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Detection of transience in eroding landscapes

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Abstract

Past variations in climate and tectonics have led to spatially and temporally varying erosion rates across many landscapes. In this contribution I examine methods for detecting and quantifying the nature and timing of transience in eroding landscapes. At a single location, cosmogenic nuclides can detect the instantaneous removal of material or acceleration of erosion rates over millennial timescales using paired nuclides. Detection is possible only if one of the nuclides has a significantly shorter half-life than the other. Currently, the only practical way of doing this is to use cosmogenic in-situ $^{14}$C alongside a longer lived nuclide, such as $^{10}$Be. Hillslope information can complement or be used in lieu of cosmogenic information: in soil mantled landscapes, increased erosion rates can be detected for millennia after the increase by comparing relief and ridgetop curvature. This technique will work as long as the final erosion rate is greater than twice the initial rate. On a landscape scale, transience may be detected based upon disequilibria in channel profiles or ridgetops, but transience can be sensitive to the nature of transient forcing. Where forcing is periodic, landscapes display differing behavior if forcing is driven by changes in base level lowering rates versus changes in the efficiency of either channel or hillslope erosion (e.g. driven by climate change). Oscillations in base level lowering lead to basin averaged erosion rates that reflect a long term average erosion rate despite
strong spatial heterogeneity in local erosion rates. This averaging is reflected in \(^{10}\)Be concentrations in stream sediments. Changes in hillslope sediment transport coefficients can lead to large fluctuations in basin averaged erosion rates, which again are reflected in \(^{10}\)Be concentrations. The variability of erosion rates in landscapes where both the sediment transport and channel erodibility coefficients vary is dominated by changes to the hillslope transport coefficient.

**Introduction**

Gone are the days when geomorphologists thought of landscapes as experiencing a period of ‘rejuvenation’ followed by a dignified, if not dull, period of gradual adjustment (c.f., Davis, 1899). We now understand the Earth’s crust to be constantly in motion, with faults rupturing and plates buckling under tectonic stresses, all leading to surface deformation (e.g., Kirby and Whipple, 2010). In addition, there is now growing recognition that the mantle also has a role to play, as large scale convection, mantle plumes and diapirs are thought to lead to vertical displacements over geologic timescales (e.g., Rohrman and van der Beek, 1996; Saunders et al., 2007; Braun, 2010; Hartley et al., 2011; Moucha and Forte, 2011). In addition, plutonism can lead to density differences that drive uplift (e.g., Braun et al., 2011). Isotope records show beyond doubt that our planet’s climate varies wildly and sometimes abruptly, with ice sheets growing and shrinking (e.g., Dansgaard et al., 1993) and sea levels rising and falling by tens of meters (e.g., Lambeck and Chappell, 2001).
Geodynamic and climatic activity plays a fundamental role in shaping our planet’s terrestrial surface. Geomorphologists have increasingly turned their attention to quantifying the effects of tectonic and climatic change on rivers and hillslopes. In the last few decades, new developments have led to intensified research into the nature and speed of landscape adjustment. Several provocative physical experiments have reproduced morphologies that resemble large catchments and even mountain ranges, but that displayed unexpected landscape dynamism. The tank experiments of Hasbargen and Paola (2000) and the sandboxes of Lague et al. (2003) and Bonnet et al. (2009) featured landscapes that would equilibrate to a steady sediment flux rate. The flux rate in these experiments was defined as the sediment removed from the system averaged over the time necessary to erode through the highest point on the landscape. The geometry of these experimental landscapes, however, was anything but steady. Divides moved, knickpoints migrated and the organisation of the drainage network varied vigorously through time.

Numerical modelling has also stimulated interest in the transient state of landscapes. Like analogue experiments, numerical models have allowed workers to see virtual mountain ranges grow and adjust to changing climate (as approximated by changing precipitation and erodibility coefficients) and tectonics (as approximated by changing uplift rates or lateral displacement). Many early models were constructed with the aim of predicting the topographic outcome of so-called geomorphic transport laws (sensu Dietrich et al., 2003). This mirrored early analytical work by pioneers such as Culling (1960), who linked constitutive equations linking sediment transport with topographic forms. Culling famously
showed why hilltops are convex, in an elegant mathematical demonstration of Gilbert’s (1909) earlier hypothesis. Whereas the early focus of many modelling studies was to recreate digital topography that resembled natural topography, there has been a movement in recent years to use models to test, or at least falsify, hypotheses about past landscape evolution in the face of changing tectonic or environmental forcing.

One feature of early models was that an established drainage network and its ridge network tended to stay in a fixed position (e.g., Howard, 1994). These results contrasted with widespread observation of landforms interpreted to result from drainage capture (e.g., Davis, 1889; Bishop, 1995), and also contrasted with experimental models. One early model that did predict evolving drainage divides was that of Smith et al. (1997), which predicted the splitting of divides, as observed in the tank experiments of Hasbargen and Paola (2000). More recent models have tried to account for changing drainage areas as hillslopes adjust to transient forcing, leading to models predicting a much more dynamic drainage and ridgetop network (Pelletier, 2004; Castelltort et al., 2012; Goren et al., 2014). Research based on recent metrics that detect disequilibrium across drainage divides supports the widespread presence of highly dynamic landscapes, even in tectonically quiescent settings such as the Appalachians (Willett et al., 2014).

Within the context of changing tectonics and climate, and the potential complication of autogenic landscape variability (e.g., Jerolmack and Paola, 2010; Coulthard and Van de Wiel, 2013), one of the major challenges in geomorphology is to try to reconstruct past
changes based on current information (e.g., Wobus et al., 2010; Whittaker, 2012).

Geomorphologists can rarely work with a time series of landscapes: geomorphic change may occur over thousands to millions of years and, barring the invention of time travel, we will, in most cases, need to work with current landscape properties in order to reconstruct past changes. However, there are a number of recent advances that may give us some insight into how the landscape arrived at its current configuration.

