On the Performance of Relay Aided Millimeter Wave Networks

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

On the Performance of Relay Aided Millimeter Wave Networks

Sudip Biswas, Satyanarayana Vuppala, *Member, IEEE*, Jiang Xue, *Member, IEEE* and Tharmalingam Ratnarajah, *Senior Member, IEEE*

***Abstract*—In this paper, we investigate the potential benefits of deploying relays in outdoor millimeter-wave (mmWave) networks. We study the coverage probability from sources to a destination for such systems aided by relays. The sources and the relays are modeled as independent homogeneous poisson point processes (PPPs). We present a relay modeling technique for mmWave networks considering blockages and compute the density of active relays that aid the transmission. Two relay selection techniques are discussed, namely best path selection and best relay selection. For the first technique, we provide a closed form expression for end-to-end signal to noise ratio (SNR) and compute the best random relay path in a mmWave network using order statistics. Moreover, the maximum end-to-end SNR of random relay paths is investigated asymptotically by using extreme value theory. For the second technique, we provide a closed form expression for the best relay node having the maximum path gain. Finally, we analyze the coverage probability and transmission capacity of the network and validate them with simulation results. Our results show that deploying relays in mmWave networks can increase the coverage probability and transmission capacity of such systems.**

***Index Terms*—mmWave Networks, Poisson Point Processes, Relay, Extreme Value Theory**

I. Introduction

In recent years, the explosive growth of mobile data traffic has led to an ever-growing demand for much higher capacity and lower latency in wireless networks. This has culminated in the development of the fifth generation (5G) wireless communication systems, expected to be deployed by the year 2020, with key goals of data rates in the range of Gbps, billions of connected devices, lower latency, improved coverage and reliability and low-cost, energy efficient and environment-friendly operation. To meet the ever-increasing demands in wireless traffic, and keeping in mind that the current wireless spectrum is almost saturated, it is imperative to shift the paradigm of cellular spectrum to a new range of frequencies. In this regard, millimeter wave (mmWave) bands with significant amounts of unused or moderately used bandwidths appear to be a viable way to move forward. With bands of 20-100 GHz available for communication, mmWave has the potential to be the cornerstone in the design of 5G networks.

This work was supported by the UK Engineering and Physical Sciences Research Council (EPSRC) under grant number EP/L025299/1. The work of S. Biswas and S. Vuppala was also supported by the Seventh Framework Programme for Research of the European Commission under grant number HARP-318489.

S. Biswas, S. Vuppala, J. Xue and T. Ratnarajah are with the Institute for Digital Communications, the University of Edinburgh, King's Building, Edinburgh, UK, EH9 3JL. E-mails: [Sudip.Biswas; S.Vuppala; J.Xue; T.Ratnarajah]@ed.ac.uk. Correspondent author: S. Vuppala.

In [1], the authors explore mmW frequency bands to design a 5G enhanced Local Area Network (eLAN). While [2] proposes a general framework to analyze the coverage and rate performance of mmWave networks, [3] proposes a tractable mmWave cellular network model and analyzes the coverage rate. However, one must remember that mmWave cellular communication is heavily dependent on the propagation environment. MmWave signals are affected by several environmental factors such as O_2 absorption and atmospheric conditions. and cannot penetrate through obstacles like buildings, concrete walls, vehicles, trees etc. Due to these limitations, such bands were not considered suitable for cellular transmission for a long time. However, recent studies and measurements have revealed that the significant increase in omnidirectional path loss can be compensated by the proportional increase in overall antenna gain with appropriate beamforming. The performance of mmWave cellular systems was analyzed in [4] using real time propagation channel measurements. Blockage effects and angle spreads were also incorporated in [5] to analyze mmWave systems. Generally in a communication system, path losses are computed for both line-of-sight (LOS) and non-line-of-sight (NLOS) measurements. It was stated in [6] that the blockages cause substantial differences in the LOS and NLOS path loss characteristics. Hence, it is very important to appropriately model the LOS and NLOS links in mmWave networks. Furthermore, the measurements for path loss were carried out for 73 GHz frequency in [7] and [8].

In conventional communication systems, relay aided transmission has been regarded as an effective way to increase the coverage probability, throughput and transmission reliability of the networks [9]. While [10] considers the deployment of relays as a network infrastructure without a wired backhaul connection, [11] explores the potential of deploying relays to design a cost effective network. The use of relays can be a promising solution for mmWave systems to combat the blockage effects and path losses that are encountered in mmWave networks. In this regard, multiple relays can be deployed between the sources and the destination of a transmission link. Performance evaluation of relay aided networks has been widely studied in [12], [13]. Recently, cooperative relaying has been proposed in order to extend the coverage, increase the capacity and to provide cost effective solutions. In [14], authors have studied the coverage probability of relay aided cellular networks with different association criteria between the base station and mobile station. It has been shown that coverage probability highly depends on path loss exponents and density of relays. Similarly, the achievable transmission capacity has been analyzed in relay assisted device-to-device

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

networks in [15].

Recently, the performance of Decode-and-Forward and Amplify-and-Forward strategies with high gain antenna arrays was characterized in [16]. The numerical results proved that directional antennas are useful for multi-hop relays. Hence, it is implicit that relays can prove to be an important tool in the design of mmWave cellular systems because coverage in such systems is a more acute problem, given the large difference between LOS and NLOS propagation characteristics.

All together, it has been clearly shown in literature that relays are beneficial and provide larger coverage and higher data rates. Furthermore, several strategies have been proposed in literature for relay aided transmission, namely amplify and forward, decode and forward and demodulate and forward [9], [10], [12], [15].

Stochastic geometry approaches have recently gained significant attention to develop tractable models to analyze the performance of wireless networks [17]. In this approach, the wireless network is abstracted to a convenient point process that is used to capture the network properties. A poisson point process (PPP) is the most popular and tractable point process to model the locations of users and base stations in wireless networks. [18] models the base stations as a PPP and determines the aggregate coverage probability. Heterogeneous networks with a similar base station modelling were studied in [19]. Inspired by the stochastic geometry approach to analyze the performance of conventional cellular systems, we design a framework for evaluation of the coverage and rate performance in mmWave networks. However, applying the results of conventional cellular systems to mmWave is non-trivial due to their differences in propagation characteristics and the use of highly directional beamforming. Directional beamforming was applied in [20] by considering a simplified path loss model. While in [21] a blockage model for mmWave is used to analyze the rate and coverage area of such systems, a distance dependent path loss model along with antenna gain parameters are considered in [3] to characterize the propagation environment in mmWave systems. Furthermore, we would like to refer the readers to [1]–[3], [21] which develop several mathematical frameworks to model the propagation characteristics of mmWave networks.

In this paper, we incorporate relays to aid mmWave networks in order to provide better coverage and decrease blockage effects on the transmission link. We consider a stochastic geometry approach to characterize the spatially distributed relays and the sources. It is assumed that the sources and the relays in the mmWave network follow two PPPs but are independent of each other. Most works on relay-aided networks assume that the number of relays in the network is fixed and known. However, such fixed type network relays may not be suitable for practical outdoor environments when a network topology dynamically changes. Due to the fact that some relays are in outage because of blockages in the network, we consider the subset of relays which has lesser path loss. This consideration leads to a marked Poisson process. In general, however, one must contend with the mathematical challenges of working with such point processes.

Furthermore, several relay selection techniques have been proposed in literature for relay aided transmission, namely random relay, best relay and optimal relay [9], [10], [12], [15]. However, we conform to two strategies for tractable analysis, namely random relay and best relay. The motivation behind the use of a random relay selection is to capture blockage effects on performance of active set of relays. Specifically, the end-to-end SNR is characterized using amplify and forward technique where the relay obtains a noisy version of the signal transmitted by the source in presence of blockages and then amplifies its received signal and re-transmits it to the destination again in presence of blockages. After finding a best random path, one will be able to provide a bound on the active relays which can participate in the communication. These relay nodes are the ones that are minimally affected by blockages. In this paper, we also consider the best relay selection in order to study the trade off between performance and complexity of random relay selection techniques in mmWave networks.

Specifically, we will investigate the coverage probability and the transmission capacity of relay-assisted mmWave networks using stochastic geometry tools. The analysis presented here adds valuable insights to related recent works [3] and [22] on the impact of blockages in mmWave random networks.

The main contributions of this paper can be summarized as in the following points:

- We have presented a relay modeling technique in mmWave networks considering blockages, in which we compute the density of active relays that aid the transmission.
- A closed form expression for end-to-end SNR is provided and the best random relay path in a mmWave network using order statistics is calculated.
- To investigate the asymptotic increase in the number of transmission paths, extreme value theory is used and accordingly the maximum end-to-end SNR of random relay paths is found to approach the Gumbel distribution.
- We have also provided the closed form expression of the SNR distribution for the best relay having maximum path gain in such a network.
- Finally, an analysis on the coverage probability and the transmission capacity of relay aided mmWave networks is provided. It is shown that relays improve the received SNR for mmWave networks for a specific coverage probability.

Notations: We use upper and lower case to denote cumulative distribution functions (CDFs) and probability density functions (PDFs) respectively. $\mathbb{R}$ denotes the real plane while $\mathbb{Z}^+$ denotes the plane for real and positive integers. The probability is denoted by $\mathbb{P}[\cdot]$. All other symbols will be explicitly defined wherever used.

The rest of the paper is organized as follows. Section II gives the mathematical preliminaries to aid our analysis while Section III describes the system model. The conditions for relay transmission in mmWave networks are presented in Section IV and the SNR analysis of the relay schemes used in the paper is presented in section V. In Section VI, we present the coverage probability and transmission capacity and section VII gives the simulation results. Finally, we conclude the paper in Section VIII.

II. Mathematical Preliminaries

In this paper, we extensively use log-normal random variables to model the shadowing effects caused due to random blockages in a mmWave network. A few important results are presented in this section for better understanding of the paper. However, we avoid the proofs of any results provided here as they are well known in literature of probability theory.

