{i} + \mathbf{n}, \quad (4)$$

where $\mathbf{H} \in \mathbb{C}^{n_r \times (M+1)}$ is channel matrix and $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$ denotes the noise vector at the receiver. The receiver is assumed to be a standard multiple antenna receiver. The noise model is assumed to be additive white Gaussian noise (AWGN) with zero mean and unit variance.

B. Signal Transmission Using an EA Transmitter

In order to radiate the signal, the currents at the antenna element need to be varied based on the transmission signals. In the case of the standard multiple antenna system, the currents are varied by varying the input voltage at each antenna element. However, in the case of an EA, the loads at the parasitic elements and the voltage feeding at the active element needs to be varied. As discussed previously, for a certain transmission signal, it is possible that the voltage and loadings are such that they lead to a negative input resistance and result in an unstable EA. Let $\hat{\mathbf{i}} = [\hat{i}_0, \hat{i}_1, \dots, \hat{i}_M]^T$ denote desired current vector corresponding to ideal signals that

is required to be transmitted by the EA. Utilizing (4), the input impedance, Z_{in} , can be calculated for $\hat{\mathbf{i}}$ as

$$Z_{in} = Z_{00} + \frac{\sum_{m=1}^M Z_{0m} \hat{i}_m}{\hat{i}_0}. \quad (5)$$

If for some signals, $\hat{\mathbf{i}}$, the input resistance becomes negative, we propose that instead of transmitting the ideal signal, a signal closely approximating the ideal signal, which does not lead to a negative input resistance, is transmitted. In this way, the EA system does not need to be redesigned based on the transmission signal. As mentioned previously, we denote this EA with pre-processing as EA-P.

III. SIGNAL PROCESSING FOR AN EA-P

In this section, we obtain the current vectors approximating the ideal current vectors that leads to a positive input resistance. One can easily obtain the corresponding values of the voltage and the loads for an EA-P by using the approximate currents.

A. Problem Formulation

The problem to obtain the configuration for the EA-P, i.e. obtaining the values of the voltage and the loads, for each transmission signal can be formulated as an optimization problem to minimize the MSE between the currents corresponding to the ideal and approximate transmission signals, under the practical consideration of positive input resistance, and is given as

$$\min_{v_0, z_1, z_2, \dots, z_M} \left\| \hat{\mathbf{i}} - \mathbf{i} \right\|_2^2, \quad (6a)$$

$$\text{st. } \Re \left\{ Z_{00} + \frac{\sum_{m=1}^M Z_{0m} i_m}{i_0} \right\} \geq \xi, \quad (6b)$$

$$\mathbf{i} = (\mathbf{Z} + \mathbf{Z}_L)^{-1} \mathbf{v}, \quad (6c)$$

where $\mathbf{i}$ denotes the approximate current vectors. The objective of this optimization problem is to find the voltage feeding v_0 and the loads $z_1, z_2, \dots, z_M$ to minimize MSE between the ideal and approximate current vectors. As input resistance is required to be positive, a positive variable, ξ , is introduced in constraint (7b) to make sure that the input resistance is positive. Choosing $\xi > 0$ ensures stability of the EA. The impact of value of ξ on SER performance will be discussed in Subsection IV-C. A constraint on the real and imaginary part of impedance values can be considered. However, running extensive simulations, we found that the range of load values lies between $[-500, 500] \Omega$. The load values in this range are acceptable for antenna design [9] and therefore this constraint is omitted in this work. As the

configuration variables $\mathbf{Z}_L$ and $\mathbf{v}$ only appear in the (6c), the optimization problem in (6) can be solved in two steps. The first step is to obtain the approximate currents, $\mathbf{i}$, which can be obtained by solving the optimization problem with only constraint (6b). Once $\mathbf{i}$ is obtained, the loads and the voltage feeding can be calculated using (6c).

If for a certain ideal current vectors $\hat{\mathbf{i}}$, the corresponding input resistance is greater than ξ , the EA can radiate the corresponding signal without any issue and in this case $\mathbf{i} = \hat{\mathbf{i}}$ and no optimization is required. The set for current vectors which yield input resistance greater than ξ can be represented as

$$\mathbb{I}_{P\xi} = \left\{ \hat{\mathbf{i}} \left| \Re \left(Z_{00} + \frac{\sum_{m=1}^M Z_{0m} \hat{i}_m}{\hat{i}_0} \right) \geq \xi \right. \right\}. \quad (7)$$

Similarly, the unsupported set $\mathbb{I}_{N\xi}$ is the complement of $\mathbb{I}_{P\xi}$.

B. Alternate representation for the input resistance constraint

In order to simplify the constraint (6b), let R_m and X_m denote the real part and the imaginary part of Z_{0m} , respectively. $\hat{w}_{2m+1}$ and $\hat{w}_{2m+2}$ denote the real part and the imaginary part of $\hat{i}_m$, respectively. Similarly, w_{2m+1} and w_{2m+2} for i_m , respectively. As all these variables are complex, we reformulate them from the complex domain into the real domain by separating their real and imaginary parts. Then the ideal and approximate current vectors can be written as new vectors $\hat{\mathbf{w}} = [\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4, \dots, \hat{w}_{2M+1}, \hat{w}_{2M+2}]^T$ and $\mathbf{w} = [w_1, w_2, w_3, w_4, \dots, w_{2M+1}, w_{2M+2}]^T$, respectively.

Proposition 1 The input resistance can be expressed as

$$\Re \left\{ Z_{00} + \frac{\sum_{m=1}^M Z_{0m} i_m}{i_0} \right\} = \frac{\mathbf{w}^T \mathbf{A} \mathbf{w}}{w_1^2 + w_2^2}, \quad (8)$$

where the matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} R_0 & 0 & \frac{R_1}{2} & -\frac{X_1}{2} & \dots & \frac{R_M}{2} & -\frac{X_M}{2} \\ 0 & R_0 & \frac{X_1}{2} & \frac{R_1}{2} & \dots & \frac{X_M}{2} & \frac{R_M}{2} \\ \frac{R_1}{2} & \frac{X_1}{2} & 0 & 0 & \dots & 0 & 0 \\ -\frac{X_1}{2} & \frac{R_1}{2} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{R_M}{2} & \frac{X_M}{2} & 0 & 0 & \dots & 0 & 0 \\ -\frac{X_M}{2} & \frac{R_M}{2} & 0 & 0 & \dots & 0 & 0 \end{bmatrix}. \quad (9)$$

Proof: The proof is provided in Appendix A. ■

As ξ is positive, by substituting (8) into (6b), we get $\mathbf{w}^T \mathbf{A} \mathbf{w} \geq \xi (w_1^2 + w_2^2) = \xi (\mathbf{w}^T \mathbf{G} \mathbf{w})$, where

$\mathbf{G} = \text{diag}(1, 1, 0, \dots, 0)$. The constraint (6b) can be expressed as $\mathbf{w}^T \mathbf{B} \mathbf{w} \geq 0$, where, $\mathbf{B} = \mathbf{A} - \xi \mathbf{G}$. The optimization problem to obtain the approximate current vectors can be then reformulated as

$$\min_{\mathbf{w}} \quad \|\hat{\mathbf{w}} - \mathbf{w}\|_2^2, \quad (10a)$$

$$\text{st.} \quad \mathbf{w}^T \mathbf{B} \mathbf{w} \geq 0. \quad (10b)$$

The convexity of (10) depends on the matrix $\mathbf{B}$ as the objective function (10a) is convex.

Proposition 2 The optimization problem in (10) is non-convex as $\mathbf{B}$ is an indefinite matrix. The eigenvalues of $\mathbf{B}$ are given by (11).

Proof: The proof is provided in Appendix B. ■

C. Reformulation of the Nonconvex-Constrained Optimization Problem

For the problem in (10), the objective function is quadratic convex and the constraint is a quadratic non-convex inequality. As strong duality holds for (10) [17], its solution can be obtained by solving its dual problem. The Lagrangian of this optimization problem is

$$\begin{aligned} L(\mathbf{w}, \delta) &= \|\hat{\mathbf{w}} - \mathbf{w}\|_2^2 + \delta (-\mathbf{w}^T \mathbf{B} \mathbf{w}) \\ &= \mathbf{w}^T (\mathbf{I} - \delta \mathbf{B}) \mathbf{w} - 2\mathbf{w}^T \hat{\mathbf{w}} + \hat{\mathbf{w}}^T \hat{\mathbf{w}}, \end{aligned} \quad (12)$$

where δ is Lagrange multiplier, and $\mathbf{I}$ is an identity matrix of size $2(M+1)$. The stationarity condition is

$$\nabla_{\mathbf{w}} L(\mathbf{w}, \delta) = 2(\mathbf{I} - \delta \mathbf{B}) \mathbf{w} - 2\hat{\mathbf{w}} = 0. \quad (13)$$

As $\mathbf{B}$ is a square matrix, the optimal value, $\mathbf{w}(\delta)$, which satisfies (13) is

$$\mathbf{w}(\delta) = (\mathbf{I} - \delta \mathbf{B})^{-1} \hat{\mathbf{w}}. \quad (14)$$

The dual function of (10) can be expressed using the Lagrangian as

$$g(\delta) = \inf_{\mathbf{w}} (\mathbf{w}^T (\mathbf{I} - \delta \mathbf{B}) \mathbf{w} - 2\hat{\mathbf{w}}^T \mathbf{w}) + \hat{\mathbf{w}}^T \hat{\mathbf{w}}. \quad (15)$$