Two advances that have refreshed the study of geomorphology are the measurement and interpretation of cosmogenic radionuclides (CRNs; e.g., Bierman et al., 1994) and the rapid expansion of the quality and availability of high resolution topographic data (e.g., Slatton et al., 2007). While the application of CRNs to geomorphic research is now considered mature, improving instrument sensitivity and analytical techniques are extending the range of application (Rood et al., 2010). New measurement techniques, notably for in-situ cosmogenic $^{14}$C, are opening opportunities for querying landscapes. In-situ cosmogenic $^{14}$C has only been measured reliably in laboratories in the last 5 years (e.g., Fulop et al., 2010; Hippe et al., 2013; Goehring et al., 2014) but offers new opportunities to examine landscape transience.

High resolution topography, primarily generated using airborne light detection and ranging (lidar), has allowed us to quantify topography on sub-meter scale. It is not unusual for modern airborne lidar campaigns to collect dozens, or even hundreds of square kilometres at point cloud densities of $>10$ pts m$^{-2}$. This presents unprecedented
opportunities for geomorphologists and ecologists alike (e.g., Tarolli, 2014; Passalacqua et al., 2015): not only do these data allow sensing of the plant canopy (e.g., Dubayah and Drake, 2000; Lefsky et al., 2002), but they also allow geomorphologists to observe and quantify landscape features at the process scale, such as evidence of biotic activity (e.g., Roering et al., 2010; Gabet et al., 2014), fault scarps (e.g., Sherrod et al., 2004; Arrowsmith and Zielke, 2009), headwater channels (e.g., Passalacqua et al., 2010; Orlandini et al., 2011; Pelletier, 2013; Clubb et al., 2014), bedrock outcrop (e.g. DiBiase et al., 2012; Milodowski et al., 2015) and other relevant features.

While sedimentary archives may be used to detect landscape transience (e.g., Schaller et al., 2002; Balco and Stone, 2005; Armitage et al., 2011; Charreau et al., 2011; Marshall et al., 2015), I wish to focus here on eroding landscapes where sediment export precludes the use of such archives to detect transience. In this contribution I aim to show that landscape transience may be both detected and quantified in the absence of sedimentary deposits using topography and cosmogenic nuclides.

Detection of change in a single soil profile using in-situ cosmogenic nuclides

One of the most revolutionary developments in the last 30 years in geomorphology is the widespread adoption of methods based on in-situ cosmogenic radionuclides (CRNs); since the seminal work of Lal (1991), hundreds of authors have used the concentration of CRNs to infer past erosion rates. Applications of CRNs in geomorphic studies frequently use an assumption that erosion rates are constant in time (a review of the many
applications can be found in Dunai, 2010 and Granger et al., 2013), but fewer authors have sought to examine how CRN concentrations may be used in transient settings. Several authors have examined how CRN concentrations may be expected to vary in soils in the face of either periodic (e.g., Heimsath, 2006) or stochastic (Lal, 1991; Small et al., 1999; Musikar, 2009; Parker and Perg, 2005; Heimsath, 2006; Schaller and Ehlers, 2006) variations in erosion rates. A key concept in these studies is that the concentration of CRNs can be inverted for an erosion rate. This erosion rate is ‘apparent’ because it depends on the assumptions used in the inversion and may not reflect the actual erosion rate. For example, an apparent erosion rate may be calculated using Lal’s (1991) equation for steady state erosion, but this equation assumes erosion rates and cosmogenic production rates that do not vary in time. This contrasts with the actual erosion rate, by which I mean the erosion rate that has actually occurred, or in other words the amount of material removed divided by the time over which this removal is averaged, and in addition the apparent erosion rate may not reflect the instantaneous erosion rate, i.e., the erosion rate that is currently occurring in the landscape.

One of the key findings of studies investigating the effect of transient erosion rates on CRN concentrations is that the apparent erosion rates derived from CRN concentrations will reflect the time integrated erosion rate. In addition the lag between instantaneous and apparent erosion rates will increase with greater amplitude variations in erosion rate and shorter periods of periodic forcing or time between stochastic events. Alternatively, low frequency variations in erosion rate and low amplitude variations in erosion rate will result in apparent erosion rates that more closely reflect the instantaneous erosion rate.
The focus of most studies examining the effect of erosion transience has focused on the accuracy of inferred erosion rates in the face of erosion variability, but few have commented on using CRN concentrations to infer past changes in erosion rates. Lal (1991) famously constructed plots of the ratio between $^{10}\text{Be}$ and $^{26}\text{Al}$ concentration against $^{10}\text{Be}$ concentration to demonstrate the existence of a ‘steady state island’ in this parameter space: samples plotting outside this steady state island had a more complex erosion and exposure history. This principle is used extensively in the dating of surfaces and burial dating (e.g., Granger 2006). In eroding landscapes, however, we can narrow the range of exposure histories since it is a reasonable assumption that on sloping ground, material is continuously eroding and unlikely to experience prolonged periods of exposure or burial. With these constraints in mind, I would like to examine the possibility of inferring past erosional changes in eroding landscapes using CRN concentrations.