Definition: A log-normal random variable X with parameters μ and σ is defined as

$$X = e^{\mu+\sigma Z}, \tag{1}$$

where μ and σ are the mean and standard deviation of the variable's natural logarithm respectively and Z is a standard normal variable. The PDF of a log-normal distribution is given by

$$f_X(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\, e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}} \tag{2}$$

and the CDF is given by

$$\begin{aligned} F_X(x;\mu,\sigma) &= \int_0^x f_X(p;\mu,\sigma)\mathrm{d}p, \\ &= \frac{1}{2}\,\mathrm{erfc}\left(-\frac{\ln x-\mu}{\sigma\sqrt{2}}\right) = Q\left(\frac{\ln x-\mu}{\sigma}\right), \end{aligned} \tag{3}$$

where erfc is the complementary error function, and Q is the cumulative distribution function of the standard normal distribution. We give the following lemmas in no particular order which will aid our subsequent analyses.

Lemma 1: Let $X_j \sim \ln\mathcal{N}(\mu_j,\sigma_j^2)$ be n statistical independent log-normally distributed variables, and $Y = \prod_{j=1}^n X_j$, then Y is also log-normally distributed with parameters $\sum_{j=1}^n \mu_j$, and $\sum_{j=1}^n \sigma_j^2$.

Lemma 2: Let $X_j \sim \ln\mathcal{N}(\mu_j,\sigma_j^2)$ are independent log-normally distributed variables with varying σ and μ parameters, and $Y = \sum_{j=1}^n X_j$. Then the distribution of Y has no closed form expression, but can be reasonably approximated by another log-normal distribution Z with parameters [23]

$$\mu_Z = \ln\left[\sum e^{\mu_j+\sigma_j^2/2}\right] - \frac{\sigma_Z^2}{2}, \tag{4}$$

$$\sigma_Z^2 = \ln\left[\frac{\sum e^{2\mu_j+\sigma_j^2}(e^{\sigma_j^2}-1)}{(\sum e^{\mu_j+\sigma_j^2/2})^2}+1\right]. \tag{5}$$

Lemma 3: Let $X \sim \ln\mathcal{N}(\mu,\sigma^2)$, then $aX \sim \ln\mathcal{N}(\mu + \ln a,\ \sigma^2),\ a \in \mathbb{R}$.

*Lemma 4:*If $X \sim \ln\mathcal{N}(\mu,\sigma^2)$, then $\frac{1}{X} \sim \ln\mathcal{N}(-\mu,\ \sigma^2)$.

III. System Model

In this section, we illustrate our system model for a relay assisted mmWave network. We focus on the communication from multiple sources to a destination aided by multiple relays in the presence of blockages. The destination is assumed to be located at the origin $\mathcal{O}$. We term the direct link between a source and the destination or a relay and the destination as connection link. The link between a source and a relay is termed as the relay link. The specifics of the model are described below.

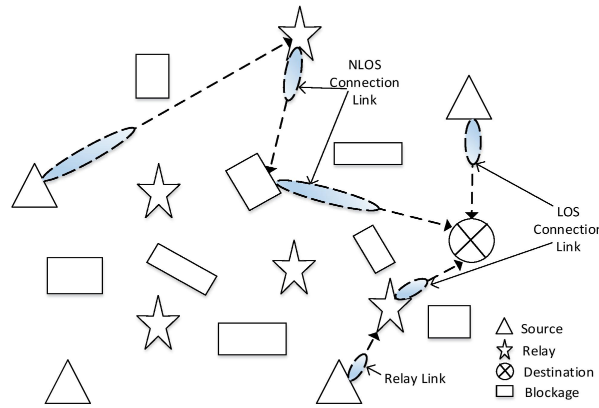


Fig. 1: An illustration of an outdoor mmWave network setup aided by relays.

A. Network Modeling

We consider a relay-aided mmWave ad hoc network consisting of multiple sources transmitting to a typical destination (reference point) as shown in Fig. 1. The sources in the network are modeled as points in $\mathbb{R}^2$ which are distributed uniformly as a homogeneous PPP Φ_S with intensity λ_S. The relays are also modeled as points of a uniform PPP, denoted by Φ_R, with density λ_R in $\mathbb{R}^2$.

B. Path Loss Modeling

It is well known that shadow fading heavily depends on the site-specific details of an environment. More specifically, path loss dependent shadow fading is typically a result of regression analysis on a signal level measurement represented on a distance dependent path loss scatter plot. In other words, a path loss law is fitted to the measurement, and the residual error of the model fit is called shadow fading. The path loss can be modeled in several ways from practical data accumulated from field measurements. In this paper, for analytic tractability, we use the alpha plus beta model (based on the traditional free space path loss model) given in [1], which takes into consideration the log-normal shadowing. Accordingly, in a mmWave transmission, the path loss (in dB) associated with the transmission between any two nodes x_i and x_j can be given as

$$L(x_i,x_j) = \beta + 10\alpha\log_{10}||x_i-x_j|| + \mathcal{X}_\mathcal{N}, \tag{6}$$

where $||x_i-x_j||$ is the distance between the ith and jth nodes with $\{i,j\} \in \mathbb{Z}^+$ and $\mathcal{X}_\mathcal{N} \sim \mathcal{N}(0,\sigma^2)$. However, it is to be noted that the sources and the relays can be either LOS or NLOS. Let the path loss at a fixed small reference distance be β. Then for such a model, α can be physically interpreted as the path loss exponent. Moreover, the parameters (α,β) can be looked upon as the floating intercept and slope of the best linear fit data. In that case, it may not be necessary to attribute (α,β) with any specific physical interpretation. The deviation in fitting (in dB) is modelled as a Gaussian random variable

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

$\mathcal{X}_{\mathcal{N}}$ (Lognormal in linear scale) with zero mean and variance σ^2. Accordingly, α, β and σ^2 are altered for each of the two scenarios.

According to [1], [24], the alpha plus beta model can be compared to the free space path loss model for a certain range of distances (30m-200m). For millimeter wave networks, due to path loss sensitivity, the typical communication range falls under 200m. Therefore, considering the alpha plus beta model is a viable approximation for such high frequency communications.

In mmWave networks, small scale fading does not have as much impact on transmitted signals as compared to lower frequency systems. However, blockages and shadowing are more significant in such systems. It is extensively mentioned in literature [1], [5] that in mmWave analysis, small scale fading can be ignored. Hence, ignoring fading and considering only shadowing, the probability density function of $\mathcal{X}_{\mathcal{N}}$ in (6) can be defined as

$$\mathcal{X}_{\mathcal{N}} \sim f_{\mathcal{X}_{\mathcal{N}}}(x;\mu_c,\sigma_c) = \frac{1}{x\sqrt{2\pi}\sigma_c}\exp\bigg(-\frac{(\log x-\mu_c)^2}{2\sigma_c^2}\bigg), \tag{7}$$

where the parameters, μ_c and σ_c^2 follows from [3] and $x > 0$.

C. Directional Beamforming Modeling

Due to the small wavelength of mmWaves, directional beamforming can be exploited for compensating the path loss and additional noise. Accordingly, antenna arrays are deployed at the source, relays and the destination. In our model, we assume all the sources and the relays to be equipped with directional antennas with sectorized gain pattern. Let θ be the beamwidth of the main lobe. Then the antenna gain pattern for a source, relay or destination node about some angle ϕ is given as [2]

$$G_q(\theta) = \left\{ \begin{array}{ll} G_q^{\max} & \text{if}|\phi| \le \theta \\ G_q^{\min} & \text{if}|\phi| \ge \theta \end{array} \right\}, \tag{8}$$

where $q \in \mathrm{S,R,D}$, $\phi \in [0,2\pi)$ is the angle of boresight direction, $G_q^{(max)}$ and $G_q^{(min)}$ are the array gains of main and side lobes, respectively. The user gain pattern can also be modeled similarly. Hereinforth, for simplicity we assume the antenna beams of the connection link and the relay link to be aligned. Hence, the total gain on a desired connection link is $G^{\max}$ and the relay link is $(G^{\max})^2$.

D. Blockage Modeling

Blockages in the network are usually concrete buildings which cannot be penetrated by mmWaves. We consider the blockages to be stationary blocks which are invariant with respect to directions. Different researchers have tried to model blockages with varied level of success based on different geographical scenarios. [2] uses the PPP based random blockage model, where $e^{-\beta r}$ is considered to be the probability of LOS with β being the blockage density and r the distance between the transmitting and receiving nodes. Another model that has been considered in literature is a fixed LOS probability model as was depicted in [3]. Leveraging the modeling of blockages from this later model, we consider a two state statistical model for each and every link. The link can be either LOS or NLOS. LOS link occurs when there is a direct propagation path between a source and the destination while NLOS occurs when the link is blocked and the destination receives the signal through reflection from a blockage. Let the LOS area within a circular ball of radius r_D be centered around the reference point. Then, if the LOS link is of length r, the probability of the connection link to be LOS is given by p_{LOS} if $r < r_D$ and 0 otherwise. Similarly, the NLOS probability is represented by p_{NLOS}. The parameters r and r_D are dependent on the geographical and deployment scenario of the network. The analytical results derived in this paper are based on the blockage model proposed in [3] and the numerical analyses are done based on the data accumulated by [2] and [3].

E. SNR Modeling

Recent studies on mmWave networks [1], [3], [5], state that mmWave networks in urban settings are more noise limited - in contrast to conventional cellular networks, which are usually strongly interference limited. This is due to the fact that in the presence of blockages, the signals received from unintentional sources are close to negligible. In such densely blocked scenarios (typical for urban settings), SNR provides a good enough approximation to signal to interference plus noise ratio (SINR) for directional mmWave networks. Additionally, such an assumption also aids us in deriving closed form expressions and hence, interference at the destination is ignored hereinafter.

In order to characterize the SNR distribution, we assume a two slot synchronous communication throughout the paper. While the active relay nodes are allowed to receive from the sources in the first time slot, the destination is allowed to receive from the active relay nodes and the sources in the second time slot. We also assume that all relays co-operate with each other while transmitting and are deployed with a guard zone[1].

First Time Slot: Consider that the relay nodes are served by the sources during this time slot. The SNR at any specific relay, R can then be formulated as

$$\gamma_{\mathrm{SR}}^i = \frac{P_{\mathrm{S}}(G^{\max})^2\mathcal{X}_{\mathcal{N}}r_{\mathrm{SR}}^{-\alpha_i}}{N_0}, \tag{9}$$

where P_{S} is the transmit power of the source, ${r_{\mathrm{SR}}}^2$ is the length of the link from the source to relay, α is the path loss exponent, $i \in \{\mathrm{LOS},\ \mathrm{NLOS}\}$ and N_0 is the noise power.