After substituting (14) into (15), it can be noticed that $g(\delta) = -\infty$ if matrix $(\mathbf{I} - \delta \mathbf{B})$ is not a positive semidefinite matrix or $\hat{\mathbf{w}}^T$ is not in the range of $\mathbf{I} - \delta \mathbf{B}$. Therefore, the dual problem can be expressed as

$$\max_{\delta} \quad -\hat{\mathbf{w}}^T (\mathbf{I} - \delta \mathbf{B})^{-1} \hat{\mathbf{w}} + \hat{\mathbf{w}}^T \hat{\mathbf{w}}, \quad (16a)$$

$$\text{st.} \quad \mathbf{I} - \delta \mathbf{B} \in \mathbf{S}_+^{2(M+1)}, \quad (16b)$$

$$\hat{\mathbf{w}} \in \mathbb{R}(\mathbf{I} - \delta \mathbf{B}), \quad (16c)$$

$$\delta \geq 0, \quad (16d)$$

where $\mathbf{S}_+^n$ denotes the set of symmetric positive semidefinite $n \times n$ matrices, and $\mathbb{R}(\mathbf{I} - \delta \mathbf{B})$ denotes the range of $\mathbf{I} - \delta \mathbf{B}$. This can be further reformulated to obtain the closed-form expression of the optimal value of δ .

$$\begin{cases} \zeta_1 = \zeta_2 = (R_0 - \xi) - \sqrt{(R_0 - \xi)^2 + \sum_{m=1}^M (R_m^2 + X_m^2)} < 0 \\ \zeta_3 = \zeta_4 = (R_0 - \xi) + \sqrt{(R_0 - \xi)^2 + \sum_{m=1}^M (R_m^2 + X_m^2)} > 0 \\ \zeta_n = 0 \quad \text{for } n = 5, 6, \dots, 2(M+1) \end{cases} \quad (11)$$

Proposition 3 The dual problem can be reformulated as

$$\min_{\delta} \quad \frac{r_1}{1 - \delta\zeta_1} + \frac{r_2}{1 - \delta\zeta_3}, \quad (17a)$$

$$\text{st.} \quad -\delta \leq 0, \quad \delta \leq 1/\zeta_3, \quad (17b)$$

where $r_1 = g_1^2 + g_2^2$ and $r_2 = g_{2M+1}^2 + g_{2M+2}^2$, here $\mathbf{g} = [g_1, g_2, \dots, g_{2M+1}, g_{2M+2}]^T = \mathbf{Q}^T \hat{\mathbf{w}}$ and the columns of $\mathbf{Q}$ are eigenvectors of matrix $\mathbf{B}$ when the corresponding eigenvalues are in ascending order.

Proof: The proof is provided in Appendix C. ■

D. Solution of the Optimization Problem

As the dual optimization problem is convex, the Lagrangian of (17) is expressed as

$$L_{\delta}(\delta, \sigma_1, \sigma_2) = \frac{r_1}{1 - \delta\zeta_1} + \frac{r_2}{1 - \delta\zeta_3} - \sigma_1\delta + \sigma_2\left(\delta - \frac{1}{\zeta_3}\right) \quad (18)$$

where σ_1 and σ_2 are Lagrange multipliers associated with the two inequality constraints. The stationarity condition of the KKT conditions is given by

$$\nabla_{\delta} L_{\delta}(\delta, \sigma_1, \sigma_2) = \frac{\zeta_1 r_1}{(1 - \delta\zeta_1)^2} + \frac{\zeta_3 r_2}{(1 - \delta\zeta_3)^2} - \sigma_1 + \sigma_2 = 0. \quad (19)$$

The remaining Karush-Kuhn-Tucker (KKT) conditions with respect to the complementary slackness conditions are

$$\sigma_1 \geq 0, \sigma_2 \geq 0, \delta \geq 0, \delta - \frac{1}{\zeta_3} \leq 0, \quad (20a)$$

$$\sigma_1\delta = 0, \sigma_2\left(\delta - \frac{1}{\zeta_3}\right) = 0. \quad (20b)$$

Proposition 4 After solving the optimization problem in (17) based on (19) and (20), the result of optimal value of δ is given as

$$\delta = \begin{cases} 0 & \text{for } \hat{\mathbf{i}} \in \mathbb{I}_{P\xi} \\ \frac{\sqrt{-\zeta_1 r_1} - \sqrt{\zeta_3 r_2}}{\zeta_3 \sqrt{-\zeta_1 r_1} - \zeta_1 \sqrt{\zeta_3 r_2}} & \text{for } \hat{\mathbf{i}} \in \mathbb{I}_{N\xi} \end{cases} \quad (21)$$

Proof: The proof is provided in Appendix D. ■

Substituting (21) into (14), the value of approximate transmit current vectors is given in the following theorem.

Theorem 1 The entry i_m , $m = 0, 1, \dots, M$ of the approximate current vectors $\hat{\mathbf{i}} = (i_0, i_1, \dots, i_M)$, for an

EA-P, can be obtained as $i_m = w_{2m+1} + jw_{2m+2}$, where w_k is the k -th entry of vector $\mathbf{w}$. The value of $\mathbf{w}$ is

$$\begin{cases} \mathbf{w} = \hat{\mathbf{w}} & \text{for } \hat{\mathbf{i}} \in \mathbb{I}_{P\xi} \\ \mathbf{w} = \left(\mathbf{I} - \frac{\sqrt{-\zeta_1 r_1} - \sqrt{\zeta_3 r_2}}{\zeta_3 \sqrt{-\zeta_1 r_1} - \zeta_1 \sqrt{\zeta_3 r_2}} \mathbf{B} \right)^{-1} \hat{\mathbf{w}} & \text{for } \hat{\mathbf{i}} \in \mathbb{I}_{N\xi} \end{cases}, \quad (22)$$

where $\hat{\mathbf{w}}$ is obtained by separating the real and imaginary parts of ideal current vectors $\hat{\mathbf{i}} = (\hat{i}_0, \hat{i}_1, \dots, \hat{i}_M)$, and its k -th entry is

$$\hat{w}_k = \begin{cases} \Re\left(\hat{i}_{\frac{k-1}{2}}\right) & k = 1, 3, \dots, 2M+1 \\ \Im\left(\hat{i}_{\frac{k-2}{2}}\right) & k = 2, 4, \dots, 2M+2 \end{cases} \quad (23)$$

where $\Im\left(\hat{i}_{\frac{k-2}{2}}\right)$ denotes the imaginary part of $\hat{i}_{\frac{k-2}{2}}$.

Theorem 1 gives the expression to calculate the approximate current vectors. When the input resistance from the ideal current vectors is no less than ξ , the ideal current vectors can be realized by the EA-P without any optimization operation. Otherwise, the EA-P transmits the approximate current vectors which are obtained from Theorem 1.

The loads and feeding, for the EA-P, can be calculated based on the approximate current vectors as [14],

$$v_0 = z_0 i_0 + \sum_{j=0}^M Z_{0j} i_j, \quad (24a)$$

$$z_m = -\frac{\sum_{j=0}^M Z_{mj} i_j}{i_m}, \quad m = 1, 2, \dots, M. \quad (24b)$$

From these expressions, it can be noted that the load values depends on the normalized current vectors and MC matrix, and the input power to the active element is controlled by the feeding. When the normalized current vectors are fixed, the load values are independent of the feeding circuit.

The steps to obtain the configuration for the EA-P for every transmit signal are summarized in Algorithm 1.

E. Discussion

1) *Complexity:* For the computation of the approximate currents, the EA-P algorithm involves only matrix operations and no iteration is required. The eigenvalues and corresponding eigenvectors of the MC matrix need to be computed only once. In other words, steps 1 to 3 in Algorithm 1 need to be calculated only once and can be used for the subsequent signal transmissions. Thus, the complexity from step 1 to 3 is negligible. Steps 4

Algorithm 1 The EA configuration for $\hat{\mathbf{i}}$ Input: Required ideal current vectors $\hat{\mathbf{i}}$ Output: EA-P loads and voltage feeding for $\hat{\mathbf{i}}$,Step 1: if $\hat{\mathbf{i}} \in \mathbb{I}_{P\xi}$ {Goto Step 6.} else {Goto Step 2.}Step 2: Obtain $\hat{\mathbf{w}}$ by separating real and imaginary parts of $\hat{\mathbf{i}}$. Calculate $\mathbf{B}$ from $\mathbf{B} = \mathbf{A} - \xi \mathbf{G}$ and (9).Step 3: Calculate ζ_1 , ζ_3 , and $\mathbf{Q}$ from (11) and eigenvalue decomposition of $\mathbf{B}$.Step 4: Calculate $\mathbf{g}$ by $\mathbf{g} = \mathbf{Q}^T \hat{\mathbf{w}}$.Step 5: Calculate r_1 and r_2 by $r_1 = g_1^2 + g_2^2$ and $r_2 = g_{2M_t-1}^2 + g_{2M_t}^2$.Step 6: Calculate $\mathbf{i}$ from (22).Step 7: Calculate the corresponding loads $z_1, z_2, \dots, z_M$ and feeding v_0 as (24).

to 7 of Algorithm 1 are required to be performed for each signal transmission. Therefore, from the complexity perspective, step 4 involves a matrix multiplication, step 6 has a matrix multiplication and a inversion operation, and step 7 involves two inner products. When the ideal signal cannot be supported by EA, it requires the total of $n^3 + 5n^2 + 3n - 2$ floating point (flop) operations [24], [25].