I begin with a general statement of conservation of a nuclide for a steadily eroding surface (e.g., Granger and Smith, 2000, Schaller et al., 2002; Vermeesch, 2007):

$$C_i = C_{i,0} e^{(t-t_0)\lambda_i} + P_{i,SLHL}S_{tp} \sum_{j=0}^{n} \left[ \frac{S_i F_{i,j} \Gamma_{i,j}}{\varepsilon - \Gamma_{i,j} A_i} e^{-(d+\varepsilon t)/\Gamma_{i,j}} \left( e^{(t-t_0)\varepsilon/\Gamma_{i,j}} - e^{(t-t_0)\lambda_i} \right) \right],$$  \hfill (1)

where $C_i$ is the concentration of nuclide $i$, $C_{i,0}$ (typically reported in atoms per gram) is the initial concentration of this nuclide, $t$ and $t_0$ are the current and initial time, respectively, $\lambda_i$ is the decay coefficient ($T^{-1}$, dimensions henceforth denoted in [M]ass, [L]ength, and [T]ime in square brackets), $P_{i,SLHL}$ is the production rate of nuclide $i$ at sea level and high latitude (in units atoms g$^{-1}$ yr$^{-1}$), $S_{lp}$ is a dimensionless scaling factor due to topographic shielding, the subscript $j$ refers to the production mechanism (this could be either nucleonic spallation or various muogenic pathways), $S_j$ is a dimensionless scaling factor that accounts for a number of effects such as changing production rates as a function of altitude or pressure, shielding from snow and self-shielding (e.g., Vermeesch, 2007), $F_{i,j}$ is a dimensionless factor that determines the proportion of nuclide production due to each mechanism, $\Gamma_{i,j}$ [M L$^{-2}$] is the attenuation length (typically reported in g cm$^{-2}$), $\varepsilon$ is erosion rate in mass equivalent units (typically g cm$^{-2}$ yr$^{-1}$) and $d$ is a mass per unit area that is related to the depth by:

$$d = \int_{\zeta - h}^{\zeta} \rho(z) \, dz,$$

(2)

where $\zeta$ [L] is the elevation of the surface, $h$ [L] is depth below the surface, and $\rho$ [M L$^{-3}$] is the density of the rock or regolith as a function of elevation. The quantity $d$ is called the shielding depth.
Results of experiments conducted by Heisinger *et al.* (2002a, b) suggested that muons could contribute significantly to CRN production at depth, but measurements from field sites now suggest that production from muons is smaller than suggested by experiments (Braucher *et al.*, 2013); for example Braucher *et al.* (2013) reported that muogenic production of $^{10}\text{Be}$ accounted from ~0.5% of the total production, with the remainder produced by nucleonic spallation. With only nucleonic spallation, equation (1) reduces to:

$$C_i = C_{i,0} e^{(t-t_0)\lambda_i} + P_{i,SLHL} S_t \left[ e^{-(d+et)/\Gamma} \left( e^{(t-t_0)\varepsilon/\Gamma} - e^{(t-t_0)\lambda_i} \right) \right],$$  \hspace{1cm} (3)$$

where $S_t$ [dimensionless] is a combined scaling term that takes into account production scaling and snow, self and topographic shielding.

Equation (3) may be solved for the steady state concentration if $t$ approaches infinity and the initial concentration is zero (e.g., Lal, 1991):

$$C_{i,SS} = \frac{P_{i,SLHL} S_t \Gamma}{\varepsilon + \Gamma \lambda_i} e^{-\frac{d}{\Gamma}}.$$  \hspace{1cm} (4)$$
Equation (4) can then be solved for the apparent erosion rate, $\varepsilon_{app}$ (in units g cm$^{-2}$ yr$^{-1}$), which one would infer from CRN concentration if the system were assumed to be in steady state:

$$\varepsilon_{app} = \frac{e^{-\tau P_{isLHL}S_t \Gamma - C_l \Gamma \lambda_i}}{C_l}.$$  \hspace{1cm} (5)

My aim is to exploit equations (3-5) to gain information about past erosion rates, focusing on two scenarios.

**Scenario 1: Instantaneous removal of mass**

The first scenario is one in which there is a constant background erosion rate but some thickness of material is removed instantaneously from the surface. This scenario mimics a landslide, or alternatively is an approximation for a period of intense soil loss such as that experienced in ancient Greece (e.g., van Andel *et al.*, 1990) or Rome (e.g., Judson, 1968), and builds on a similar analysis in Lal (1991).

The initial time, $t_0$, is set to zero and represents the moment the mass is removed (this could be either bedrock or soil or a combination of the two, as long as mass removal is instantaneous). The time $t$ represents the time since mass removal. The initial
concentration of the nuclide will be determined by the steady state concentration (equation 4), but the depth must be adjusted to account for removal of mass, such that the adjusted depth, \(d_{adj} = d + \epsilon t + d_{br}\), where \(d_{br}\) is the depth of mass removal. This can be inserted into equation (4), which can be substituted in as the initial concentration in equation (3), and this concentration can be inserted into equation (5) to yield the apparent erosion rate after mass removal:

\[
\epsilon_{app,br} = \left[ \epsilon e^{d_{br} + \epsilon t} + \Gamma \lambda_i \left( e^{d_{br} / \Gamma} - 1 \right) \right] \left[ 1 + e^{d_{br} / \Gamma} \left( e^{t[\Gamma + \lambda_i]} - 1 \right) \right]^{-1}.
\] (6)

It is perhaps useful here to explain how equations (3-6) might be practically applied. Consider a situation in which the worker has no information about past changes in erosion rates (i.e., there is no lake sediment record downstream, no historic erosion rate data, etc.). The goal then is to use only the concentration of nuclides to determine past transience. If erosion rates are constant in time, the apparent erosion derived from equation (5) will be equal to the true erosion rate. However, if erosion rates are transient, nuclide concentrations will reflect some averaging of past erosion. I seek a way to diagnose if there has been transience by either quantifying changes in nuclide concentrations with depth or quantifying differences between concentrations of different nuclides. Equation (6) predicts an apparent erosion rate (that is, the erosion rate one calculates assuming steady erosion) if nuclide concentrations are the result of mass removal. That is, equation (6) is used to explore if nuclide concentrations at different
depths or from different nuclides report the same apparent erosion rate (in which case
the hypothesis that erosion is steady cannot be rejected) or if they report different
apparent erosion rates.

Inspection of equation (6) yields a significant result: the apparent erosion rate is not a
function of depth. This means that after mass removal, regardless of where in the soil or
regolith column samples are extracted, the apparent erosion rate is the same. This is
important because it means that the depth profile of a single CRN is of no use in
identifying landscape transience under the mass removal scenario.