Second Time Slot: Consider that the destination, D is served by a source with or without the help of relay R during this time slot[3]. Then the SNR at the destination D receiving signal

[1]The guard zone resembles a specific SNR which must be fulfilled in order for the relay node to be deployed. This is explained in Section III of this paper.

[2]r_{AB} is the distance between the A-th and B-th nodes.

[3]This model of considering the destination to receive the signal from the source as well as relay in the second time slot can be useful when considering a maximal ratio combining scheme at the destination which would take into consideration both the signals from the relay and the source provided that the strength of the signal is above a certain threshold.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

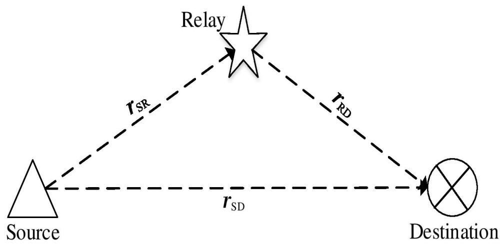


Fig. 2: Topology of a relay assisted network link.

only from the source, S can be given as

$$\gamma_{\mathrm{SD}}^{i} = \frac{P_{\mathrm{S}} G^{\max} \mathcal{X}_{\mathcal{N}} r_{\mathrm{SD}}^{-\alpha_i}}{N_0}. \tag{10}$$

Similarly, the SNR at the destination D receiving signal only from the relay, R can be given as

$$\gamma_{\mathrm{RD}}^{i} = \frac{P_{\mathrm{R}} G^{\max} \mathcal{X}_{\mathcal{N}} r_{\mathrm{RD}}^{-\alpha_i}}{N_0}, \tag{11}$$

where P_{R} is the transmit power of the relay. Note that for simplicity, we have omitted the subscript 'max' from G in all our subsequent discussions. Hereinafter, for analytical tractability, we consider that the transmitted power at the source and relay is the same and given as P.

Now, considering that the source transmits to the destination only through the aid of the relay, the coverage probability of such a relay-aided transmission link with a target SNR, T is given by

$$\mathcal{P}_{\mathrm{R}}^{c} = 1 - \mathbb{P}\{\gamma_{\mathrm{SR}} < T\}\mathbb{P}\{\gamma_{\mathrm{RD}} < T\}. \tag{12}$$

IV. Relay aided mmWave transmission

Fig. 2 shows an example of a transmission from a source to a destination through the aid of a relay. With the assistance of relays, it is possible to act on the constraints of path loss in a mmWave network and also extend the communication distance while improving the quality of communication. In this section we characterize the conditions for relay aided transmission in mmWave communication networks.

Relay cooperation takes place if and only if the SNR at the destination from the source through a direct link is not good enough and falls below a certain threshold. In order to avoid the aid of relay, we define a required outage constraint γ_{out} for the source-to-destination link as

$$\mathcal{P}_{\mathrm{out}} = \mathbb{P}\left\{\gamma_{\mathrm{SD}}^{i} < \gamma_{\mathrm{out}}\right\}. \tag{13}$$

A. Preliminaries on Active Relays

Due to the impact of blockages, some of the relay nodes may not be available or capable to support the transmission from source node to destination node and only a subset of the relay nodes may participate in the communication. In this subsection we give an insight on such active relays which are available to aid the communication from the source to the destination.

Consider the distribution of relays follows a terrain according to its coverage probability, which depends on the blockages and deployment constraints. Hence, the distribution is far from being spatially uniform. Such conditions are clearly distinct from the random and uniformly distributed network assumptions that lead to a Poison number of nodes per unit area $i.e.$, the PPP model – commonly adopted in current literature [12]–[14]. Some recent works such as [25], [26] focus on the impact of topological models on random networks. To elaborate, in [25] HCPPs are proposed to model networks with carrier-sensing multiple access (CSMA) techniques, and in [26] the coverage probability of cellular systems are analyzed under PPP, HCPP and Strauss Process (SP) models. These models are further compared against field data, which demonstrate that indeed HCPP and SP lead to significantly more accurate results than the PPP model commonly used earlier. All in all, it is now an established fact that as far as the topological models for random networks are concerned, the PPP alone is not sufficient, and hence alternative models need to be considered. Motivated by such recent results [25]–[27], we consider the Matérn HCPP model in order to characterize the distribution of active relays in the following analysis[4].

Since we model the distribution of relays in our network with a MHCPP, it is worthwhile to mention here some properties of MHCPP. In the MHCPP Type I, all the points obtained from a stationary PPP of intensity λ_p are retained if and only if they are at a distance of at least d from all other points. Whereas, in MHCPP Type II model, points are obtained by deleting the primary points that co-exist within a distance less than the hard core distance from another primary point having a lower mark.

For a MHCPP model, which is generated from a homogenous PPP, Φ_p, with intensity λ_p and repulsive distance d, the intensity λ_m of the MHCPP is given by [25], [27]

$$\lambda_m = \frac{1 - \exp\left(-\lambda_p \pi d^2\right)}{\pi d^2}. \tag{14}$$

Consequently, the probability of a point being retained from Φ_p is

$$\mathcal{P} = \frac{\lambda_m}{\lambda_p} = \frac{1 - \exp\left(-\lambda_p \pi d^2\right)}{\lambda_p \pi d^2}. \tag{15}$$

These hardcore models (Type I and Type II) of point processes are not directly applicable to fading and blockage environments. This is due to the fact that the density of active number of nodes depends on random fading gains and blockage processes. To tackle the impact of fading, [27] extends the hard core process analysis for the case of Rayleigh fading and [25] derives the active number of transmitters under generalized fading channel by employing MHCPP Type II model. In this paper, we leverage the results from [25] and incorporate additional blockage effects. It is a well known fact that the characterization of non-PPP models (general topologies) via the Laplace Functional and probability generating

[4]The HCPP is considered in this paper to find the density of active relays only.

functionals is in reality a challenging problem. Therefore, the hard-core point processes are quite difficult to analyze due to the simple reason that their probability generating functionals do not exist [27]–[29]. However, it has been argued in [27], [28] that the nodes further away from the hard core distance, d can still be modeled as a PPP. Furthermore, it has been shown in [29] that MHCPP type II is better approximated with a PPP rather than Type I. Hence, we take into account such an approximation for analytical tractability and consider that the distribution of relay nodes follows a PPP, while the density of the relay nodes is approximated by that of a modified hard-core PPP with density $\bar{\lambda}_{\mathrm{R}}$.

B. Density of Active Relays

In this subsection, we aim to find the intensity of active relays by generalizing the traditional MHCPP for blockage environments in mmWave. To overcome underestimation flaw, in [25], authors made an assumption of a bounded region, a circle with a deterministic radius, where the nodes contribute to the event of interest. In our model, the contribution of each relay node to the event of interest will be Bernoulli distributed with a parameter that accounts for both shadowing and blockage process. The procedure to find active density of relay nodes follows similar steps as in [25]. However, the neighborhood success probability varies due to the addition of blockage process in our system.

Let Φ_{R} be the primary point process and $\bar{\Phi}_{\mathrm{R}}$ be the generalized MHCPP. In order to generalize the traditional MHCPP with respect to SNR, the hard-core distance d is replaced with the received SNR. A relay node is retained in $\bar{\Phi}_{\mathrm{R}}$ if and only if it has the lowest mark in its neighborhood set of relays $N(x_i)$ which is determined by dynamically changing the random-shaped region defined by instantaneous path gains.

Lemma 5: Let $\mathcal{P}_\zeta$ be the neighborhood success probability. Now, if the retaining probability of a relay node is $\mathcal{P}_{\mathrm{R}} = \frac{1-e^{-N\mathcal{P}_\zeta}}{N\mathcal{P}_\zeta}$ with the expected number of nodes in the disc N, then the intensity of active number of relays is given by $\bar{\lambda}_{\mathrm{R}} = \lambda_{\mathrm{R}}\mathcal{P}_{\mathrm{R}}$ [25, Theorem 4.1].

Therefore, in order to find the retaining probability, $\mathcal{P}_{\mathrm{R}}$ in Lemma 5, one must compute the neighborhood success probability, $\mathcal{P}_\zeta$. As mentioned earlier, the neighborhood set of any relay node is determined by bounding the observation region by $\mathcal{B}_{x_i}(r_d)$, where r_d is a sufficiently large distance, such that the probability for a relay located beyond r_d to become a neighbor of x_i is a very small number, ϱ. Therefore,

$$\mathbb{P}\left\{\frac{P\left(G^{\max}\right)^2\mathcal{X}_{\mathcal{N}}}{||x_i-x_j||^\alpha} > \gamma_{\mathrm{R}} \,|\, ||x_i-x_j|| > r_d\right\} \leq \varrho, \quad (16)$$

where γ_{R} is the minimum required target SNR.

Hence, r_d can be determined as

$$r_d = \left(\frac{P\left(G^{\max}\right)^2}{\gamma_{\mathrm{R}}} F^{-1}_{\mathcal{X}_{\mathcal{N}}}(\varrho)\right)^{1/\alpha}, \quad (17)$$

where, F^{-1} denotes the inverse of the CDF of $\mathcal{X}_{\mathcal{N}}$.

Then the neighborhood success probability within the bounded region can be defined as

$$\mathcal{P}_\zeta = \mathbb{P}\{\gamma_{x_i,x_j} \geq \gamma_{\mathrm{R}} | x_j \in \mathcal{B}_{x_i}(r_d)\}. \quad (18)$$

Therefore, considering blockages (18) can be written as

$$\begin{aligned}\mathcal{P}_\zeta &= \sum_{i\in\text{LOS,NLOS}} p_i \int_0^{r_d} \left(1 - F_{\mathcal{X}_{\mathcal{N}}}\left(\frac{\gamma_{\mathrm{R}} r^{\alpha_i}}{P\left(G^{\max}\right)^2}\right)\right) r\mathrm{d}r,\\ &= \sum_{i\in\text{LOS,NLOS}} p_i \int_0^{r_d} \left[1 - Q\left(\frac{\log\left(\frac{\gamma_{\mathrm{R}} r^{\alpha_i}}{P\left(G^{\max}\right)^2}\right) - \mu_c}{\sigma_c}\right)\right] r\mathrm{d}r,\end{aligned} \quad (19)$$

where, $Q(.)$ is the cumulative distribution function of the standard normal distribution. A closed form expression for $\mathcal{P}_\zeta$ is given in (20) on the top of next page.