2) *Matching*: For maximum power transfer, impedance matching is necessary. The generator/source impedance needs to be matched to the overall input impedance of the EA. In [14], a dynamic impedance matching circuit is proposed to compensate the impedance mismatch between the source and input impedance of the EA.

When considering dynamic matching circuit, the optimization problem will not be changed. The load values are independent of the feeding circuit and the matching circuit from Theorem 1 and (24b) as mentioned in the Subsection III-D. Therefore, after matching the impedance, one only needs to calculate the new feeding voltage [14]. As shown in Fig. 2, the input voltage to ESPAR is $V_{in} = \sum_{j=0}^M Z_{0j} i_j$. As the matching impedance is $Z_{dm} = \Re\{Z_{in}\} - \Im\{Z_{in}\}$, the new feeding voltage can be calculated as

$$V_0^{dy} = \frac{V_{in}}{Z_{in}} (Z_{dm} + Z_{in})$$

$$= \frac{\sum_{j=0}^M Z_{0j} i_j}{Z_{00} + \frac{\sum_{m=1}^M Z_{0m} i_m}{i_0}} \left(2\Re \left\{ Z_{00} + \frac{\sum_{m=1}^M Z_{0m} i_m}{i_0} \right\} \right) \quad (25)$$

3) *Stability and implementation of Parasitic loads*: After solving the optimization problem, the loads z_1 to

Fig. 2: antennas matching

z_M might lead to a negative resistance value. A stable circuit for generating parasitic loads with a negative real part has been proposed in [26], [27]. The parasitic elements might require some gain to produce the required currents². However, this is not the aim of this work and it is assumed that the parasitic elements are efficiently implemented and stable. As in [16], a system structure is unstable if the input port impedance has a negative real part. Therefore, provided that the parasitic loads with a negative real part are stable, the resulting EA structure will be stable if the input resistance is positive. Thus, it is necessary to satisfy this condition of stability for an EA. If this is not guaranteed, the EA will be unstable and oscillation is possible [16]. From the expression (24), the loads z_1 to z_M might lead to a negative resistance value. If there exists a load value z_m at the m -th parasitic element, its real part is negative, $\Re(z_m) < 0$. From (24b), we have

$$\Re(z_m) = \Re \left(-\frac{\sum_{j=0}^M Z_{mj} i_j}{i_m} \right) = -\Re \left(\frac{\sum_{j=0}^M Z_{mj} i_j}{i_m} \right) < 0, \quad (26)$$

As this is an active load, the input impedance at the m -th parasitic element is $Z_{in}^{(m)} = \frac{V_{in}^{(m)}}{i_m} = \frac{\sum_{j=1}^M Z_{mj} i_j}{i_m}$, and its real part is $\Re(Z_{in}^{(m)}) = \Re \left(\frac{\sum_{j=1}^M Z_{mj} i_j}{i_m} \right)$. From (26), we have $\Re(Z_{in}^{(m)}) > 0$. Thus, this guarantees the stability of the system while z_1 to z_M might have negative real parts.

IV. APPLICATION OF THE ALGORITHM IN VARIOUS COMMUNICATION SCENARIOS

In this section, we analyse and compare the performance of the EA-P, EA-S, and the standard multiple

²The parasitic elements can be implemented in multiple ways, eg. one approach is using varactor diodes. A T- or pi-network of variable capacitors can be designed to implement complex loads for load modulation transmitter as in [26]. For a single antenna transmitter, a data rate of 1.2 Gbit/s with Schottky diodes has already been demonstrated in a prototype [27]. Also, transmitters with dynamic load modulation were proposed in [28]. Moreover, the parasitic elements can also be implemented digitally using a network of pindodes [29]. Implementation for active resistance is discussed by networks of the complementary metal oxide semiconductor (CMOS) technology in [25].

antenna transmitters in various communication scenarios.

A. Each Antenna Element Transmits the Same Signal

First, we consider the simplest scenario in which all the antenna elements transmit the same signal s , i.e. $\hat{i} = [s, s, \dots, s]^T$. In this case, substituting $\hat{i}$ in (5), yields the input resistance, R_{in} as

$$R_{in} = \Re \left(Z_{00} + \frac{\sum_{m=1}^M Z_{0m}s}{s} \right) = R_0 + \sum_{m=1}^M R_m. \quad (27)$$

From (27), it can be noted that the input resistance is independent of the transmitted signal and only depends on the MC matrix. Therefore, any EA characterized by a MC matrix which satisfies the condition, $R_0 + \sum_{m=1}^M R_m > 0$, can be employed to transmit the same signal from every antenna element. If this condition is generally satisfied in a practical EA system, radiating the same signal is generally feasible. Thus, in this scenario, the EA-P gives the same performance as EA-S and the standard multiple antenna transmitter.

B. Spatial Multiplexing

Next we consider the scenario in which each antenna element, transmits a different signal. In this case, $\hat{i} = [s_0, s_1, \dots, s_M]^T$ where s_m denotes the signal at the $(m+1)$ -th antenna element. The input resistance can be expressed as

$$R_{in} = \Re \left(Z_{00} + \frac{\sum_{m=1}^M Z_{0m}s_m}{s_0} \right). \quad (28)$$

From (28), it can be noted that unlike the previous scenario, whether a signal can be transmitted not only depends on the MC matrix but also on the transmission signal. The transmission signal depends on the modulation scheme employed. As an example, assume that the modulation scheme is N -PSK. The signal at the $(m+1)$ -th element can be denoted as $s_m = \exp(j\phi_m)$, where $\phi_m = \frac{2\pi n_m}{N}$, $n_m \in \{0, 1, \dots, (N-1)\}$ is the phase of the signal at the $(m+1)$ -th antenna element and N is the order of the PSK signal constellation [9]. In this case, the input resistance can be expressed as (29).

Therefore, in order to transmit different N -PSK signals for each antenna element, the MC matrix of the EA is required to satisfy

$$R_{in} > 0, \quad \text{for } n_m, n_0 \in \{0, 1, \dots, N-1\}. \quad (30)$$

In the special case of BPSK modulation, when the transmit signal set is $\{\exp(j0), \exp(j\pi)\}$, the input resistance

can be simply expressed as $R_{in} = R_0 + \sum_{m=1}^M a_m R_m$,

where, $a_m = \begin{cases} 1 & \text{for } \phi_m = \phi_0 \\ -1 & \text{for } \phi_m \neq \phi_0 \end{cases}$. Thus, the BPSK signal can be transmitted by the EA when it satisfies the condition $R_{in} = R_0 + \sum_{m=1}^M a_m R_m > 0$, for all $a_m \in \{1, -1\}$. These conditions, for BPSK and N -PSK modulation, might not be satisfied for some EAs, and therefore in such cases, EA-P can be employed to achieve transmission of those signals.

Numerical Simulation: As an example, in Fig. 3, we plot the input resistance against the phase of the transmission signal for an EA with two antenna elements. In the simulations, only the transmitter is assumed to be equipped with an EA and a standard multiple antenna receiver is considered. Without loss of generality, the number of antennas at the receiver, n_r , is assumed to be one and the channel is Rayleigh faded. The modulation scheme is 16-PSK. In Fig. 3, the green colored region denotes $R_{in} > 0$ and the red region denotes $R_{in} \leq 0$. The black points are all possible phase combinations of the two 16-PSK signals.

Fig. 3(a) is for the EA with MC matrix given by $Z^{(1)}$, and Fig. 3(b) is for the EA with MC matrix given by $Z^{(2)}$ and $Z^{(3)}$. These three MC matrices for 2-element ESPAR in the IE3D antenna software are

$$\begin{aligned} \mathbf{Z}^{(1)} &= \begin{bmatrix} 45.12 - j16.60 & 42.39 - j29.51 \\ 42.39 - j29.51 & 21.12 - j29.64 \end{bmatrix}, \\ \mathbf{Z}^{(2)} &= \begin{bmatrix} 52.81 - j11.09 & 40.27 - j20.75 \\ 40.27 - j20.75 & 52.81 - j11.09 \end{bmatrix}, \\ \mathbf{Z}^{(3)} &= \begin{bmatrix} 465.4 - j659.5 & -24.06 + j34.93 \\ -24.06 + j34.93 & 21.12 - j157.2 \end{bmatrix}. \end{aligned}$$

These have been designed in [14], [28].