If a single nuclide cannot reveal information about transience, what options are available
to detect transience? I will show below that a single nuclide also fails to provide
information about transience in the second scenario, featuring a step change in erosion
rate. All is not lost, however. Nuclides with differing decay coefficients equilibrate to local
conditions at different rates; this is the principle behind the steady state island plots of Lal
(1991) and various burial dating techniques (e.g., Granger et al., 2012). For both
scenarios, the differing apparent erosion rates derived from two nuclides can be used to
reveal information about landscape transience.
Nuclides that decay more rapidly will adjust more quickly to changes in erosion rate (Lal, 1991). Thus, if mass is instantaneously is removed from the surface, the nuclide with more rapid decay will have a greater perturbation to its apparent erosion rate. Following Lal’s (1991) lead, we may look for greater apparent erosion rates in shorter lived nuclides to detect changes in erosion rates. I will thus compare $\epsilon_{app,br}$ for two different nuclides calculated using equation (6), as a function of the background erosion rate ($\epsilon$), the depth of the mass removal ($d_{br}$) and the time since mass removal ($t$).

To be conservative, I assume that any apparent erosion rate has 10% uncertainties attached. Thus, to plausibly detect transience, the ratio between the two nuclides must be greater than 20%. Note that in some cases, production rate uncertainties exceed this value (e.g., Balco et al., 2008), but in these examples the samples will have the same production rate (since we are sampling effectively the same particle) so production uncertainties will result in absolute but not relative uncertainties. We can also consider a plausible range of background erosion rates. Erosion rates greater than 1 mm yr$^{-1}$ are widely considered to be rapid; this equates to $\sim 0.25$ g cm$^2$ yr$^{-1}$ of bedrock lowering for typical rock densities ($\sim 2.5$ g cm$^{-3}$). There are landscapes with faster erosion rates but, as we will momentarily see, even background erosion rates of 1 mm yr$^{-1}$ are beyond our current analytical limits for detecting transience. The lower limit of erosion is of course zero, but the vast majority of sloping terrains (even ones located on very low relief cratons) are eroding faster than 0.001 mm yr$^{-1}$ (e.g., Portenga and Bierman, 2011, although bare rock surfaces tend to erode more slowly; see their Figure 2a).
First, consider two commonly measured nuclides, $^{10}$Be and $^{26}$Al. These have decay coefficients of $500 \times 10^{-9}$ yr$^{-1}$ (Chmeleff et al., 2010; Korschinek et al., 2010) and $980 \times 10^{-9}$ yr$^{-1}$ (Nishiizumi, 2004), respectively, and both have a value of 160 g cm$^{-2}$ (Balco et al., 2008). Even in a slowly eroding landscape ($0.0026$ g cm$^{-2}$ yr$^{-1}$, equivalent to $0.01$ mm yr$^{-1}$ in material with density 2.6 g cm$^{-3}$) where $500$ g cm$^{-2}$ of material is removed (this is roughly equivalent to a ~2 m thick layer of bedrock) and the surface is sampled one year after block removal, the difference in the apparent erosion rates is only 2\% ($\varepsilon_{app}$ for $^{10}$Be = 0.064 g cm$^{-2}$ yr$^{-1}$ versus $\varepsilon_{app}$ for $^{26}$Al = 0.062 g cm$^{-2}$ yr$^{-1}$). The difference gets yet smaller for thinner blocks and faster background erosion rates. Thus, it is virtually impossible to detect removal of a block in an eroding landscape using paired $^{10}$Be and $^{26}$Al. In addition, apparent erosion rates from stable nuclides (e.g., $^{21}$Ne) cannot be differentiated from apparent erosion rates derived from $^{10}$Be and $^{26}$Al.

Now, consider $^{10}$Be and in-situ $^{14}$C ($\lambda = 1.21 \times 10^{-4}$ yr$^{-1}$, Bowman, 1990). Several laboratories are now capable of measuring cosmogenic in-situ $^{14}$C (Fulop et al., 2010; Hippe et al, 2013; Goehring et al., 2014), which is a promising nuclide due to its short half-life, as I will demonstrate. We can plot the ratio of apparent erosion rates for these two nuclides after block removal, this is shown for two different background erosion rates and two block removal depths in Figure 1. When $50$ g cm$^{-2}$ is removed, it can be detected given our conservative threshold of a 20\% difference for background erosion rates of 0.026 g cm$^{-2}$ yr$^{-1}$, which is an approximate bedrock erosion rate of ~0.1 mm yr$^{-1}$ (Figure
1a). At these rates, a mass removal of 50 g cm\(^{-2}\) can be detected for approximately 100 years after the mass removal. If the background erosion rate is slower, change can be detected for thousands of years. If a mass of 150 g cm\(^{-2}\) is removed, one can detect this removal for a millennium if background erosion rates are 0.026 g cm\(^{-2}\) yr\(^{-1}\) or slower (Figure 1b). If background erosion rates are too rapid, however, mass removal cannot be detected; with background erosion rates of 0.26 g cm\(^{-2}\) yr\(^{-1}\), no thickness of mass removal results in apparent erosion rates between \(^{10}\)Be and \(^{14}\)C exceeding the threshold of 20% erosion rate difference. This method can therefore mass block removal in slowly eroding landscapes, but the nuclide ratios are not unique: we should be able to tell if mass removal has occurred but we cannot know both the time and depth of removal.