Using (20), we can derive the generalized MHCPP process of the relays and their active nodes which can withstand the blockage effects in the network to transfer the information with less outage probability. In practical scenarios, selecting a relay from an observation (or defined) region with a small neighborhood set of relays is optimal. Since the computational complexity increases with number of relays, a carefully designed region can be taken into consideration.

From the above analysis, it is clear that the achievable capacity of relay assisted link depends on the distance between the relay and the reference point. Assume that our communication is taking place within radius r_d, then source-destination pair should select the optimal relay with distance less than r_d. In the subsequent section, we discuss relay selection techniques based on the best end-to-end SNR[5] and minimum path loss.

Here, we follow two strategies for tractable analysis, namely, random relay and best relay while taking into consideration the blockage effects. The random relay selection technique is used to capture the blockage effects on the performance of active set of relays while, the best relay selection is studied in order to weigh on the trade off between performance and complexity of random relay selection techniques in mmWave networks.

V. SNR ANALYSIS OF THE RELAY SCHEMES

In this section, we analyze the SNRs of two relay selection techniques in order to determine the best technique suitable for a mmWave communication. In the first technique we select the path with the best SNR from a set of random paths. The random paths can be looked upon as the end-to-end SNR from the source to the destination through the aid of relay. In the second case we select the best relay first based on the minimum path loss and then use that relay to transmit the signal to the destination from the source.

A. Best Path Selection

In this subsection, in order to select any random path, we first select a random relay and then compute the end-to-end

[5] The end-to-end SNR signifies the total SNR from source to the destination through the aid of relay using amplify and forward technique [13].

$$\mathcal{P}_\zeta = \frac{1}{4}{r_d}^2 \sum_{i\in\{LOS,\, NLOS\}} p_i \left[-\exp\left(\frac{2\left(\alpha_i\mu_c+\sigma_c^2\right)}{\alpha_i^2}\right)\left(\frac{\gamma_R}{P(G^{\max})^2}r_d^{\alpha_i}\right)^{\frac{-2}{\alpha_i}} \operatorname{erf}\left(\frac{\alpha_i\mu_c-\alpha\log\left(\frac{\gamma_R}{P(G^{\max})^2}r_d^{\alpha_i}\right)+2\sigma_c^2}{\sqrt{2}\alpha_i\sigma_c}\right) + \operatorname{erf}\left(\frac{\mu_c-\log\left(\frac{\gamma_R}{P(G^{\max})^2}r_d^{\alpha_i}\right)}{\sqrt{2}\sigma_c}\right)\right]. \tag{20}$$

SNR[6] distribution of that path. Subsequently, we select the path with the best SNR distribution from an asymptotic point of view (when the number of links tend to infinity in a dense network) by using extreme value theory.

As stated before, any node can receive a signal either through LOS or NLOS link. We now compute the SNR distribution accounting for both the LOS and NLOS links. Thus the achievable SNR between the source and the destination can be given as[7]

$$\gamma_{\text{SD}} = \gamma_{\text{SD}}^{\text{LOS}} p_{\text{LOS}} + \gamma_{\text{SD}}^{\text{NLOS}} p_{\text{NLOS}}, \tag{21}$$

where $\gamma_{\text{SD}}^{\text{LOS}}$ and $\gamma_{\text{SD}}^{\text{NLOS}}$ are the LOS and NLOS SNRs respectively for the links from source to destination and p_{LOS} and p_{NLOS} are the probabilities that the links are LOS and NLOS respectively. Similarly, the achievable SNR between the source and relay and the relay and destination are given respectively as

$$\gamma_{\text{SR}} = \gamma_{\text{SR}}^{\text{LOS}} p_{\text{LOS}} + \gamma_{\text{SR}}^{\text{NLOS}} p_{\text{NLOS}} \text{ and} \tag{22}$$

$$\gamma_{\text{RD}} = \gamma_{\text{RD}}^{\text{LOS}} p_{\text{LOS}} + \gamma_{\text{RD}}^{\text{NLOS}} p_{\text{NLOS}}. \tag{23}$$

Considering the LOS regime, the SNR distribution can be formulated as

$$\begin{aligned} F_{\gamma_{SD}^{\text{LOS}}}(z) &= \mathbb{P}\left\{\frac{P\,G^{\max}\mathcal{X}_\mathcal{N}}{r^{\alpha_0}N_0} < z\right\}, \\ &= \mathbb{P}\left\{\mathcal{X}_\mathcal{N} < \frac{zr^{\alpha_{\text{LOS}}}N_0}{P\,G^{\max}}\right\}, \\ &= Q\left(\frac{\log\left(\frac{zN_0r^{\alpha_{\text{LOS}}}}{P\,G^{\max}}\right)-\mu_{\text{SD}}^{\text{LOS}}}{\sigma_{\text{SD}}^{\text{LOS}}}\right). \end{aligned} \tag{24}$$

Using Lemma 3, the distribution of $\gamma_{\text{SD}}^{L} p_{\text{LOS}}$ can now be expressed as

$$F_{\gamma_{SD}^{\text{LOS}}}(z) = Q\left(\frac{\log\left(\frac{zr^{\alpha_i}}{P\,G^{\max}}\right)-(\mu_{\text{SD}}^{\text{LOS}}+p_{\text{LOS}})}{\sigma_{\text{SD}}^{\text{LOS}}}\right). \tag{25}$$

Similarly the $\gamma_{\text{SD}}^{\text{NLOS}}$ can be characterized. Therefore, now the total SNR can be calculated using equation (21). However, $\gamma_{\text{SD}}^{\text{LOS}}$ and $\gamma_{\text{SD}}^{\text{NLOS}}$ are two independent log-normally distributed variables with different μ and σ parameters. In this scenario, the distribution of the total SNR γ_{SD} has no closed form expression, but it can be approximated by another log-normal distribution using Lemma 2 with parameters μ_{SD} and σ_{SD}^2.

In order to capture the blockage effects on both sides of relay (Source-to-Relay and Relay-to-Destination), we consider the end-to-end SNR to find the path with the best SNR distribution.

For practical systems, the relay gain is given by $\mathcal{G}^2 = (1/(P(G^{\max})^2\mathcal{X}_\mathcal{N}r^{\alpha_i}+N_0))$. However, assuming the ideal relaying gain[8] i.e., $\mathcal{G}^2 = (1/(P(G^{\max})^2\mathcal{X}r^{\alpha_i}))$, the end-to-end SNR of the link through the aid of relay can now be given as [13], [30]

$$\hat{\gamma}_{\text{SRD}} = \frac{\gamma_{\text{SR}}\gamma_{\text{RD}}}{\gamma_{\text{SR}}+\gamma_{\text{RD}}}, \tag{26}$$

where the subscript SRD stands for the path from the source to the relay to the destination.

Proposition 1. *The end-to-end SNR $\hat{\gamma}_{\text{SRD}}$ is log-normally distributed with new parameters $\hat{\mu}_{\text{SRD}}$ and $\hat{\sigma}_{\text{SRD}}$.*

Proof. Let $X = \gamma_{\text{SR}}\gamma_{\text{RD}}$ and $Y = \gamma_{\text{SR}}+\gamma_{\text{RD}}$, then in order to prove that $Z = \frac{X}{Y}$ is log-normally distributed, it is sufficient to prove that Z is a log-normal random variable with parameters μ_Z and σ_Z. A detailed proof is given in Appendix A. □

Proposition 2. *Let $\bar{\gamma} = \max\{\hat{\gamma}_{\text{SRD}}\}$. Then the probability distribution of the best path from source to the destination which exhibits the maximum end-to-end SNR can be given as*

$$F_{\bar{\gamma}} = \prod_{i=1}^{n} F_{\hat{\gamma}_{\text{SRD}_i}} = (F_{\hat{\gamma}_{\text{SRD}_i}})^n, \tag{27}$$

where $n = K \times N$ gives the total number of paths available for a given K number of sources and N number of relays.

Proof. See Appendix B. □

Asymptotic analysis: We now investigate the asymptotic behavior of the distribution of the maximum SNR $\hat{\gamma}$ of the best relay path with the help of extreme value theory. This is to obtain insights into coverage in very dense networks. In general, extreme value theory is used to deal with extreme values, such as maxima or minima of distributions when the number of random variables increases asymptotically. Let φ_is be the realizations of a random variable $\bar{\varphi}$, where φ_is are independent and identically distributed with $i = 1, 2, \ldots, n$. By extreme value theory [31], if there exist constants $a \in \mathbb{R}, b > 0$, and some non-degenerate distribution function

[6] We would like to refer the readers to [13], [30] for an elaborate description on this technique.

[7] Since we model the links between the sources and the destination as LOS and NLOS which are independent of each other, we leverage the notion of mark from stochastic geometry to further split the Poisson Point Processes into two independent LOS and NLOS sub processes.

[8] The adoption of the ideal relaying gain is mainly for analytical tractability and can act as a tight upper bound for the practical relaying gain. This method is widely used in literature [13], [30] to approximate relay gains.

$F(k)$ such that the distribution of $\frac{\bar{\varphi}_{max}-a}{b}$ scales to $F(k)$, then $F(k)$ converges to one of the three standard extreme value distributions: Gumbel, Frechet and Weibull distributions, where $\bar{\varphi}_{max} = \max(\varphi_1, \varphi_2, \ldots, \varphi_n)$. There are only three possible non-degenerate limiting distributions for maxima, which can be expressed as

1) $F_1(k) = e^{-e^{-k}}, \quad -\infty < k < \infty$
2) $F_2(k) = e^{-k^{-\alpha}}u(k), \quad \alpha > 0$
3) $F_3(k) = \begin{cases} e^{-(-k)^{\alpha}}, & \alpha > 0,\ k \leq 0 \\ 1, & k > 0 \end{cases}$

where $u(k)$ is the step function.