It can be observed from the figure that some combinations of the 16-PSK signals cannot be supported using $Z^{(1)}$. For this EA, by employing EA-P, the transmission of these signal combinations is possible. Whereas, for the EA with $Z^{(2)}$ and $Z^{(3)}$, all possible signal combinations can be supported. Comparing $Z^{(1)}$, $Z^{(2)}$, and $Z^{(3)}$, it can be noted that the EA, whose MC matrix has higher self impedance at the feeding element can support more signal combinations. In the Fig. 3(a) and Fig. 3(b), the MC matrix $Z^{(1)}$, $Z^{(2)}$, and $Z^{(3)}$ is obtained using the IE3D antenna design software. Furthermore, the feasible regions for linear EAs with different antenna spacing are shown the Fig. 3(c) and 3(d), respectively. The MC matrices, in this case, are obtained using the induced electromotive force method (IEFM) for different antenna spacing [29]. The feasible region for two different antenna spacing $d = \frac{\lambda}{16}$ and $d = \frac{\lambda}{4}$ for linear EAs are shown in the Fig. 3(c) and Fig. 3(d), respectively. It can

$$R_{in} = \Re \left(Z_{00} + \sum_{m=1}^M Z_{0m} \exp(j(\phi_m - \phi_0)) \right) = R_0 + \sum_{m=1}^M \left(R_m \cos \frac{2\pi(n_m - n_0)}{N} - X_m \sin \frac{2\pi(n_m - n_0)}{N} \right). \quad (29)$$

Fig. 3: The feasible region for different EAs when 16-PSK is transmitted

observed from Fig. 3(c) and Fig. 3(d), the proportion of supported signal combinations decreases as the antenna spacing is reduced. Therefore, in this scenario, for signal transmission, the EA-P algorithm becomes indispensable.

The SER performance of the system where the transmitter transmits different data streams from each antenna is shown in Fig. 4 and Fig. 5. For simplicity, we denote the results from standard multiple antenna as SMA in the simulation figures. The SER is plotted against signal-to-noise ratio (SNR)³. The modulation scheme considered in these figures is 16-PSK and 16-QAM, respectively. As any positive ξ can guarantee signal transmission using an EA, without loss of generality, ξ is set equal to one. Three EAs with MC matrices, $\mathbf{Z}^{(1)}$, $\mathbf{Z}^{(2)}$, and $\mathbf{Z}^{(3)}$ respectively, are considered and the SER performance of EA-P, EA-S, and the standard multiple antenna transmitters are compared. In the case of the standard multiple antenna transmitter, $\hat{\mathbf{i}}$ is transmitted exactly. For the EA-P transmitter, the transmission policy is given in Theorem 1. For a fair comparison of SER with respect to transmit SNR, the transmitted signal $\hat{\mathbf{i}}$ is normalized by $\frac{\hat{P}}{P}$, where $\hat{P}$ and P are power based on ideal signals and practical signals, respectively, and the normalization factor is controlled by varying the voltage feeding. In the case of EA-S, no signal is stably transmitted when input

³The radiated power is proportional to the current i.e. $P \propto |\hat{\mathbf{i}}|^2$ [15]. Ignoring the proportionality constant, we define average radiated power as $P = \mathbb{E} \{ \hat{\mathbf{i}}^H \hat{\mathbf{i}} \}$. And the SNR is defined as the ratio of the radiated power to the noise power i.e. $SNR = P/N_0$.

Fig. 4: SER performance comparison of the EA transmitter and standard multiple antenna transmitter in the case of spatial multiplexing scheme under 16-PSK

resistance is negative and transmitted otherwise. It can be observed in Fig. 4, that as the EA with MC matrix, $\mathbf{Z}^{(2)}$, and $\mathbf{Z}^{(3)}$, can supported all signal combinations for 16-PSK modulation, and the performances of the EA-P, EA-S and standard multiple antenna transmitters are similar. However, for the EA with MC matrix, $\mathbf{Z}^{(1)}$, all possible signal combinations cannot be supported using the EA-S. Therefore, in this case, the SER performance of EA-S degrades significantly compared with the standard multiple antenna transmitter. However, it can be observed that by employing the proposed EA-P, performance similar to that of the standard multiple antenna transmitter can be achieved. In Fig. 5, the results are shown for 16-QAM modulation scheme. It can be observed that in this case, the EAs with both $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$, do not support all possible signal combinations and the performance of EA-S transmitter is significantly degraded. However, nearly the same SER performance can be achieved as in the case of the standard multiple antenna transmitter by using the proposed EA-P.

This also highlights the usefulness of our scheme. All signal combinations could be transmitted by using the EA with MC matrix, $\mathbf{Z}^{(2)}$, for 16-PSK. But once the modulation scheme changed to 16-QAM, some signal combinations became infeasible. But the same EA can be used to transmit signals with any modulation scheme by using the proposed EA-P.

Fig. 5: SER performance comparison of the EA transmitter and standard multiple antenna transmitter in the case of spatial multiplexing scheme under 16-QAM

C. Transmit Diversity

One of the main advantages of multiple antennas is that they provide diversity gain which results in significant performance improvement. There are different approaches to achieve transmit diversity gain depending on the CSI availability at the transmitter.

1) *Transmitter without CSI:* When the transmitter does not possess CSI, the diversity gain can be achieved by using space-time block codes (STBC). One of the simplest STBC that gives diversity gain is the Alamouti code. In the case of a two elements EA, given a signal vector $[s_0, s_1]^T$, the ideal port current vectors is $\hat{\mathbf{i}} = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}$. The corresponding input resistances for these two symbol periods are

$$\begin{aligned} R_{in}^{(1)} &= \Re \left(Z_{00} + \frac{Z_{01}s_1}{s_0} \right) = R_0 + \Re \left(\frac{Z_{01}s_1}{s_0} \right), \\ R_{in}^{(2)} &= \Re \left(Z_{00} + \frac{Z_{01}s_0^*}{-s_1^*} \right) = R_0 + \Re \left(\frac{Z_{01}s_0^*}{-s_1^*} \right). \end{aligned} \quad (31)$$

The supported signal vector set $[s_0, s_1]^T$, which can be exactly transmitted according to Alamouti code by a EA, is

$$\mathbb{S}_{P\xi} = \left\{ [s_0, s_1] \mid R_{in}^{(1)} > 0, R_{in}^{(2)} > 0 \right\}. \quad (32)$$

Again, in this case, whether a signal can be supported depends on the MC matrix and also the transmission signals. In the case of N -PSK modulation, where the transmission symbols s_0 and s_1 have phases ϕ_0 and ϕ_1 , respectively. The input resistances are

$$\begin{aligned} R_{in}^{(1)} &= R_0 + R_1 \cos(\phi_1 - \phi_0) - X_1 \sin(\phi_1 - \phi_0), \\ R_{in}^{(2)} &= R_0 - R_1 \cos(\phi_1 - \phi_0) + X_1 \sin(\phi_1 - \phi_0). \end{aligned} \quad (33)$$

Fig. 6: SER performance comparison of the EA transmitter and standard multiple antenna transmitter in the case of Alamouti scheme under 16-QAM.

It can be noted that the input resistances vary with the MC matrix and the phases of transmitted signals. When the transmitter uses BPSK, then the MC matrix is required to satisfy that $R_{in} = R_0 \pm R_1 > 0$.

Numerical Simulation: The SER performance of the system where the transmitter employs Alamouti code is shown in Fig. 6. The modulation scheme is 16-QAM. Three EAs with MC matrices, $\mathbf{Z}^{(1)}$, $\mathbf{Z}^{(2)}$, and $\mathbf{Z}^{(3)}$, respectively, are considered and the SER performance of EA-P, EA-S and the standard multiple antenna transmitters are compared. It can be observed that the performance for EA-S system with $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ degrades significantly as the EAs with MC matrices, $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ do not support all possible signal combinations. However, again by using EA-P, the SER is improved and nearly the same performance as that of the standard multiple antenna transmitter can be achieved.

2) *Transmitter with CSI:* When the transmitter has CSI, transmit diversity can be achieved by precoding using the CSI. One such precoding scheme, called MRT, is considered here. Assuming that symbol s is to be transmitted, the ideal port current vectors for the transmitter is $\hat{\mathbf{i}} = \frac{\mathbf{h}}{\|\mathbf{h}\|}$, where $\mathbf{h} = [h_0, h_1, \dots, h_M]^H \in \mathbb{C}^{(M+1) \times 1}$ and h_m denotes the channel from the $(m+1)$ -th element of EA transmitter to the signal antenna receiver [30].

The corresponding input impedance can be obtained as

$$Z_{in} = Z_{00} + \frac{\sum_{m=1}^M Z_{0m} \hat{i}_m}{\hat{i}_0} = Z_{00} + \frac{\sum_{m=1}^M Z_{0m} h_m}{h_0}. \quad (34)$$

From (34), it can be noted that the input impedance is the function of channel and the MC matrix only, and that it is independent of the data symbols. This implies that, for the EA-P, in the case of a block fading channel, in which the channel is constant over a block

Fig. 7: 5-element circular ESPAR antenna

of symbols, the signal only needs to be preprocessed once in the beginning of the block. Moreover, from (34), it can be noted that the input resistance is random as the channel is random. Therefore, in this case, a measure of suitability of an EA can be the probability that $R_{in} > 0$, i.e. $\mathcal{P} = \Pr(R_{in} > 0)$. An EA can be selected which has the highest probability $\mathcal{P}$. Note that $\mathcal{P} = \Pr(R_{in} > 0) = 1 - F_{R_{in}}(0)$, where $F_{R_{in}}$ denotes the cumulative distribution function (CDF) of R_{in} .