Scenario 2: Step change in erosion rate

The second scenario that I investigate is a situation where there has been a step change in the erosion rate, from \(\varepsilon_{\text{old}}\) to \(\varepsilon_{\text{new}}\). The initial concentration is determined by equation (4). This concentration is then inserted into equation (2), and solved for the apparent erosion rate, resulting in:
\[ \varepsilon_{app,sc} = \left[ \Gamma \lambda_t (\varepsilon_{old} - \varepsilon_{new}) \right. \\
\left. + \varepsilon_{new} e^{\frac{\varepsilon_{new}}{\Gamma + \Gamma \lambda_t}} \right] \left[ \varepsilon_{new} + \varepsilon_{old} \left( e^{\frac{\varepsilon_{new}}{\Gamma + \Gamma \lambda_t}} - 1 \right) + \Gamma \lambda_t e^{\frac{\varepsilon_{new}}{\Gamma + \Gamma \lambda_t}} \right]^{-1}. \]  

Similar to equation (6), equation (7) is not a function of depth: one cannot use the depth profile of a single nuclide to distinguish between a steady state profile and one that has experienced a step change in erosion rate. Again, multiple nuclides must be used to identify a site that has experienced a change in erosion rates; I explore the sensitivity of a two-nuclide system as a function of the old and new erosion rates and the time since the change in erosion rate \((t)\).

As in the case of block removal, apparent erosion rates derived from \(^{10}\)Be and \(^{26}\)Al are not sufficiently different to allow identification of transient erosion rates. In addition, apparent erosion rates from stable nuclides (e.g., \(^{21}\)Ne) cannot be differentiated from apparent erosion rates derived from \(^{10}\)Be and \(^{26}\)Al. Even if erosion rates increase by a factor of 10 in a slowly eroding landscape, the difference between apparent erosion rates inferred from these two nuclides never exceeds 5%. To gain some insight into transient processes we must use a nuclide pair with a greater difference in decay rates, so once again I turn to \(^{10}\)Be and in-situ \(^{14}\)C.
First, we can examine the effect of acceleration in erosion rate (Figure 2a). Again I invoke a conservative detection threshold of >20% difference in the apparent erosion rates inferred from in-situ $^{10}$Be and $^{14}$C concentrations. A doubling of erosion rate is detectable using the $^{10}$Be and $^{14}$C pair for an initial erosion rate of 0.0026 g cm$^{-2}$ yr$^{-1}$ between 500 and 2000 years after the acceleration. Doubling of the erosion rate is not detectable if the original erosion rate is 0.026 g cm$^{-2}$ yr$^{-1}$. This implies that a doubling of erosion rate is only detectable at very slow background erosion rates. On the other hand, a five times acceleration in erosion rate is detectable with original erosion rates at both 0.0026 g cm$^{-2}$ yr$^{-1}$ and 0.026 g cm$^{-2}$ yr$^{-1}$, with the former being detectable after ~400 years and the latter being detectable after ~2000 years (Figure 2a).

Similar detection limits are found if erosion rate decreases; in this case, the apparent erosion rate inferred from $^{14}$C is lower than that of $^{10}$Be (Figure 2b). The major difference between acceleration and deceleration in erosion is that if erosion rates decrease, they cannot be detected for a long time (compared to accelerated erosion); for an erosion rate of 0.0026 g cm$^{-2}$ yr$^{-1}$ and a reduction in the erosion rate by a factor of 5, one must wait ~2000 years before transience can be detected using the $^{10}$Be and $^{14}$C pair.

Unfortunately, because of the self-similarity of CRN depth profiles, one cannot differentiate between block removal and a step change with a depth profile, nor can one uniquely find both the timing of acceleration or removal, so these techniques are limited to detecting if a perturbation has occurred to the erosion rate in the past, and it can
constrain the upper limit of how long ago this perturbation occurred. CRNs, however, are not the only means of detecting transience within the geomorphologist’s toolkit.

Detection of hillslope transience from topography

In many cases, one might wish to look for evidence of landscape transience across multiple hillslopes. One strategy is to look for a transition between low relief and high relief surfaces, which may be interpreted as separating slowly eroding from rapidly eroding portions of the landscape (e.g., Schoenbohm et al., 2004; Gallen et al., 2011; Anderson et al., 2012; Prince and Spotila, 2013). If changes in hillslope erosion rates are driven by the propagation of knickpoints up the channel network, one might expect to find a pattern of hillslope disturbance in which the proportion of the hillslope affected by the greater erosion rate increases downstream of the channel knickpoint (e.g., Mudd and Furbish, 2007; Hurst et al., 2012). Differentiating zones of rapid erosion from zones of slow erosion on a hillslope is not always trivial. In some cases a clear break in slope is visible (e.g., Rheinhardt et al., 2007), but in many cases a change along a profile is difficult to quantify since hillslopes, even at steady state, will have gradients that increase downhill (e.g., Culling, 1960). In addition, in most rapidly eroding landscapes, hillslopes tend to approach a critical slope angle (e.g., Roering et al., 2001; Binnie et al., 2007; DiBiase et al., 2010) and thus at high erosion rates, hillslope gradients become insensitive to erosion rates.
An alternative to searching for a break in slope is to use the relief structure in the landscape, in combination with information about ridgetops, to detect landscape transience. Roering et al. (1999) observed that in rapidly eroding landscapes in the Oregon Coast Range, soil-mantled hillslopes tended to become planar away from hilltops; this topography was consistent with a sediment flux law that predicted as hillslope gradients approached a critical slope, $S_c$ [dimensionless], sediment flux ($q_s$ [L$^2$ T$^{-1}$]) would rapidly increase. This flux law mirrored one earlier proposed by Andrews and Bucknam (1987):

$$\tilde{q}_s = -\frac{D \nabla\zeta}{1 - \left(\frac{\nabla\zeta}{S_c}\right)^2},$$

where $D$ [L$^2$ T$^{-1}$] is a sediment transport coefficient and the arrow indicates that sediment flux is a vector quantity (also recall $\zeta$ denotes surface elevation). Combining this flux law with a statement of mass conservation, Roering et al. (2007) noted that on ridgetops, where topographic gradients are low, erosion rates should be linearly proportional to ridgetop curvature, $C_{HT}$ [L$^{-1}$] (that is, the second derivative of surface topography, $\nabla^2\zeta$) in steadily eroding landscapes. In addition, Roering et al. (2007) found the steady solution of surface topography in one dimension, and were able to demonstrate that all steadily denuding hillslopes obeying equation (8) should fall on the nondimensional curve,
The quantity $R^*$ is a dimensionless relief: it is the relief ($R$ [L]) from channel to ridgetop scaled by the critical slope $S_c$ and the hillslope length $L_H$. The density subscripts $r$ and $s$ refer to rock and soil densities, respectively. It can also be interpreted as the mean topographic gradient of the hillslope divided by the critical slope. The quantity $E^*$ is a dimensionless erosion rate. Roering et al. (2007) went on to demonstrate that sites in the Oregon Coast Range and Gabilan Mesa, California, plot in $E^*$ vs $R^*$ space along the curve.