Proposition 3. *Let $\bar{\gamma} = \max(\hat{\gamma}_{\mathrm{SRD}_1}, \hat{\gamma}_{\mathrm{SRD}_2}, \ldots, \hat{\gamma}_{\mathrm{SRD}_n})$ denote the maximum end-to-end SNR where $\hat{\gamma}_{\mathrm{SRD}_i}s$ are independent and identically distributed and $n \in \mathbb{Z}^+$. Then, the distribution of $\bar{\gamma}$, $\mathcal{F}_n$ converges to reduced type 1 asymptotic distribution, $F_1(k)$ given as*

$$\mathcal{F}_n(a_n k + b_n) = e^{(-e)^{(-k)}}, \tag{28}$$

where

$$a_n = \beta_n \sigma e^{\hat{\mu}_{\mathrm{SRD}} + \kappa_n \hat{\sigma}_{\mathrm{SRD}}} \tag{29}$$

and

$$b_n = e^{\hat{\mu}_{\mathrm{SRD}} + \kappa_n \hat{\sigma}_{\mathrm{SRD}}}, \tag{30}$$

with $\kappa_n = \frac{2\beta_n^2 - (2\log\beta_n - \log 2 + \log 4\pi)}{2\beta_n}$ and $\beta_n = \sqrt{2\log n}$.

Proof. The proof of this proposition follows from proposition 1 where it was proved that $\bar{\gamma}$ follows lognormal distribution. The distribution of $\bar{\gamma}$, $\mathcal{F}_n(k)$ belongs to the domain of attraction of the limiting distribution, if it results in one limiting distribution for extreme. The limit law for $\mathcal{F}_n(a_n k + b_n)$ when $\mathcal{F}(n)$ has the lognormal law is $F_1(k)$. This can be verified by ascertaining that the Von-Mises criterion is satisfied. The von Mises condition [31], [32] associated with the quantity $\bar{\gamma} = \max(\hat{\gamma}_{\mathrm{SRD}_1}, \hat{\gamma}_{\mathrm{SRD}_2}, \ldots, \hat{\gamma}_{\mathrm{SRD}_n})$ requires that

$$\lim_{k\to\infty} \frac{d}{dk}\left[\frac{1 - F_{\hat{\gamma}_{\mathrm{SRD}}}(k)}{f_{\hat{\gamma}_{\mathrm{SRD}}}(k)}\right] = 0, \tag{31}$$

which indicates that $\bar{\gamma}$ follows a Gumbel Distribution. Similarly, our result follows from [33], where it was also verified that the limit law for a distribution function when it follows lognormal law is of type $F_1(k)$. The parameters a_n and b_n are derived in Appendix C. □

B. Best Relay Selection

The motivation behind the use of best and random relay selection is to study the trade off between performance and complexity of relay selection techniques in mmWave networks. The active relays which can participate in the communication are the ones that are minimally affected by blockages. Such a relay with the least path loss can be considered to be the best relay.

Proposition 4. *The SNR distribution for the best relay can be given as*

$$F_{\gamma_{best}}(t) = \exp\left(-\sum_{i\in\mathrm{LOS,NLOS}} \frac{p_i}{\alpha_i} 2\pi\lambda \left(\frac{P(G^{\max})^2}{N_0}\right)^{\frac{2}{\alpha_i}} \times \int_t^{\infty} y^{\frac{-2}{\alpha_i}-1}\, \Xi_{\left(\frac{2}{\alpha_i}\right)}(y/r_d)\mathrm{d}y\right), \tag{32}$$

where $\Xi_j(y) = \exp(\sigma^2 j^2/2 + \mu j) Q\left(-\frac{\sigma^2 - \log(y) + \mu}{\sigma}\right)$ is the j-th truncated moment of $\mathcal{X}$.

Proof. See Appendix D. □

Hence, using the above proposition, we select the best relay from a set of active relays which are obtained as stated in section III. At this point it is worthwhile to mention that compared to the decode and forward relaying technique, the amplify and forward relaying may amplify the noise as well. Considering a NLOS condition (dense blockage environment), best relay scheme may not be suitable in amplify and forward systems as it will select the best among the worst channels and amplify the noise. In such a condition, decode and forward relay is advantageous over amplify and forward although it has higher complexity. Hence, here we use the decode and forward technique to transmit the signal from the relays[9].

VI. Coverage Probability and Transmission Capacity

The relays which are located at larger distances can suffer from large path loss and incur high maintenance costs. Thus, the relay selection method should be carefully designed in order to achieve higher coverage rates. In this section we analyze the performance of our system based on two performance metrics, namely the coverage probability and transmission capacity. The coverage probability is defined as the probability that the destination is able to receive a signal with some threshold SNR T, i.e., $P_c = \mathbb{P}[\gamma > T]$. That is, the probability of coverage is the complementary cumulative distribution function (CCDF) of the SNRs over the network. On the other hand, the transmission capacity of a network can be defined as the achievable rate of successful transmission per unit area, given the constraints of certain connection outage. This metric is of interest since the characterization of the capacity of every individual active link in a large random network is impractical. Mathematically, the transmission capacity of a relay aided system is defined as

$$\tau = (\lambda_{\mathrm{R}} + \lambda_{\mathrm{S}})\mathcal{R}(1-\epsilon), \tag{33}$$

where $\mathcal{R}$ is the rate of a random end-to-end link defined as

$$\mathcal{R} = \log_2(1+T)\mathbb{P}(\gamma > T), \tag{34}$$

where T is the minimum threshold SNR. $\mathbb{P}(\gamma > T)$ follows from Proposition 3 and Proposition 4 depending on the relay selection scheme. However, for the case of decode and forward technique, the average rate, $\mathcal{R}$ is calculated as in [9].

[9]We would like to refer the readers to [9]–[11] for an elaborate description on this technique.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

VII. Simulation Results

In this section, we validate the system model and also verify the results mentioned in the propositions. In general, the computations are done through Monte Carlo simulations which is then used to validate the analytical results[10].

We consider the mmW bandwidth of 2 GHz and carrier frequency 73 GHz. Unless stated otherwise, most of the values of the parameters used are inspired from literature mentioned in the references [2], [3]. For the system guidelines, we mention these parameters and their corresponding values in Table I.

Fig. 3 shows the variation of the active number of relays with respect to the intensity of the relays before thinning for different blockage outage probabilities. In order to find the active number of relays, we need to find the retaining probability which can be evaluated by (20). For a given blockage probability and given density of the relays, one can identify the required number of active relays in order to meet the transmission requirement in the mmWave network.

In Fig. 4 we show the comparison of the coverage probability of the SNR among three links, namely the direct link between the source and the destination, the best path link from the source to the destination through the aid of relays and any random path link which also takes relay into consideration. It is evident from the figure that relay aided transmission has a better coverage probability when compared to a direct link between a source and a destination. It can also be seen that the best end-to-end link has a better coverage probability compared to any random link. Furthermore, we would like to stress on the fact that there is a steep fall on the coverage probability due to the shadowing effects caused by blockages.

Fig. 5 and Fig. 6 show the coverage probability for LOS and NLOS relay links respectively. The LOS scenario arises when we consider that all NLOS links are completely attenuated due to blockages and vice versa. In other words the path loss exponent for such links is very large for the respective scenarios and hence these links can be ignored when calculating the coverage probability of the system. The direct link from source to destination without the aid of the relay for NLOS is shown in the figure just for the sake of comparison. It is evident from the figures that relay aided transmission has better coverage probability to a direct link between the source and destination.

TABLE I: Simulation Parameters

Notation	Parameter	Values
r_d	Radius of the bounded region	200 meters
λ_s	Density of source nodes	0.001
p_{LOS}	LOS probability	0.12
$G_{\max}$	Antenna Gain	18dB
α	Path loss exponent	LOS-2, NLOS-4
P	Node transmit power	1 Watt
N_0	Noise power	Thermal noise + 10dB noise figure.
r_{SD}	Link distance	35 meter

[10]The parameters considered for simulation in this paper have been taken from recent mmWave studies [1], [3], [5].

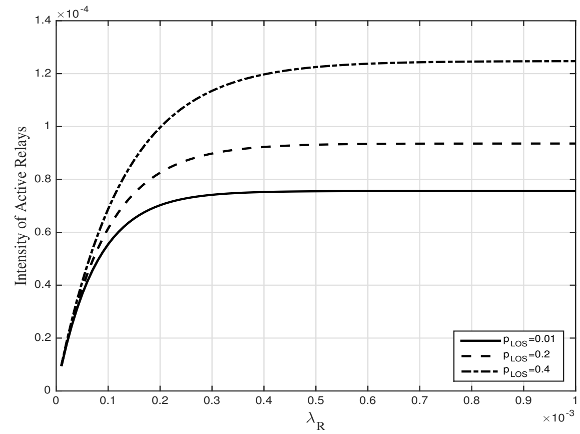


Fig. 3: Intensity of active relays versus λ_R. The minimum required target SNR was kept at 5dB.

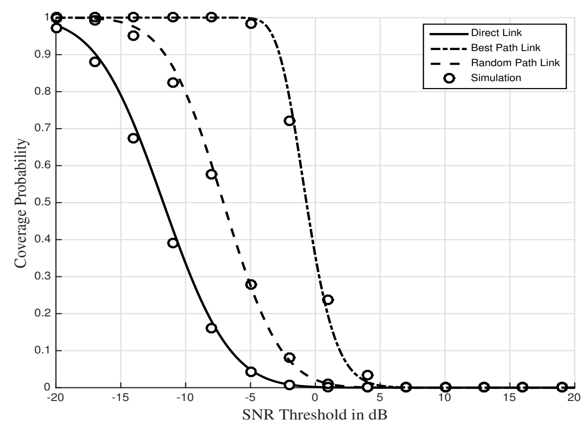


Fig. 4: Comparison of the SNR coverage among the direct link, best path link and any random link from the source to the destination.

In Fig. 7 we give insights into the coverage probability of the system in very dense networks. This figure is an attempt at validating Proposition 3 where we state that when the number of SNR links tend to infinity the distribution tends toward the non-degenerate limiting distribution $F_1(k)$. From the figure it can be seen that as we increase the value of n, the curves converges towards the asymptotic curve which represents the Gumbel distribution. Increasing n can be looked upon as increasing the density of the nodes which in turn increases the coverage probability of the system.