As an example, the CDF of R_{in} is derived for a circular antenna array. As an example, a 5-element ESPAR is shown in Fig. 7. In the case of a circular antenna array, $Z_{0m} = Z_{01}$, for $m = 2, 3, \dots, M$ [31]. We denote the real and imaginary part of Z_{01} by R_1 and X_1 , respectively. Substituting R_1 and X_1 into (34), yields the input resistance as

$$R_{in} = \Re \left(Z_{00} + Z_{01} \frac{\sum_{m=1}^M h_j}{h_0} \right) \\ = R_0 + R_1 \Re \left(\frac{\sum_{m=1}^M h_j}{h_0} \right) - X_1 \Im \left(\frac{\sum_{m=1}^M h_j}{h_0} \right). \quad (35)$$

Let $\kappa_t = \frac{\sum_{m=1}^M h_m}{h_0}$. Denoting the real and imaginary parts of κ by r_κ and x_κ , respectively, the input resistance can be expressed as $R_{in} = R_0 + R_1 r_\kappa - X_1 x_\kappa$. As r_κ and x_κ are the random variables, in order to obtain the CDF of R_{in} , the distributions of r_κ and x_κ are required. For a Rayleigh faded channel, $h_j \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$, for all $j \in \{0, 1, \dots, M\}$, where $\mathcal{CN}$ denotes a complex Gaussian distribution. As h_m is a complex Gaussian, the sum of h_m also follows complex Gaussian distribution, $\sum_{m=1}^M h_j \sim \mathcal{CN}(0, M\sigma^2 \mathbf{I})$. This implies that the random variable κ is a ratio of two independent complex Gaussian variables, and the joint PDF of the r_κ and x_κ can

be given as [32]

$$f(r_\kappa, x_\kappa) = \frac{M}{\pi} (r_\kappa^2 + x_\kappa^2 + M)^{-2}. \quad (36)$$

The CDF for R_{in} can be obtained by solving

$$F_{R_{in}}(r_{in}) = \iint_{R_{in} \leq r_{in}} f(r_\kappa, x_\kappa) dr_\kappa dx_\kappa. \quad (37)$$

Substituting R_{in} and (36) into (37), we have

$$F_{R_{in}}(r_{in}) \\ = \iint_{R_0 + R_1 r_\kappa - X_1 x_\kappa \leq r_{in}} \frac{M}{\pi} (r_\kappa^2 + x_\kappa^2 + M)^{-2} dr_\kappa dx_\kappa. \quad (38)$$

After solving the integrals in (38), the CDF of R_{in} can be obtained and is given in the proposition below.

Proposition 5 *The CDF of input resistance for a circular EA is given as*

$$F_{R_{in}}(r_{in}) = \frac{1}{2} + \frac{r_{in} - R_0}{2\sqrt{M(X_1^2 + R_1^2) + (r_{in} - R_0)^2}}. \quad (39)$$

The PDF of input resistance is given as

$$f_{R_{in}}(r_{in}) = \frac{1}{2\sqrt{M(X_1^2 + R_1^2) + R_0^2}} \\ - \frac{(r_{in} - R_0)^2}{2(M(X_1^2 + R_1^2) + (r_{in} - R_0)^2)^{3/2}} \quad (40)$$

The probability that a circular EA can transmit an exact MRT precoded signal is

$$\mathcal{P} = \Pr(R_{in} > 0) = 1 - F_{R_{in}}(0) = \frac{1}{2} + \frac{R_0}{2\sqrt{M(X_1^2 + R_1^2) + R_0^2}}. \quad (41)$$

Proof: The proof is provided in Appendix E. ■

From this proposition, it can be noted that the probability that a circular EA can support an exact MRT precoded signal, increases as the self-impedance of the active element increases. Furthermore, this probability decreases as the number of elements increases or the mutual coupling impedance between the active element and parasitic elements increases. These factors show that the probability increases as the mutual coupling is reduced. However, the basic work principle of an EA is the mutual coupling that exists due to the small distance between the elements, therefore, a relatively large mutual coupling is desired for EA operation. Thus, in this case, the probability $\mathcal{P}$ will decrease and our proposed EA-P will become crucial to achieve signal transmission using the EA.

Numerical Simulation: In Fig. 8 and Fig. 9, the CDF of R_{in} , plotted using (39), is compared with the

Fig. 8: The CDF of input resistance

Fig. 9: The CDF of input resistance with respect to different antenna spacings

CDF obtained via Monte Carlo simulation for different EAs. It can be observed that the analytical results match well with the simulation results for MC matrix $\mathbf{Z}^{(1)}$, $\mathbf{Z}^{(2)}$, and $\mathbf{Z}^{(3)}$ in Fig. 8. The probability on $R_{in} < 0$ decreases as R_0 increases. The CDFs are compared under different element numbers and antenna spacing for circular linear EAs in Fig. 9, it can be noted that reducing the number of the elements or increasing antenna spacing reduces the probability on $R_{in} < 0$.

The SER performances, in the case of MRT, are compared for circular EAs with different number of elements and different spacings in Fig. 10. It can be observed from Fig. 10 that the SER performance varies with the number of antenna elements and the antenna spacing. Increasing the number of the elements or reducing antenna spacing reduces the SER. It can be observed for the EA-S that the SER performance are very poor and an error floor is obtained. In addition, the error floor can be avoided using EA-P and especially at low SNRs, the SER

Fig. 10: SER performance comparison of the EA transmitter and standard multiple antenna transmitter in the case of maximum ratio transmission under 16-QAM.

performance of the EA-P transmitter is similar to that of the standard multiple antenna transmitter. Thus, the SER performance can be improved significantly using EA-P. However, at high SNRs there is a slight loss in performance as the number of elements is increased. This happens because the proportion of unsupported signals increases with increasing antenna elements.

In Fig. 11, the SER performances of the EA-P is shown with varying ξ^4 . It can be observed that the SER increases with the increasing ξ . This happens due to a stronger requirement on the input resistance and results in an approximate signal that is more different from the ideal signal.

V. CONCLUSION

ESPAR is a reconfigurable antenna in which the parasitic loads are varied to achieve arbitrary signal transmission. For some signals, varying the parasitic loads results in a negative input resistance. A negative input resistance implies that the antenna structure is unstable to transmit these signals. In this work, we showed that instead of those actual signals, one can transmit approximate signals, which satisfy the input resistance constraint, without much loss in performance. We obtained a simple closed-form expression for calculating the approximate signals which can be easily implemented at the transmitter. Our simulation results, conducted for various communication scenarios, show

⁴The value of ξ affects the input power from the active element, and therefore, it will also affect the efficiency of an EA. However, as this work focus on the stability and feasibility of an EA, the efficiency of an EA will be discussed in future work.

Fig. 11: SER performance comparison of the 3 elements EA transmitter with varying ξ under 16-QAM.

that the EA-P transmitter gives significant improvement over the performance of the EA-S transmitter and performs nearly the same as the standard multi-antenna system.

APPENDIX A PROOF OF PROPOSITION 1

Proof: From (5), the input resistance is (42).

Eq. (8) follows as a simple consequence of rewriting the quadratic form in (42) in the form of an appropriate matrix expansion. ■

APPENDIX B PROOF OF PROPOSITION 2

Proof: As $\mathbf{B}$ is a real symmetric matrix and it can be diagonalized as $\mathbf{B} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$, where $\mathbf{\Lambda}$ is a real diagonal matrix and its elements are eigenvalues of $\mathbf{B}$, the columns of the real orthogonal matrix $\mathbf{Q}$ are corresponding eigenvectors of $\mathbf{B}$. For the sake of simplification, the eigenvalues $\mathbf{\Lambda}$ are set in ascending order. Let ζ denote the eigenvalues of $\mathbf{B}$, the matrix $\mathbf{B} - \zeta\mathbf{I}$ can be written as

$$\mathbf{B} - \zeta\mathbf{I} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}, \quad (43)$$

where

$$\begin{aligned} \mathbf{B}_{11} &= \begin{bmatrix} R_0 - \xi - \zeta & 0 \\ 0 & R_0 - \xi - \zeta \end{bmatrix}, \\ \mathbf{B}_{12} &= \begin{bmatrix} \frac{R_1}{2} & -\frac{X_1}{2} & \cdots & \frac{R_M}{2} & -\frac{X_M}{2} \\ \frac{X_1}{2} & \frac{R_1}{2} & \cdots & \frac{X_M}{2} & \frac{R_M}{2} \end{bmatrix}, \\ \mathbf{B}_{21} &= \mathbf{B}_{12}^T, \\ \mathbf{B}_{22} &= \text{diag}(-\zeta, -\zeta, \dots, -\zeta). \end{aligned}$$

As $\mathbf{B}_{22}$ is invertible, the determinant of the matrix $\mathbf{B} - \zeta\mathbf{I}$ can be rewritten as (45).

Setting $\det(\mathbf{B} - \zeta\mathbf{I}) = 0$, the eigenvalues of $\mathbf{B}$ are (11). We can find that ζ_1 and ζ_2 are negative whereas ζ_3 and ζ_4 are positive eigenvalues, thus $\mathbf{\Lambda}$ is $\text{diag}(\zeta_1, \zeta_2, 0, \dots, 0, \zeta_3, \zeta_4)$. The matrix $\mathbf{B}$ is not a positive or a negative semidefinite matrix. The constraint (10b) is nonconvex and the optimization problem (10) is a nonconvex problem. ■

APPENDIX C PROOF OF PROPOSITION 3

Proof: From proposition 2, $\mathbf{B}$ is a real symmetric matrix and it can be eigenvalue decomposed as $\mathbf{B} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$. Here, $\mathbf{\Lambda} = \text{diag}(\zeta_1, \zeta_2, 0, \dots, 0, \zeta_3, \zeta_4)$ is a diagonal matrix whose diagonal elements are the eigenvalues of $\mathbf{B}$ in ascending order. $\mathbf{Q}$ is an orthogonal matrix whose columns are the corresponding eigenvectors. Then $\mathbf{I} - \delta\mathbf{B} = \mathbf{Q}(\mathbf{I} - \delta\mathbf{\Lambda})\mathbf{Q}^T$. The eigenvalues of $(\mathbf{I} - \delta\mathbf{B})$ are $1 - \delta\zeta_1, 1 - \delta\zeta_2, 1, \dots, 1, 1 - \delta\zeta_3, 1 - \delta\zeta_4$ and the columns of $\mathbf{Q}$ are corresponding eigenvectors of $(\mathbf{I} - \delta\mathbf{B})$. Therefore, the feasible set of the optimization problem (16) is

$$\mathbb{C} = \{\delta | 0 \leq \delta \leq 1/\zeta_3\}, \quad (46)$$

and it can be rewritten as the optimization conditions as (17b).