\[
R^* = \frac{1}{E^*} \left[ \sqrt{1 + (E^*)^2} - \ln \left( \frac{1}{2} \left[ 1 + \sqrt{1 + (E^*)^2} \right] \right) - 1 \right],
\]

(9)

where

\[
R^* = \frac{R}{S_c L_H},
\]

(10)

and

\[
E^* = \frac{2(\rho_r/\rho_s)L_H}{DS_c} = -\frac{2C_{HT} L_H}{S_c}.
\]

(11)
predicted by equation (9). Hurst et al. (2012) showed that in a landscape in the Northern Sierra Nevada of California, ridgetop curvature was linearly proportional to erosion rate and that, across the range of erosion rates in the study area (0.01 to 0.25 mm yr\(^{-1}\)), the \(E^*\) vs \(R^*\) values plotted, within error, on the curve described by equation (9). This suggests equation (8) is consistent with the topography of these landscapes, corroborating the findings of Roering et al. (1999), Roering (2008) and Grieve et al. (2016).

What happens, then, if channel incision rates change? We do have some idea of how long it takes for hillslopes to adjust to changes in channel incision rates. One measure of the time it takes a hillslope to adjust to changing channel incision rates is a hillslope’s relaxation time, a concept borrowed from the physics of heat diffusion (e.g., Carslaw and Jaeger, 1959). The relaxation time measures the time a diffusion-like system (such as molecular diffusion, diffusion of thermal energy, diffusion of pore pressure within saturated groundwater systems, or the diffusion-like behavior of surface elevation on creep-dominated hillslopes) equilibrates to a steady state after a perturbation. The formal definition of the relaxation time is the time it takes a system under steady forcing, e.g., a hillslope with a steadily eroding channel at its base, to return to within \(e^{-1}\) (which is approximately 0.37) of the steady condition (in the hillslope case measured by erosion rate). For example, if a hillslope was eroding at 1 mm yr\(^{-1}\) and the channel then began eroding at 2 mm yr\(^{-1}\), the relaxation time would be the time required for the average hillslope erosion rate to reach 1.63 mm yr\(^{-1}\).
Fernandes and Dietrich (1997) used numerical simulations to show that for hillslopes where $q_s = -D\rho_s S$, where $S$ is the topographic gradient, the relaxation time is proportional to $L_H^2/D$, although they used a threshold of 90% of the steady erosion rather than the e-folding timescale to define the relaxation time. Mudd and Furbish (2007) later showed through analytical solution that the formal relaxation time is exactly $4L_H^2/(D\pi^2)$. The relaxation time can be reduced in landscapes where sediment flux is nonlinearly proportional to topographic gradient (e.g., Roering et al., 2001) or is proportional to the product of gradient and soil thickness (e.g., Mudd and Furbish, 2007) relative to landscapes where sediment flux is linearly proportional to topographic gradient. In general, the relaxation time is strongly related to hillslope length and the sediment transport coefficient.

Now consider the following scenario. Imagine a wave of channel incision passes the base of a hillslope. The time it takes for a signal of channel incision to reach the hilltop is a function of the flux law: Mudd and Furbish (2007) showed that it takes $1/9$ of the relaxation time for the hilltop to be affected after channel perturbation where sediment flux is linearly proportional to slope; nonlinear slopes such as those described by equation (8) can respond yet more quickly (Roering et al., 2001). In both cases, however, there is a delay between increased channel incision and any response of the hilltop. The result of this delay is that hilltop curvature will lag behind hillslope relief in a transient landscape. One can calculate an apparent $E^*$ value for a hilltop, regardless of whether it is at steady state or not: $E_{\text{app}}^* = 2C_{HT} L_H / S_c$. If the channel incision rate has increased, the hilltop-based apparent dimensionless erosion rate ($E_{\text{app}}^*$) should be less than that predicted by the
steady state curve described by equation (9) for a given dimensionless relief, $R^*$ (Figure 3), since the hilltop will not yet have adjusted to the new erosion rate. If channel incision rates decrease, $E^*_{\text{app}}$ will be greater than the $E^*$ predicted by the steady state curve. Thus, increases and decreases in channel incision rates can be detected by hillslopes lying above and below the steady state curve in $E^*$ vs $R^*$ space, respectively (Hurst et al., 2013a; Figure 3).

Hurst et al. (2013a) tested this hypothesis along the Dragon’s Back Pressure Ridge, a landform that lies along the San Andreas Fault (SAF) in California. It is the result of deformation and uplift caused by the fault motion advecting the local sedimentary formation (the Paso Robles formation, made of weakly consolidated sediments) over an offset in the SAF that remains stationary in relation to the North American Plate (Hilley and Arrowsmith 2008). Detailed field mapping and palinspastic reconstruction of the deformed beds by Hilley and Arrowsmith (2008) has resulted in a uniquely well-constrained uplift field. Due to the motion of the fault, small catchments running perpendicular to the fault pass over and then away from this uplift field such that it is possible to quantify a space for time substitution on changing basin uplift as a function of position along the pressure ridge (Hilley and Arrowsmith 2008).

Hurst et al. (2013a) found that catchments that had recently passed over the zone of maximum uplift plotted above the steady state curve described by equation (9), and as catchments moved away from the zone of uplift their hillslopes adjusted towards the
steady-state line by increasing their $E_{\text{app}}^*$ before relief declined taking them below the steady state curve. This exposition of topographic hysteresis demonstrated the utility of comparing hillslope relief to hilltop curvature in order to identify landscapes that are growing from those that are static or waning.