Fig. 8 shows the trade offs of the coverage probability of the SNR among three links, namely the direct link, the best path link and the best relay link. It can be seen from the figure that the best relay transmission scheme out performs the other two links. However, the best relay scheme has a high implementation complexity, since it requires high signalling overheads and channel state information from all potential relays. For systems with limited computational capabilities, the best path link is a viable option, but at the expense of reduced coverage.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

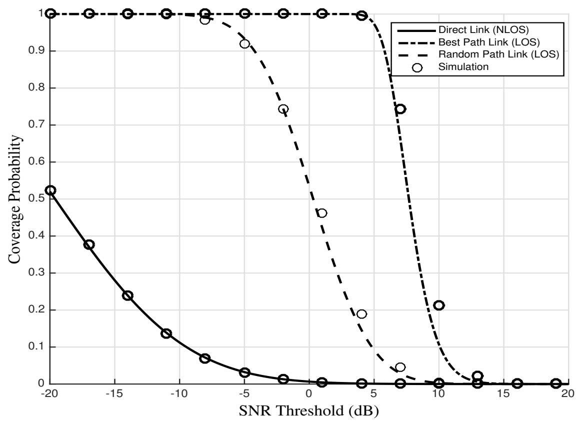


Fig. 5: Comparison of the SNR coverage between the best path link and any random link from the source to the destination for LOS scenario.

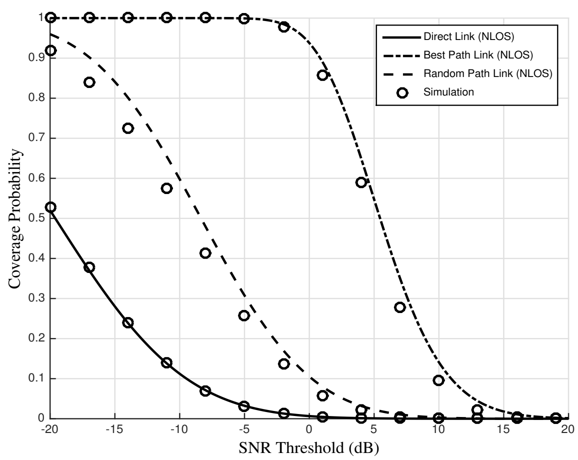


Fig. 6: Comparison of the SNR coverage between the best path link and any random link from the source to the destination for NLOS scenario.

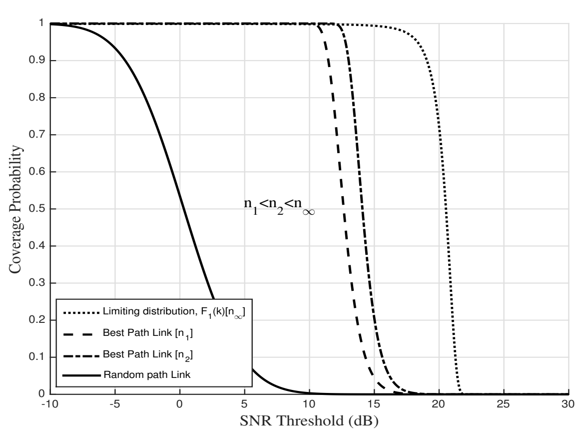


Fig. 7: Comparison of the SNR coverage among the best path links when the number of links increase asymptotically.

Fig. 9 gives the comparison between our model and a general blockage model for e.g., the ones considered in [2],

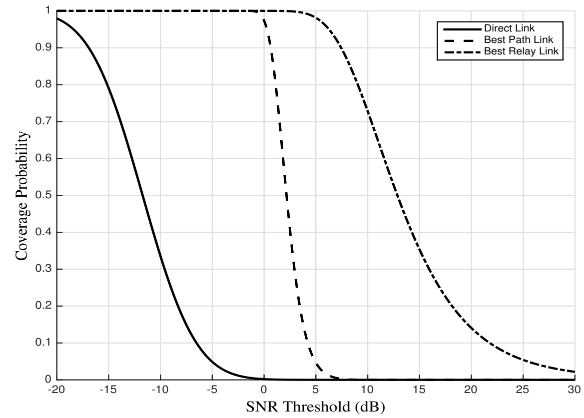


Fig. 8: Coverage probability comparison between the direct link, the best path link and best relay link from the source to the destination.

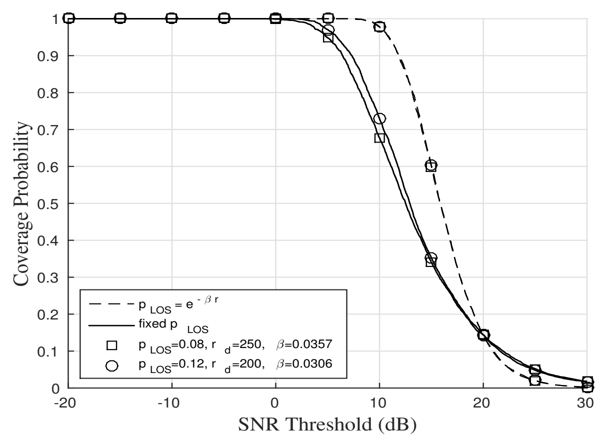


Fig. 9: Coverage probability comparison of different blockage models under best relay strategy.

[22]. It is evident from the figure that for a given relay and blockage density, performance gap of the coverage probability considering the best relay strategy between our model (fixed p_{LOS}) and the $e^{-\beta r}$ model [2] is minimal. This is comparable to the model considered in [2], [22]. We note that the adoption of step function in our analysis enables faster calculations of the coverage probability, as it simplifies expressions for the evaluation of the numerical integrals. In dense mmWave networks, the error due to such an approximation (LOS step model) is generally small and simplifies the dense network analysis. The step function approximation generally provides a lower bound of the SINR distribution corresponding to $e^{-\beta r}$ blockage model and the errors due to the approximation become smaller when the base station density increases.

In Fig. 10 we compare the transmission capacity between the direct link and the best path link from the source to the destination generated through the aid of relays. In this case, we have considered the low complexity case of the best path link. The figure shows the existence of an optimal SNR threshold

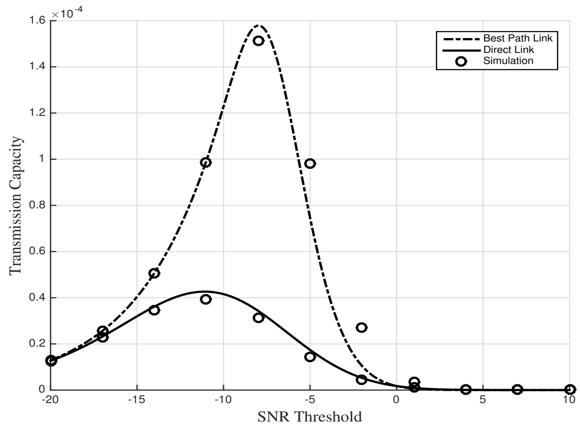


Fig. 10: Transmission capacity comparison between the direct link and the best path link from the source to the destination generated through the aid of relays.

which depends on the operating conditions of the network. The convexity of the curve can be understood from Fig. 3 where it was seen that the active number of relays reach a saturation point after a certain density. Hence, it is quite obvious for the transmission capacity to reach a optimal point.

VIII. Conclusion and Future Work

Blockages can be quite detrimental to the performance of outdoor mmWave networks. A possible fix for this is to go around the blockages by creating alternative propagation paths with the aid of relays. Accordingly, potential benefits of deploying relays in outdoor mmWave networks were investigated in this paper. Coverage probability from sources to a destination aided by relays which were modeled as independent PPPs were studied. By considering blockages in mmWave network, a relay modeling technique was given. New relay nodes from a set of relays were derived using generalized MHCPP. These active nodes are the ones that can withstand the blockage effects in the network to transfer information with less outage probability. In practical scenarios, selecting a relay from an observation (or defined) region with a small neighborhood set of relays is quite optimal. Since the computational complexity increases with the number of relays, a carefully designed region can be taken into consideration. Relay aided transmission was seen to improve the SNR by around 5dB for a specific coverage probability. Furthermore, closed form expression for end-to-end signal to noise ratio (SNR) was provided along with the computation of the best random relay path using order statistics. In very dense networks, the number of links can be quite large. To investigate such a scenario, extreme value theory was used to analyze the maximum end-to-end SNR of random relay paths. It is quite evident from our analysis that the use of relays can be quite instrumental in increasing the coverage probability and transmission capacity of mmWave networks.

We also would like to highlight that a simplified framework was presented in this paper to make it possible for the initial analysis of relay aided mmWave transmission schemes, while gaining useful insights into how to design future mmWave cellular networks to attain higher throughput rates. Certain factors such as load distribution, that deters the performance of relay aided systems were not considered in this paper. Load distribution in mmWave relays can be handled in several ways such as: partial CSI [34] instead of full CSI, distributed relaying schemes [35] and a less centralized relay scheduling scheme [36] may be used. This will however require the design of specific beamformers for specific CSI requirements. The consideration of other realistic system parameters may change the comparison results and similar analysis could potentially be applied to the deterministic equivalent frameworks considering time-varying channels and overhead due to training of pilots. This work can possibly be a very good foundation for future works where more anomalies and impairments will be considered in order to make the comparison more thorough and exact. Another direction of future work is to not only compare additional blockage models but also design new blockage models that can take into consideration an augmented number of geographical locations.

Appendix A
Proof of Proposition 1

From Lemma 1, we have

$$X \sim \log\mathcal{N}(\mu_X, \sigma_X^2), \tag{35}$$

where

$$\mu_X = \mu_{\mathrm{SR}} + \mu_{\mathrm{RD}}, \tag{36}$$
$$\sigma_X^2 = \sigma_{\mathrm{SR}}^2 + \sigma_{\mathrm{RD}}^2. \tag{37}$$

Using Lemma 2, Y can be tightly approximated with another log-normal random variable with parameters

$$\mu_Y = \ln\Big[\sum e^{\mu_j + \sigma_j^2/2}\Big] - \frac{\sigma_{\mathrm{SD}}^2}{2}, \tag{38}$$
$$\sigma_Y^2 = \ln\left[\frac{\sum e^{2\mu_j+\sigma_j^2}(e^{\sigma_j^2}-1)}{(\sum e^{\mu_j+\sigma_j^2/2})^2} + 1\right]. \tag{39}$$

Again, using Lemma 1, the distribution of $\bar{\gamma}_R = \frac{X}{Y}$ can be given as another log normal variable which is the required result.