For the objective function (16a), $\hat{\mathbf{w}}^T \hat{\mathbf{w}}$ can be simply removed from the objective function as it is constant. Similarly, the objective function is reformulated as

$$\hat{\mathbf{w}}^T (\mathbf{I} - \delta\mathbf{B})^{-1} \hat{\mathbf{w}} = (\mathbf{Q}^T \hat{\mathbf{w}})^T (\mathbf{I} - \delta\mathbf{\Lambda})^{-1} (\mathbf{Q}^T \hat{\mathbf{w}})^T. \quad (47)$$

Setting $\mathbf{g} = [g_1, g_2, \dots, g_{2M+1}, g_{2M+2}]^T = \mathbf{Q}^T \hat{\mathbf{w}} \in \mathbb{R}^{2(M+1)}$, (47) can be written as

$$\hat{\mathbf{w}}^T (\mathbf{I} - \delta\mathbf{B})^{-1} \hat{\mathbf{w}} = \frac{g_1^2 + g_2^2}{1 - \delta\zeta_1} + \frac{g_{2M+1}^2 + g_{2M+2}^2}{1 - \delta\zeta_3} + \sum_{k=3}^{2M} g_k^2. \quad (48)$$

From (48), $\sum_{k=3}^{2M} g_k^2$ depends on the ideal currents $\hat{\mathbf{w}}$ and matrix $\mathbf{B}$, it is therefore independent of the optimal value δ . It can be removed from the objective function. Setting $r_1 = g_1^2 + g_2^2$, $r_2 = g_{2M+1}^2 + g_{2M+2}^2$, the objective function (16a) can be further reformulated as

$$\frac{r_1}{1 - \delta\zeta_1} + \frac{r_2}{1 - \delta\zeta_3}. \quad (49)$$

The optimization problem can be reformulated as (17) from (46) and (49). ■

$$\Re \left\{ Z_{00} + \frac{\sum_{m=1}^M Z_{0m} i_m}{i_0} \right\} = R_0 + \frac{\sum_{m=1}^M [(R_m w_{2m+1} - X_m w_{2m+2}) w_1 + (R_m w_{2m+2} + X_m w_{2m+1}) w_2]}{w_1^2 + w_2^2} \quad (42)$$

$$= \frac{1}{w_1^2 + w_2^2} \left[R_0 w_1^2 + R_0 w_2^2 + \sum_{k=1}^M (R_m w_{2m+1} w_1 - X_m w_{2m+2} w_1 + R_m w_{2m+2} w_2 + X_m w_{2m+1} w_2) \right].$$

$$\det(\mathbf{B} - \zeta \mathbf{I}) = \det(\mathbf{B}_{22}) \det(\mathbf{B}_{11} - \mathbf{B}_{12} \mathbf{B}_{22}^{-1} \mathbf{B}_{21}) = (-\zeta)^{2M-2} \left(\zeta^2 - 2(R_0 - \xi) \zeta - \sum_{m=1}^M (R_m^2 + X_m^2) \right)^2. \quad (45)$$

APPENDIX D PROOF OF PROPOSITION 4

Proof: As when $\delta = \frac{1}{\zeta_3}$ or $\delta = \frac{1}{\zeta_1}$, the objective function of (17a) cannot obtain minimal value, there are two possible cases from (20).

A. *Case 1:* $\sigma_2 = 0$, $\delta = 0$, and $\sigma_1 \geq 0$

In this case, we can rewrite (19) as $\nabla_{\delta} L_{\delta}(\delta, \sigma_1, \sigma_2) = \zeta_1 r_1 + \zeta_3 r_2 - \sigma_1 = 0$. Thus, we have $\sigma_1 = \zeta_1 r_1 + \zeta_3 r_2$, which means $\zeta_1 r_1 + \zeta_3 r_2 \geq 0$.

B. *Case 2:* $\sigma_2 = 0$, $\sigma_1 = 0$, and $\delta \geq 0$

The stationarity condition of the KKT conditions (20) can be written as $\nabla_{\delta} L_{\delta}(\delta, \sigma_1, \sigma_2) = -\frac{\zeta_1 r_1}{(1-\delta\zeta_1)^2} - \frac{\zeta_3 r_2}{(1-\delta\zeta_3)^2} = 0$.

As $\zeta_1 < 0$, $\zeta_3 > 0$, $r_1, r_2 > 0$, (20) can be further simplified as $\frac{\sqrt{-\zeta_1 r_1}}{1-\delta\zeta_1} = \frac{\sqrt{\zeta_3 r_2}}{1-\delta\zeta_3}$. Then the value of δ is $\delta = \frac{\sqrt{-\zeta_1 r_1} - \sqrt{\zeta_3 r_2}}{\zeta_3 \sqrt{-\zeta_1 r_1} - \zeta_1 \sqrt{\zeta_3 r_2}}$.

In this case, we have $\frac{\zeta_1 r_1 + \zeta_3 r_2}{(1-\delta\zeta_1)^2} < \frac{\zeta_1 r_1}{(1-\delta\zeta_1)^2} + \frac{\zeta_3 r_2}{(1-\delta\zeta_3)^2} = 0$. Then, we have $\zeta_1 r_1 + \zeta_3 r_2 < 0$.

C. Discussion

In this subsection, we apply logic to prove that the case 1 and the case 2 are logically equivalent to the statement that the ideal signal can be transmitted by EA and the statement that the ideal signal cannot be supported by EA, respectively. Let C_1 and C_2 denote the statement case 1 and case 2, respectively. D_P for the statement $\hat{\mathbf{i}} \in \mathbb{I}_{P\xi}$ and D_N for the statement $\hat{\mathbf{i}} \in \mathbb{I}_{N\xi}$. D_1 for $\hat{\mathbf{w}}_T^T \mathbf{B} \hat{\mathbf{w}} \geq 0$ and D_2 for $\hat{\mathbf{w}}_T^T \mathbf{B} \hat{\mathbf{w}} < 0$. D_3 for $\zeta_1 r_1 + \zeta_3 r_2 \geq 0$ and D_4 for $\zeta_1 r_1 + \zeta_3 r_2 < 0$.

From (10b), we have $D_P \Leftrightarrow D_1$ and $D_N \Leftrightarrow D_2$, where $\Leftrightarrow$ means if and only if.

Now, we can see the practical meaning of these two cases. Substituting ideal currents into constraint (10b), we have $\hat{\mathbf{w}}_T^T \mathbf{B} \hat{\mathbf{w}} = (\mathbf{Q}^T \hat{\mathbf{w}})^T \mathbf{\Lambda} \mathbf{Q}^T \hat{\mathbf{w}} = \mathbf{g}^T \mathbf{\Lambda} \mathbf{g} = \zeta_1 r_1 + \zeta_3 r_2$. Thus, we have $D_1 \Leftrightarrow D_3$ and $D_2 \Leftrightarrow D_4$.

In the case 1, as $\sigma_1 \geq 0$, $\zeta_1 r_1 + \zeta_3 r_2 \geq 0$. Thus, $C_1 \Rightarrow D_3$ and then $C_1 \Rightarrow D_P$. If D_P is true, then we have $\zeta_1 r_1 + \zeta_3 r_2 \geq 0$ as $D_P \Leftrightarrow D_1 \Leftrightarrow D_3$. Since there are only two cases of solutions, we have C_1 is true, which means $D_P \Rightarrow C_1$. Thus, we have $C_1 \Leftrightarrow D_P$, which implies that case 1 is equivalent that the ideal currents $\hat{\mathbf{i}}$ can be supported by EA. Similarly, we have $C_2 \Leftrightarrow D_N$, the ideal currents cannot be implemented by ESPAR antenna. Then we have

$$\delta = \begin{cases} 0 & \text{for } \hat{\mathbf{i}} \in \mathbb{I}_{P\xi} \\ \frac{\sqrt{-\zeta_1 r_1} - \sqrt{\zeta_3 r_2}}{\zeta_3 \sqrt{-\zeta_1 r_1} - \zeta_1 \sqrt{\zeta_3 r_2}} & \text{for } \hat{\mathbf{i}} \in \mathbb{I}_{N\xi} \end{cases} \quad (50)$$

APPENDIX E PROOF OF PROPOSITION 5

Proof: From (38), the proposition can be proved in these two cases.