These results beg the question: just how much of an increase or decrease in channel incision is required for transience to be detected using the $E^*$ vs $R^*$ technique? Using the one dimensional model of Hurst et al. (2013a), I have explored both the magnitude and timing of a hillslope’s departure from the steady state $E^*$ vs $R^*$ curve for different initial and final channel erosion rates. The model starts from a steady state hillslope profile, and then channel incision undergoes a step change.

Figure (4a) shows the maximum difference in dimensionless relief that is measured on the hillslope and that is predicted by equation (9) as the hillslope responds to a change in channel incision. As a point of reference, the standard errors in observed $R^*$ values from the Dragon’s Back Pressure Ridge (DBPR) reported by Hurst et al. (2013a) are plotted: these errors are due to the natural variability in relief along the DBPR ridgelines. Differences in measured and predicted $R^*$ that are less than this error cannot be resolved, so this gives some indication of how great a perturbation in incision rate is required before changes in $R^*$ can be resolved. This error is not universal: each landscape will have its own variability in ridgeline relief (e.g., Gabet et al., 2015); the DBPR errors are plotted simply as a point of reference. Figure (4a) indicates that reductions in channel incision
are more difficult to resolve than landscapes where channel incision rates have increased.

In addition, it is easier to resolve landscape transience if the initial channel incision rate is greater, which is somewhat counterintuitive. The reason for this is that a doubling of erosion rate from initially low erosion does not lead to as large of an increase in $R^*$. 

For relatively fast initial channel incision rates, a doubling of the channel incision rate should be resolvable from the difference between predicted and measured $R^*$. For parameter values of $\rho_l/\rho_s = 2$, $D = 0.01 \text{ m}^2 \text{ yr}^{-1}$, $L_H = 25 \text{ m}$ and $S_c = 1$ (these are values similar to DBPR), an $E^*$ of 1 equates to an incision rate of $0.4 \text{ mm yr}^{-1}$, an erosion rate that is frequently achieved in tectonically active landscapes (see, for example, the erosion rate compilation of Portenga and Bierman, 2011).

Figure (4b) shows the time needed to reach the maximum difference between measured and predicted $R^*$, in units of dimensionless time, $t^*$. Time is scaled by $t^* = (D/L_H^2) t$, where $t$ is dimensional time. The time required to reach the maximum difference in measured and predicted $R^*$ varies between approximately $t^* = 0.01$ and $t^* = 0.1$ after the perturbation. This result should not come as a surprise because $t^*$ is scaled by a time similar to the relaxation time; Mudd and Furbish (2007) showed that it takes approximately $1/9$ of the relaxation time for a change in channel incision to reach the divide. For $D = 0.01 \text{ m}^2 \text{ yr}^{-1}$ and $L_H = 25 \text{ m}$, $t^* = 0.1$ equates to 6250 years. This suggests that in rapidly eroding landscapes, the $E^* \text{ vs } R^*$ method could be used to identify changes in channel
incision, perhaps brought about by changing tectonic activity, that has occurred in the last few millennia.

How reliable is the assumption of constant transport coefficients in the face of varying climate?

In the previous section, the models used to quantify landscape transience based on topographic data and other landscape features relied on an assumption that the sediment transport coefficient, $D$, could be independently quantified. A number of strategies have been used to calculate the transport coefficient. Two commonly used techniques for calculating $D$ are: i) to compare measurements of flux against topographic gradients (e.g., McKean et al., 1993; Heimsath et al. 2005; Jungers et al., 2009) or ii) to compare long term erosion rates to topography (e.g., Roering et al., 1999; Heimsath et al. 1999; Small et al. 1999; Roering, 2008, Riggins et al., 2011; Hurst et al., 2012). Often these methods rely on fluxes or erosion rates derived from in-situ cosmogenic nuclides, which average erosion rates over thousands of years. In addition, topography also evolves over millennia. A critical question therefore is this: if the sediment transport coefficient has changed due to, for example, climate induced vegetation changes, what then does the transport coefficient inferred from today’s topography represent?

I attempt to constrain the meaning of a topographically derived transport coefficient by running simulations that involve a step change in the transport coefficient, $D$. The
analogue for this step change is the case of a landscape that experiences a vegetation
shift, for example from grassland to forest (e.g., Roering et al., 2004; Hughes et al., 2009).

The hillslope evolves based on the nonlinear sediment flux law described by equation (9);
the numerical implementation is the same as that of Hurst et al. (2013a). In addition, the
concentration of $^{10}$Be being removed from the hilltop is calculated using equation (3). This
concentration is inserted into equation (5), yielding the apparent erosion rate, i.e. the
erosion rate one would calculate if one assumed steady erosion. An apparent sediment
transport coefficient is then calculated using the relationship (Hurst et al., 2012):

$$D_{\text{app}} = \frac{\xi_{\text{app}} \rho_T}{C_{HT} \rho_s}$$

The error in the transport coefficient, $D$, is then calculated as a function of time since the
step change. These errors are plotted in Figure 5. Immediately after the step change in
$D$, errors are large because topography reflects the old transport coefficient. As time
passes, however, topography adjusts to the new transport coefficient until errors are
small.

We can examine Figure 5 in the context of estimating $D$ at a field site. How much time
must elapse after a change in $D$ so that the estimate of $D$, calculated using CRN-derived
erosion rates and topographic curvature, is a reasonable approximation for the current
value of $D$ in the landscape? In Figure 5, the time is scaled by dimensionless time, $t^*$ (see previous section), which is calculated based on the transport coefficient after its step change. As a visual aid, I have indicated the 10% error; errors in cosmogenically derived erosion rates are on the order of 10% and given the noise of hilltop curvature (e.g., Hurst et al., 2012; Hurst et al., 2013b) we would likely only be able to estimate $D$ to within 10% even if the landscape was in perfect steady state. It should be noted that the $t^*$ values reported here assume a fixed hillslope length, which might change if only one side of the divide is "pushed" by a pulse of incision (e.g., Mudd and Furbish, 2005). This is why Hurst et al. (2012) focussed on ridgetops with similar slopes on either side of the hilltop.