Appendix B
Proof of Proposition 2

Let $F_Y(y)$ denote the CDF of Y, then the CDF of the maximum of identically distributed random variables $X_1, X_2, \cdots, X_n$ can be given as

$$F_Y(y) = \mathbb{P}\{Y < y\} = \mathbb{P}\{x_1 < y, x_2 < y \cdots x_n < y\} \tag{40}$$

Therefore, $F_Y(y)$ can be obtained using order statistics [37] as follows

$$F_Y(y) = \mathbb{P}\{Y < y\} = \prod_{k=1}^{n} \mathbb{P}\{x_k < y\} = (F_{X_k}(y))^n. \tag{41}$$

Proposition 2 thus follows from (41). Furthermore, the parameters K and N can be computed from the mean of the expected number of source and relay nodes.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSTSP.2015.2509951, IEEE Journal of Selected Topics in Signal Processing

Mean of Expected Number of Source Nodes: For given values of propagation parameters in bounded region, one can obtain the expected number of source nodes present in the communication vicinity by describing the propagation process. Let $\Phi_{\mathrm{S}} = \{\frac{r^{\alpha_i} N_0}{\mathcal{X} P(G^{\max})^2}, r \in \phi\}$ be the path loss process, where $i \in \{LOS,\ NLOS\}$. Then the expected number of nodes can be given as

$$\Lambda_{\mathrm{S}}((0,t]) = 2\pi\lambda_{\mathrm{S}} \int_{\mathbb{R}^+} \mathbb{P}\left\{\frac{r^{\alpha_i} N_0}{\mathcal{X}_{\mathcal{N}} P(G^{\max})^2} < t\right\} r\mathrm{d}r \quad (42)$$

The closed form expression for the above integral follows as in [3]. The mean of the expected number of the relay nodes follows similarly with density $\bar{\lambda}_{\mathrm{R}}$.

Appendix C
Proof of Proposition 3

In order to evaluate the constant a_n and b_n we first define $\bar{\xi}_n = (\log \bar{\gamma}_n - \hat{\mu}_{\mathrm{SRD}})\hat{\sigma}_{\mathrm{SRD}}$, where $\hat{\mu}_{\mathrm{SRD}}$ and $\hat{\sigma}_{\mathrm{SRD}}$ follows from Proposition 1. We also define $\zeta_n = n[1 - F_{\hat{\xi}}(\hat{\xi}_n)]$, where $\hat{\xi}_i$ is a realization of $\bar{\xi}$ with $i \in \mathbb{Z}^+$ and $F_{\hat{\xi}}(\hat{\gamma}_{\mathrm{SRD}}) = \int_{-\infty}^{\hat{\gamma}_{\mathrm{SRD}}} (2\pi)^{\frac{-1}{2}} e^{(\frac{-\hat{\gamma}_{\mathrm{SRD}}^2}{2})} \mathrm{d}\hat{\gamma}_{\mathrm{SRD}}$.

Now, we have from [38] that as $n \to \infty$, $\bar{\xi}_n = \kappa_n - \beta_n \log \zeta$, where $\kappa_n = \frac{2\beta_n^2 - (2\log\beta_n - \log 2 + \log 4\pi)}{2\beta_n}$ and $\beta_n = \sqrt{2\log n}$. Also, $\mathbb{P}\{\zeta \leq u\} = 1 - e^{-u},\ u \geq 0$. Therefore,

$$\mathbb{P}\{\bar{\xi}_n \leq \hat{\xi}\} = e^{-e^{\frac{-(\hat{\xi}-\kappa_n)}{\beta_n}}} \quad \text{for } \infty < \hat{\xi} < \infty. \quad (43)$$

Now, from the definition of $\bar{\xi}_n$ we have

$$\mathbb{P}\{\bar{\gamma}_n \leq \hat{\gamma}_{\mathrm{SRD}}\} = e^{-\left(\hat{\gamma}^{\frac{-1}{\beta_n \hat{\sigma}_{\mathrm{SRD}}}} e^{\left(\frac{-(\hat{\mu}_{\mathrm{SRD}})}{\beta_n \hat{\sigma}_{\mathrm{SRD}}} + \frac{\kappa_n}{\beta_n}\right)}\right)} \quad \text{for } \hat{\gamma}_{\mathrm{SRD}} \geq 0. \quad (44)$$

Let k_i be a realization of a new random variable ψ_n. Then, defining

$$\psi_n = \epsilon_n(\bar{\gamma}_n - 1), \quad (45)$$

where $\epsilon_n = \frac{1}{\beta_n \hat{\sigma}_{\mathrm{SRD}}}$, we have

$$\begin{aligned} \mathbb{P}\{\psi_n < k\} &= \mathbb{P}\left\{\bar{\gamma}_n < 1 + \frac{k}{\epsilon_n}\right\} \\ &= e^{\left\{-\left(1+\frac{k}{\epsilon_n}\right)^{-\epsilon_n} e^{\left(\frac{\hat{\mu}_{\mathrm{SRD}}}{\beta_n \hat{\sigma}_{\mathrm{SRD}}} + \frac{\kappa_n}{\beta_n}\right)}\right\}}. \end{aligned} \quad (46)$$

Also, for $-\infty < k < \infty$, we have

$$\mathbb{P}\left\{\bar{\gamma}_n \leq e^{(\hat{\mu}_{\mathrm{SRD}} + \kappa_n \hat{\sigma}_{\mathrm{SRD}})}\left(1 + \frac{k}{\epsilon_n}\right)\right\} = e^{\left\{-\left(1+\frac{k}{\epsilon_n}\right)^{-\epsilon_n}\right\}}. \quad (47)$$

Now, as $n \to \infty$, $\epsilon_n \to \infty$. Therefore,

$$\lim_{n\to\infty} \mathbb{P}\left\{\bar{\gamma}_n \leq e^{\left(\frac{\hat{\mu}_{\mathrm{SRD}}}{\beta_n \hat{\sigma}_{\mathrm{SRD}}} + \frac{\kappa_n}{\beta_n}\right)^{\frac{1}{\epsilon_n}}}\left(1 + \frac{k}{\epsilon_n}\right)\right\} = e^{-e^{-k}}. \quad (48)$$

Hence, the constants a_n and b_n can respectively be identified from (48) as

$$a_n = \beta_n \hat{\sigma}_{\mathrm{SRD}} e^{\hat{\mu}_{\mathrm{SRD}} + \kappa_n \hat{\sigma}_{\mathrm{SRD}}} \quad (49)$$

and

$$b_n = e^{\hat{\mu}_{\mathrm{SRD}} + \kappa_n \hat{\sigma}_{\mathrm{SRD}}}. \quad (50)$$

Appendix D
Proof of Proposition 4

Let $\Phi = \left\{x_i = \frac{P(G^{\max})^2}{N_0} r^{-\alpha_i}\right\}$ be path gain process, where $i \in \{\mathrm{LOS},\ \mathrm{NLOS}\}$. By using Mapping theorem [39], the density function under the effect of blockages can be given as

$$\lambda(x) = \sum_{i \in \mathrm{LOS,NLOS}} \frac{p_i 2\pi\lambda}{\alpha_i} \left(\frac{P(G^{\max})^2}{N_0}\right)^{\frac{2}{\alpha_i}} x^{\frac{-2}{\alpha_i} - 1}. \quad (51)$$

Since our propagation process Φ is also effected by shadowing, using the displacement theorem [39], the updated density in bounded region can be given as

$$\hat{\lambda}(y) = \int_0^{r_d} \lambda(x)\rho(x,y)\ \mathrm{d}x, \quad (52)$$

where

$$\rho(x,y) = \frac{\mathrm{d}}{\mathrm{dy}}(1 - F_{\mathcal{X}_{\mathcal{N}}}(y/x)) = -\tfrac{y}{x^2} f_{\mathcal{X}_{\mathcal{N}}}(y/x). \quad (53)$$

Thus (52) becomes

$$\begin{aligned} \hat{\lambda}(y) &= \sum_{i \in \mathrm{LOS,NLOS}} \frac{p_i}{\alpha_i} \int_0^{r_d} 2\pi\lambda \left(\frac{P(G^{\max})^2}{N_0}\right)^{\frac{2}{\alpha_i}} x^{\frac{-2}{\alpha_i}-1} \rho(x,y)\,\mathrm{d}x \\ &= \sum_{i \in \mathrm{LOS,NLOS}} \frac{p_i}{\alpha_i} \int_0^{r_d} 2\pi\lambda \left(\frac{P(G^{\max})^2}{N_0}\right)^{\frac{2}{\alpha_i}} x^{\frac{-2}{\alpha_i}-1} f_{\mathcal{X}}(y/x)\tfrac{1}{x}\ \mathrm{d}x \\ &\overset{(z=\frac{y}{x})}{=} \sum_{i \in \mathrm{LOS,NLOS}} \frac{p_i}{\alpha_i} 2\pi\lambda \left(\frac{P(G^{\max})^2}{N_0}\right)^{\frac{2}{\alpha_i}} y^{\frac{-2}{\alpha_i}-1} \int_{y/r_d}^{\infty} z^{\frac{2}{\alpha_i}} f_{\mathcal{X}}(z)\,\mathrm{d}z. \end{aligned}$$

Using the void probability of a PPP, the path gain distribution for best relay in interval of (t,∞) can thus be given as

$$\begin{aligned} F_{\gamma_{best}}(t) &= \exp\left(-\int_t^{\infty} \hat{\lambda}(y)\mathrm{dy}\right) \\ &= \exp\left(-\sum_{i \in \mathrm{LOS,NLOS}} \frac{p_i}{\alpha_i} 2\pi\lambda \left(\frac{P(G^{\max})^2}{N_0}\right)^{\frac{2}{\alpha_i}} \right. \\ &\qquad \left. \times \int_t^{\infty} y^{\frac{-2}{\alpha_i}-1} \int_{y/r_d}^{\infty} z^{\frac{2}{\alpha_i}} f_{\mathcal{X}}(z)\,\mathrm{d}z\mathrm{dy}\right). \end{aligned} \quad (54)$$

References

[1] A. Ghosh, T. N. Thomas, M. C. Cudak, R. Ratasuk, P. Moorut, F. W. Vook, T. S. Rappaport, G. R. MacCartney, S. Shun, and S. Nie, "Millimeter-wave enhanced local area systems: A high data-rate approach for future wireless networks," *IEEE J. Select. Areas Commun.*, vol. 32, no. 6, pp. 1153–1163, June 2014.

[2] A. Thornburg, T. Bai, and R. W. Heath, "Performance analysis of mmWave ad hoc networks," can be found at http://arxiv.org/pdf/1412.0765v1.pdf, 2014.