A. *Case 1,* $R_1 > 0$

In this case, by setting $l_1 = \frac{X_1}{R_1}$, $l_2 = \frac{r_{in} - R_0}{R_1}$, the CDF of input resistance is

$$F_{R_{in}}(r_{in}) = \iint_{r_{\kappa} \leq \frac{X_1}{R_1} x_{\kappa} + \frac{r_{in} - R_0}{R_1}} f(r_{\kappa}, x_{\kappa}) dr_{\kappa} dx_{\kappa}$$

$$= \frac{M}{\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{l_1 x_{\kappa} + l_2} (r_{\kappa}^2 + x_{\kappa}^2 + M)^{-2} dr_{\kappa} dx_{\kappa}. \quad (51)$$

The indefinite integral with respect to r_{κ} can be calculated by using [33, eq. (2.110.2)] and [33, eq.

(2.01.15)] and it is

$$\int (r_\kappa^2 + x_\kappa^2 + M)^{-2} dr_\kappa = \frac{r_\kappa}{2(M + x_\kappa^2)(M + x_\kappa^2 + r_\kappa^2)} + \frac{\arctan\left(\frac{r_\kappa}{\sqrt{M + x_\kappa^2}}\right)}{2(M + x_\kappa^2)^{3/2}}. \quad (52)$$

Applying Newton-Leibniz formula, the definite integral with respect to r_κ is (53).

Substituting (53) into (51), the double integral can be separated into the sum of three integrals. The indefinite integral of the first part, $F_1(x_\kappa)$, can be calculated by using [33, eq. (2.01.15)] and [33, eq. (2.01.2)] and doing some algebraic manipulations, and it is (54).

Similarly, the indefinite integral of the second part, $F_2(x_\kappa)$, can be calculated from [33, eq. (2.264.5)] and it is

$$F_2(x_\kappa) = \int \frac{\pi/2}{2(M + x_\kappa^2)^{3/2}} dx_\kappa = \frac{\pi}{4} \frac{x_\kappa}{M\sqrt{M + x_\kappa^2}}. \quad (55)$$

The indefinite integral of third part, $F_3(x_\kappa)$, can be firstly simplified by applying the theorem of integration by parts, and then calculated by using [33, eq. (2.01.15)] and [33, eq. (2.01.2)]. After some algebraic manipulations, we have (56).

According to Newton-Leibniz formula and after some algebraic manipulations, the probability when the real part of the input impedance is less than r_{in} under the condition $R_1 \geq 0$ is

$$F_{R_{in}}(r_{in}) = \frac{1}{2} + \frac{l_2}{2\sqrt{M + l_1^2 M + l_2^2}}. \quad (57)$$

As $l_1 = \frac{X_1}{R_1}$ and $l_2 = \frac{r_{in} - R_0}{R_1}$, putting them into (57), we have (39). The PDF of R_{in} can be obtained from its CDF as (40). The probability of the real part of the input impedance is bigger than 0 under the condition $R_1 > 0$ is (41).

B. Case 2: $R_1 < 0$

From (37), in this case, the probability is

$$\begin{aligned} F_{R_{in}}(r_{in}) &= \iint_{r_\kappa > \frac{X_1}{R_1} x_\kappa + \frac{r_{in} - R_0}{R_1}} f(r_\kappa, x_\kappa) dr_\kappa dx_\kappa \\ &= \frac{M}{\pi} \int_{-\infty}^{+\infty} \int_{l_1 x_\kappa + l_2}^{+\infty} (r_\kappa^2 + x_\kappa^2 + M)^{-2} dr_\kappa dx_\kappa. \end{aligned} \quad (58)$$

Applying Newton-Leibniz formula based on (52), we

have

$$\begin{aligned} &\int_{l_1 x_\kappa + l_2}^{+\infty} (r_\kappa^2 + x_\kappa^2 + M)^{-2} dr_\kappa \\ &= \frac{\pi/2}{2(M + x_\kappa^2)^{3/2}} - \frac{\arctan\left(\frac{l_1 x_\kappa + l_2}{\sqrt{M + x_\kappa^2}}\right)}{2(M + x_\kappa^2)^{3/2}} \\ &\quad - \frac{l_1 x_\kappa + l_2}{2(M + x_\kappa^2) \left(M + x_\kappa^2 + (l_1 x_\kappa + l_2)^2\right)}. \end{aligned} \quad (59)$$

Substituting (59) into (58) and after some algebraic manipulations, the CDF and PDF when $R_1 < 0$ is (39), (40), respectively. Similarly, the probability of the real part of the input impedance is bigger than 0 under the condition $R_1 < 0$ is (41) ■

REFERENCES

- [1] A. Kalis, A. Kanatas, and C. Papadias, "A novel approach to MIMO transmission using a single RF front end," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 6, pp. 972–980, 2008.
- [2] Y. Zhou, R. Adve, and S. Hum, "Design and evaluation of pattern reconfigurable antennas for MIMO applications," *IEEE Trans. Antennas Propag.*, vol. 62, no. 3, pp. 1084–1092, March 2014.
- [3] B. A. Cetiner, H. Jafarkhani, J.-Y. Qian, H. J. Yoo, A. Grau, and F. De Flaviis, "Multifunctional reconfigurable mems integrated antennas for adaptive mimo systems," *Communications Magazine, IEEE*, vol. 42, no. 12, pp. 62–70, 2004.
- [4] D. Piazza, N. J. Kirsch, A. Forenza, R. W. Heath Jr, and K. R. Dandekar, "Design and evaluation of a reconfigurable antenna array for mimo systems," *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 3, pp. 869–881, 2008.
- [5] A. M. Sayeed and N. Behdad, "Continuous aperture phased mimo: A new architecture for optimum line-of-sight links," in *Antennas and Propagation (APSURSI), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 293–296.
- [6] J. D. Boerman and J. T. Bernhard, "Performance study of pattern reconfigurable antennas in mimo communication systems," *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 1, pp. 231–236, 2008.
- [7] A. M. Sayeed and V. Raghavan, "Maximizing mimo capacity in sparse multipath with reconfigurable antenna arrays," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 1, no. 1, pp. 156–166, 2007.
- [8] N. Gulati and K. R. Dandekar, "Learning state selection for reconfigurable antennas: A multi-armed bandit approach," *Antennas and Propagation, IEEE Transactions on*, vol. 62, no. 3, pp. 1027–1038, 2014.
- [9] O. Alrabadi, C. Papadias, A. Kalis, and R. Prasad, "A universal encoding scheme for MIMO transmission using a single active element for PSK modulation schemes," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 5133–5142, 2009.
- [10] V. Barousis, A. Kanatas, and A. Kalis, "Beam-space-domain analysis of single-RF front-end MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 60, no. 3, pp. 1195–1199, 2011.
- [11] O. Alrabadi, J. Perruisseau-Carrier, and A. Kalis, "MIMO transmission using a single RF source: Theory and antenna design," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 654–664, 2012.
- [12] C. B. Papadias, "An emerging technology: load-modulated arrays for small and large scale mimo systems," in *Presentation given in the Smart Antennas Workshop, Stanford, USA*, 2014.
- [13] V. Barousis, C. Papadias, and R. Muller, "A new signal model for MIMO communication with compact parasitic arrays," in *Communications, Control and Signal Processing (ISCCSP), 2014 6th International Symposium on*, 2014, pp. 109–113.
- [14] V. Barousis and C. Papadias, "Arbitrary precoding with single-fed parasitic arrays: Closed-form expressions and design guidelines," *IEEE Wireless Communications Lett.*, vol. PP, no. 99, pp. 1–4, 2014.

$$\int_{-\infty}^{l_1 x_\kappa + l_2} (r_\kappa^2 + x_\kappa^2 + M)^{-2} dr_\kappa = \frac{l_1 x_\kappa + l_2}{2(M + x_\kappa^2) \left(M + x_\kappa^2 + (l_1 x_\kappa + l_2)^2 \right)} + \frac{\pi/2}{2(M + x_\kappa^2)^{3/2}} + \frac{\arctan \left(\frac{l_1 x_\kappa + l_2}{\sqrt{M + x_\kappa^2}} \right)}{2(M + x_\kappa^2)^{3/2}} \quad (53)$$

$$F_1(x_\kappa) = \int \frac{l_1 x_\kappa + l_2}{2(M + x_\kappa^2) \left(M + x_\kappa^2 + (l_1 x_\kappa + l_2)^2 \right)} dx_\kappa = \frac{2l_2 \arctan \left(\frac{x_\kappa}{\sqrt{M}} \right)}{4(l_1^2 M + l_2^2) \sqrt{M}} \\ - \frac{2l_2 \arctan \left((x_\kappa + x_\kappa l_1^2 + l_1 l_2) / \sqrt{M + l_1^2 M + l_2^2} \right)}{4(l_1^2 M + l_2^2) \sqrt{M + l_1^2 M + l_2^2}} + \frac{l_1}{4(l_1^2 M + l_2^2)} \ln \frac{M + x_\kappa^2 (1 + l_1^2) + 2x_\kappa l_1 l_2 + l_2^2}{M + x_\kappa^2} \quad (54)$$

$$F_3(x_\kappa) = \int \frac{\arctan \left(\frac{l_1 x_\kappa + l_2}{\sqrt{M + x_\kappa^2}} \right)}{2(M + x_\kappa^2)^{3/2}} dx_\kappa = \frac{1}{2} \int \arctan \left(\frac{l_1 x_\kappa + l_2}{\sqrt{M + x_\kappa^2}} \right) d \frac{x_\kappa}{M \sqrt{M + x_\kappa^2}} \\ = - \frac{l_2 \arctan \left(\frac{x_\kappa}{\sqrt{M}} \right)}{2\sqrt{M} (l_1^2 M + l_2^2)} + \frac{x_\kappa \arctan \left((x_\kappa l_1 + l_2) / \sqrt{M + x_\kappa^2} \right)}{2M \sqrt{M + x_\kappa^2}} \\ + \frac{l_2 \sqrt{M + l_1^2 M + l_2^2} \arctan \left((x_\kappa + x_\kappa l_1^2 + l_1 l_2) / \sqrt{M + l_1^2 M + l_2^2} \right)}{2M (l_1^2 M + l_2^2)} + \frac{l_1 \ln \frac{M + x_\kappa^2}{M + x_\kappa^2 (1 + l_1^2) + 2x_\kappa l_1 l_2 + l_2^2}}{4(l_1^2 M + l_2^2)} \quad (56)$$

- [15] C. A. Balanis, *Antenna theory: analysis and design*. John Wiley & Sons, 2012.
- [16] D. M. Pozar, *Microwave engineering*. Wiley, 2012.
- [17] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2009.
- [18] V. Barousis, M. A. Sedaghat, R. R. Müller, and C. B. Papadias, "Massive antenna arrays with low front-end hardware complexity: An enabling technology for the emerging small cell and distributed network architectures," *CoRR*, vol. abs/1407.7733, 2014.
- [19] E. Tsakalaki, O. Alrabadi, C. B. Papadias, and R. Prasad, "Adaptive reactance-controlled antenna systems for multi-input multi-output applications," *IET microwaves, antennas & propagation*, vol. 5, no. 8, pp. 975–984, 2011.
- [20] M. T. Ivrlac and J. A. Nossek, "Toward a circuit theory of communication," *IEEE Trans. Circuits Syst.*, vol. 57, no. 7, pp. 1663–1683, 2010.
- [21] Y. Fei, Y. Fan, B. K. Lau, and J. Thompson, "Optimal single-port matching impedance for capacity maximization in compact MIMO arrays," *IEEE Trans. Antennas Propag.*, vol. 56, no. 11, pp. 3566–3575, 2008.
- [22] G. Alexandropoulos, V. Barousis, and C. Papadias, "Precoding for multiuser mimo systems with single-fed parasitic antenna arrays," in *Global Communications Conference (GLOBECOM), 2014 IEEE*, Dec 2014, pp. 3897–3902.
- [23] D. F. Kelley and W. L. Stutzman, "Array antenna pattern modeling methods that include mutual coupling effects," *IEEE Trans. Antennas Propag.*, vol. 41, no. 12, pp. 1625–1632, 1993.
- [24] R. Hunger, *Floating point operations in matrix-vector calculus*. Munich University of Technology, Inst. for Circuit Theory and Signal Processing Munich, 2005.
- [25] G. H. Golub and C. F. Van Loan, *Matrix computations*. JHU Press, 2012, vol. 3.
- [26] B. Han, V. Barousis, A. Kalis, and A. Kanatas, "Active parasitic arrays for low cost compact MIMO transmitters," in *Antennas and Propagation (EUCAP), Proceedings of the 5th European Conference on*. IEEE, 2011, pp. 3663–3667.
- [27] B. Han, V. I. Barousis, A. Kalis, C. B. Papadias, A. G. Kanatas, and R. Prasad, "A single rf mimo loading network for high-order modulation schemes," *International Journal of Antennas and Propagation*, vol. 2014, 2014.
- [28] B. Han, V. Barousis, C. Papadias, A. Kalis, and R. Prasad, "MIMO over ESPAR with 16-QAM modulation," *IEEE Wireless Communications Lett.*, vol. 2, no. 6, pp. 687–690, December 2013.
- [29] W. L. Stutzman and W. A. Davis, *Antenna theory*. Wiley Online Library, 1998.
- [30] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Veh. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct 1999.
- [31] A. Kalis, A. G. Kanatas, and C. B. Papadias, *Parasitic Antenna Arrays for Wireless MIMO Systems*. Springer, 2014.
- [32] R. Baxley, B. Walkenhorst, and G. Acosta-Marum, "Complex gaussian ratio distribution with applications for error rate calculation in fading channels with imperfect CSI," in *Global Telecommunications Conference (GLOBECOM 2010), 2010 IEEE*, Dec 2010, pp. 1–5.
- [33] A. Jeffrey and D. Zwillinger, *Table of integrals, series, and products*. Academic Press, 2007.

Lin Zhou received the dual B.Sc. degrees in Electrical and Electronic Engineering & Computer Science, and the M.Sc. degree in Electronics and Information Engineering from Huazhong University of Science and Technology, Wuhan, China, in 2004 and 2007 respectively. From 2004 to 2007, she had been with Wuhan National Laboratory for Optoelectronics, China, working on the implementation of Beyond 3G systems as an R&D engineer. From 2007 to 2012, she has

been working as a integrated circuit (IC) design engineer on computer science and telecommunication in O2micro and Fiberhome, respectively. She is currently working toward her PhD at The University of Edinburgh, Edinburgh, U.K., from 2013. Her research interests include design and modeling of compact MIMO transceivers, parasitic reconfigurable arrays and smart antennas, energy efficient MIMO design.

Fahd Ahmed Khan received the B.Sc. degree in electrical engineering from National University of Science and Technology, Pakistan, in 2007. He completed his Masters in Communications Engineering from Chalmers University of Technology, Sweden, in 2009 and earned his Ph.D. degree in electrical engineering from King Abdullah University of Science and Technology (KAUST) in Aug. 2013. He was a Research Fellow with the Institute for Digital Communications at the

University of Edinburgh, United Kingdom from Nov. 2013 to Nov. 2014. He is currently working as an Assistant Professor at the National University of Science and Technology, Pakistan. He has received the Academic Excellence Award from KAUST in 2010 and in 2011, respectively. He is also the recipient of the Best poster award at IEEE DySPAN 2012. His research interests include performance analysis of cooperative relaying networks and cognitive radio networks.

Tharmalingam Ratnarajah (A96-M05-SM05) is currently with the Institute for Digital Communications, University of Edinburgh, Edinburgh, UK, as a Professor in Digital Communications and Signal Processing. His research interests include signal processing and information theoretic aspects of 5G wireless networks, full-duplex radio, mmWave communications, random matrices theory, interference alignment, statistical and array signal processing and

quantum information theory. He has published over 260 publications in these areas and holds four U.S. patents. He is currently the coordinator of the FP7 projects HARP (3.2M) in the area of highly distributed MIMO and ADEL (3.7M) in the area of licensed shared access. Previously, he was the coordinator of FP7 Future and Emerging Technologies project CROWN (2.3M) in the area of cognitive radio networks and HIATUS (2.7Me) in the area of interference alignment. Dr Ratnarajah is a Fellow of Higher Education Academy (FHEA), U.K., and an associate editor of the IEEE Transactions on Signal Processing.

Constantinos B. Papadias is the Dean of Athens Information Technology (AIT), in Athens, Greece, where he is also Professor and Head of its Broadband Wireless and Sensor Networks (B-WiSE) Research Group. He is also Adjunct Professor at Aalborg University in Denmark. He received the Diploma of Electrical Engineering from the National Technical University of Athens (NTUA) in 1991 and the Doctorate degree in Signal Processing (highest honors) from the Ecole

Nationale Supérieure des Télécommunications (ENST), Paris, France, in 1995. He was a researcher at Institut Eurcom (1992-1995), Stanford University (1995-1997) and Bell Labs (as Member of Technical Staff from 1997-2001 and as Technical Manager from 2001-2006). He was also Adjunct Professor at Columbia University (2004-2005) and Carnegie Mellon University (2006-2011). His research interests span several areas of advanced communication systems, with emphasis on wireless, cognitive, green and next generation networks. He has published over 150 papers, one research monograph, two edited books, 5 book chapters, and has received over 5000 citations for his work. He has also made standards contributions and holds 12 patents. He was a member of the Steering Board of the Wireless World Research Forum (WWRF) from 2002-2006, a member and industrial liaison of the IEEE Signal Processing for Communications Technical Committee from 2003-2008 and a National Representative of Greece to the European Research Councils IDEAS program from 2007-2008. He has served as member of the IEEE Communications Society's Fellow Evaluation and Awards Committees, as well as an Associate Editor for the IEEE Transactions on Signal Processing, the IEEE Transactions on Wireless Communications and the Journal of Communications and Networks. He has participated in several European Commission research grants, including the recently awarded (Horizon2020) project SANSA in the area of satellite-assisted wireless backhauling and another two FP7 research projects where he acts as Technical Coordinator: HARP, in the area of remote radio heads, and ADEL, in the area of licensed shared access. His distinctions include the Bell Labs Presidents Award (2002); a Bell Labs Teamwork Award (2003); the IEEE Signal Processing Society's Young Author Best Paper Award (2003); ISI's most cited paper of the decade citation in the area of wireless networks (2006); his recognition as a Highly Cited Greek Scientist (2011); and the co-authorship of two papers that earned Best Student Paper Awards at the IEEE International Conference on Bioinformatics and BioEngineering (2013&2014). He was a Distinguished Lecturer of the IEEE Communications Society for 2012-2013. Dr. Papadias is a Member of the Technical Chamber of Greece and a Fellow of IEEE.