[3] S. Singh, M. N. Kulkarni, A. Ghosh, and J. G. Andrews, "Tractable model for rate in self-backhauled millimeter wave cellular networks," *IEEE J. Select. Areas Commun.*, vol. 33, no. 1, pp. 2196–2211, Jan. 2015.

[4] M. Akdeniz, Y. Liu, S. Rangan, and E. Erkip, "Millimeter wave picocellular system evaluation for urban deployments," in *Proc. IEEE Global Telecommunications Conference (Globecom'13)*, Dec. 2013, pp. 105–110.

[5] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Shun, S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter-wave channel modeling and cellular capacity evaluation," *IEEE J. Select. Areas Commun.*, vol. 32, no. 6, pp. 1164–1179, June 2014.

[6] S. Rajagopal, S. Abu-Surra, and M. Malmrichegini, "Channel feasibility for outdoor non-line-of-sight mmWave mobile communication," in *Proc. IEEE Vehicular Technology Conference (VTC'12 Fall)*, 2012, pp. 1–6.

[7] G. R. MacCartney and T. S. Rappaport, "73GHz millimeter wave propagation measurements for outdoor urban mobile and backhaul communications in New York city," in *Proc. IEEE International Conference on Communications*, Sydney, Austrlia, June 2014, pp. 2429–2433.

[8] S. Nie, G. R. MacCartney, S. Shun, and T. S. Rappaport, "72 GHz millimeter wave indoor measurements for wireless and backhaul communications," in *Proc. IEEE 24^{th} International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'13)*, Sept. 2013, pp. 2429–2433.

[9] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory.*, vol. 50, no. 12, pp. 3062–3080, 2004.

[10] M. Renzo, F. Graziosi, and F. Santucci, "A comprehensive framework for performance analysis of dual-hop cooperative wireless systems with fixed-gain relays over generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 5060–5074, Oct. 2009.

[11] S. W. Peters, A. Y. Panah, K. T. Truong, and R. W. Heath, "Relay architectures for 3GPP LTE-advanced," *EURASIP Journal on Wireless Communications and Networking*, p. 14pages, Jul. 2009.

[12] M. D. Renzo and W. Lu, "End-to-end error probability and diversity analysis of AF-based dual-hop cooperative relaying in a Poisson field of interferers at the destination," *IEEE Trans. Wireless Comm.*, vol. 4, no. 1, pp. 15 – 32, Jan. 2015.

[13] A. Behnad, A. M. Rabiei, and N. C. Beaulieu, "Performance analysis of opportunistic relaying in a Poisson field of amplify-and-forward relays," *IEEE Trans. Communications.*, vol. 61, no. 1, pp. 97–107, Jan. 2013.

[14] W. Lu and M. D. Renzo, "Performance analysis of relay aided cellular networks by using stochastic geometry," in *Proc. IEEE International Workshop on Computer Aided Modeling and Design of Communication Links and Networks*, Athens, Greece, Dec.

[15] Z. Lin, Y. Gao, X. Zhang, and D. Yang, "Stochastic geometry analysis of achievable transmission capacity for relay-assisted device-to-device networks," in *Proc. IEEE International Conference on Communications - Mobile nad Wireless Networking Symposium*, Sydney, Austrlia, June 2014.

[16] Z. Lin, X. Peng, F. Chin, and W. Feng, "Outage performance of relaying with directional antennas in the presence of co-channel interferences at relays," *IEEE Wireless Communicatin Letters*, vol. 1, no. 4, pp. 288 – 291, Aug. 2012.

[17] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and congnitive cellular wireless networks: A survey," *IEEE Communications Surveys and Tutorials*, vol. 15, no. 3, pp. 996 – 1019, July 2013.

[18] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[19] R. W. Heath, M. Kountouris, and T. Bai, "Modeling heterogeneous network interference using Poisson point processes," *IEEE Transactions on Signal Processing*, vol. 61, no. 16, pp. 4114 – 4126, Aug. 2013.

[20] S. Akoum, E. O. Ayach, and R. W. Heath, "Coverage and capacity in mmWave cellular systems," in *Proc. IEEE Asilomar Conference on Signals, Systems and Computers*, 2012, pp. 688–692.

[21] T. Bai, R. Vaze, and R. W. Heath, "Analysis of blockage effects on urban cellular networks," *IEEE Trans. Wireless Comm.*, vol. 13, no. 9, pp. 5070–5083, June 2014.

[22] T. Bai and R. W. Heath, "Coverage and rate analysis for millimeter-wave cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 1100–1114, Feb. 2015.

[23] L. Fenton, "The sum of Log-Normal probability distributions in scatter transmission systems," *IRE Trans. Communication. Systems*, vol. 8, no. 1, pp. 57–67, Mar. 1960.

[24] G. R. MacCartney, J. Zhang, S. Nie, and T. S. Rappaport, "Path loss models for 5G millimeter wave propagation channles in urban microcells," in *Proc. IEEE Globecom*, Dec. 2013, pp. 1–6.

[25] H. ElSawy and E. Hossain, "A modified hard core point process for analysis of random CSMA wireless networks in general fading environments," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1520 – 1534, April 2013.

[26] A.Guo and M. Haenggi, "Spatial stochastic models and metrics for the structure of base stations in cellular networks," *IEEE Trans. Wireless Comm.*, vol. 12, no. 11, pp. 5800 – 5812, Nov. 2013.

[27] H. Q. Nguyen, F. Baccelli, and D. Kofman, "A stochastic geometry analysis of dense IEEE 802.11 networks," in *IEEE International International Conference on Computer Communications*, 2007, p. 11991207.

[28] A. Hasan and J. G. Andrews, "The guard zone in wireless ad hoc networks," *IEEE Trans. Wireless Comm.*, vol. 4, no. 3, pp. 897–906, March 2007.

[29] M. Haenggi, "Mean interference in hard-core wireless networks," *IEEE Commun. Lett.*, vol. 15, no. 8, pp. 792–794, Aug. 2011.

[30] M. O. Hasna and M. S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Communications.*, vol. 52, no. 1, pp. 30–135, 2004.

[31] E. Castillo, *Extreme value theory in engineering*. Academic Press, 1988.

[32] J. Xue, T. Ratnarajah, M. Sellathurai, and Z. Ding, "Performance analysis for multi-way relaying in rician fading channels," *IEEE Trans. Communications.*, vol. 63, no. 11, pp. 4050–4062, Nov. 2015.

[33] E. J. Gumbel, *Statistics of Extremes*. Columbia University Press, 1957.

[34] C. K. Lo, R. W. Heath, and S. Vishwanath, "Opportunistic relay selection with limited feedback," in *Proc. IEEE Vehicular Technology Conference (VTC'07 Spring)*, Apr. 2007, pp. 135–139.

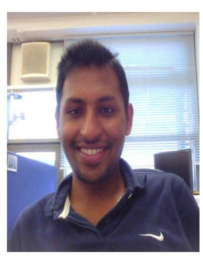
[35] H. Jiang, S. Zhang, and W. Zhou, "Outage analysis of distributed scheme on opportunistic relaying with limited CSI," *IEEE Commun. Lett.*, vol. 15, no. 9, pp. 935–937, Sept. 2011.

[36] S. Tavildar, S. Shakkottai, T. Richardson, J. Li, R. Laroia, and A. Jovicic, "FlashLinQ: A synchronous distributed scheduler for peer-to-peer ad hoc networks," in *Allerton Conf. Commun. Control Comput.*,, Allerton, IL, USA, Oct. 2010, pp. 514–521.

[37] H. A. David and H. N. Nagaraja, *Order Statistics*. Wiley, 2003.

[38] *Mathematical Methods of Statistics*. Princeton University Press, 1946.

[39] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.



Sudip Biswas received his B.Tech. degree in Electronics and Communication Engineering from the Sikkim Manipal Institute of Technology, Sikkim, India in 2010 and his M.Sc. in Signal Processing and Communications from the University of Edinburgh, Edinburgh, UK in 2013. He is currently pursuing his Ph.D. at the University of Edinburgh's Institute for Digital Communications. His research interests include various topics in wireless communications and network information theory with particular focus on stochastic geometry and possible 5G technologies such as Massive MIMO, mmWave and Full-Duplex.

Satyanarayana Vuppala (S'12-M'15) received the B.Tech. degree with distinction in Computer Science and Engineering from JNTU Kakinada, India, in 2009, and the M.Tech. degree in Information Technology from the National Institute of Technology, Durgapur, India, in 2011. He received the Ph.D degree in Electrical Engineering from Jacobs University Bremen in 2014. He is currently a post-doctoal researcher at IDCOM in University of Edinburgh. His main research interests are physical, access, and network layer aspects of wireless security. He also works on performance evaluation of mmWave systems. He is a recipient of MHRD, India scholarship during the period of 2009-2011.

Jiang Xue (S'09-M'13) received the B.S. degree in Information and Computing Science from the Xi'an Jiaotong University, Xi'an, China, in 2005, the M.S. degrees in Applied Mathematics from Lanzhou University, China and Uppsala University, Sweden, in 2008 and 2009, respectively. Dr. J. Xue reveived the Ph.D. degree in Electrical and Electronic Engineering from ECIT, the Queen's University of Belfast, U.K., in 2012. He is currently a Research Fellow with the University of Edinburgh, UK. His main interest lies in the performance analysis of general multiple antenna systems, stochastic geometry, cooperative communications, and cognitive radio.

Tharmalingam Ratnarajah (A'96-M'05-SM'05) is currently with the Institute for Digital Communications, University of Edinburgh, Edinburgh, UK, as a Professor in Digital Communications and Signal Processing. His research interests include signal processing and information theoretic aspects of 5G wireless networks, full-duplex radio, mmWave communications, random matrices theory, interference alignment, statistical and array signal processing and quantum information theory. He has published over 270 publications in these areas and holds four U.S. patents. He is currently the coordinator of the FP7 projects HARP (3.2M) in the area of highly distributed MIMO and ADEL (3.7M) in the area of licensed shared access. Previously, he was the coordinator of FP7 Future and Emerging Technologies project CROWN (2.3M) in the area of cognitive radio networks and HIATUS (2.7M) in the area of interference alignment. Dr Ratnarajah is a Fellow of Higher Education Academy (FHEA), U.K., and an associate editor of the IEEE Transactions on Signal Processing.