Figure 5 shows that under a wide range of background erosion rates and relative changes in the transport coefficient, the time to fall within 10% error usually occurs by $t^* = 0.01$. For $D = 0.001 \text{ m}^2 \text{ yr}^{-1}$, $L_H = 25 \text{ m}$, this means the apparent sediment transport coefficient will be within 10% of the actual value within 625 years, which by geological standards is short. Figure 6 shows the value of the hillslope relaxation time ($=0.4 \times t^*$) as a function of $D$ and $L_H$. Reported values of $D$ vary over several orders of magnitude but most lie between 0.01 and 0.001 $\text{ m}^2 \text{ yr}^{-1}$ (Hurst et al., 2013b). The implication of these results is that in most cases a dramatic change in vegetation would have had to occur within the past few millennia for estimates of $D$ based on CRN derived erosion rates and hilltop curvature to be in error by more than 10%. This is encouraging since the time to reduce the error between apparent and true $D$ is, for most landscapes, shorter than the time elapsed since the younger Dryas period that featured widespread vegetation changes (e.g., Schuman et al., 2002).
What effect do varying styles of transience have on the concentration of in-situ cosmogenic nuclides collected in stream sediments?

Thus far I have discussed the use of in-situ cosmogenic nuclides such as $^{10}\text{Be}$ and $^{14}\text{C}$ in single regolith profiles but in fact, many, if not most studies that constrain erosion rates, use the technique of detrital CRNs (e.g., Brown et al., 1995; Bierman et al., 1996, Granger et al., 1996). To obtain basin-wide erosion rates, sediment leaving a basin is sampled and the concentration of cosmogenic nuclides in these sediments is used to calculate erosion rates. The technique can even resolve erosion rates if they are spatially heterogeneous (see Granger et al., 1996), but one of the assumptions is that erosion rates should be constant in time in order to estimate the average erosion rate. What if erosion rates are transient? I answer this question using numerical simulations.

The model solves a simple governing equation that combines the hillslope flux law of equation (8) with the stream power law that is a simple approximation of channel incision (e.g., Howard, 1994). The resulting conservation equation is:

$$\frac{\partial \zeta}{\partial t} = -\nabla \cdot \left[ \frac{D \nabla \zeta}{1 - \left( \frac{\nabla \zeta}{S_c} \right)^2} \right] - K A^m S^n + U. \quad (13)$$

where $K$ [L$^{-1}$T$^{-1}$] is an erodibility coefficient, $A$ [L$^2$] is the drainage area, $S$ [dimensionless; L/L] is the topographic gradient and $U$ [L T$^{-1}$] is the tectonic uplift rate. For simplicity, I do not consider density conversion between rock and soil. The FASTSCAPE
algorithm of Braun and Willett (2013) is used to solve channel incision, so S is determined along lines of steepest descent to the lowest of the eight neighboring cells (the D8 flow method). For computational efficiency, a D8 scheme is also used to calculate drainage area. Equation (8) is solved with a two dimensional version of the implicit method used in Hurst et al. (2013a). The philosophy of this simple approach is that the model should capture the essence of competition between advective (e.g., fluvial) and diffusion like (e.g., hillslope) erosion processes (c.f., Perron et al., 2009).

The model has its elevation fixed on the north and south boundaries, and the east and west boundaries are periodic. The concentration of $^{10}$Be is solved within soil columns throughout the domain using equation (3). This assumes that erosion rates are constant over a model timestep (in the simulations presented here the timestep is 20 years), but erosion rates can change between timesteps. It is assumed that there is no storage of sediment within the channels: once a particle is eroded from the surface of the model it is instantaneously delivered as virtual stream sediment to be queried for the apparent erosion rate using equation (5). The contribution of each column to the collected concentration at the outlet is weighted by each column’s erosion rate: this reproduces the weighting in cosmogenic concentrations that are a consequence of greater fluxes originating from portions of the landscape that erode more quickly.

The model simulations are aimed at probing the effect of changing climatic and base level forcing on apparent erosion rates, similar to studies of Godard et al. (2013) and Braun et
al. (2015), but this study differs in its attention to the effect of landscape perturbation on cosmogenic concentrations. Tectonic variations are simulated with time varying uplift. To approximate the effect of a changing climate on the landscape, I vary the fluvial erodibility coefficient, $K$, and the hillslope sediment transport coefficient, $D$. The model was tested with both 40 kyr and 100 kyr cycles. These were to mimic the dominant climate cycles over the past few million years (e.g., Lisiecki and Raymo, 2005). There is no obvious reason to suspect tectonic activity will vary over the same timescale as climate, but uplift is varied over the same period in order to make comparisons between forcings with the same period of variation.

Two initial landscapes, formed under constant forcing, were used; these were small basins of 5 km$^2$. The landscapes have an average uplift rate of 0.2 mm yr$^{-1}$; each simulation begins with a topography adjusted to this erosion rate, with cosmogenic nuclide concentrations set to the steady state concentration (i.e., both the apparent and the actual erosion rates are 0.2 mm yr$^{-1}$ at the beginning of the simulations). All simulations are run with $m = 0.5$, $n = 1$, $S_c = 1$, and all have a mean $D = 0.005$ m$^2$ yr$^{-1}$. The first set of simulations have a mean $K = 0.00001$ yr$^{-1}$. The second set of simulations are run with higher $K$ values (0.00005 yr$^{-1}$); these are heavily dissected but the high fluvial erodibility coefficient allows tectonic signals to propagate farther into the model domain over an erosion cycle. Relief for the simulations with lower and higher $K$ values begins at ~200 and ~27 meters, respectively (Figure 7).
The apparent and actual erosion rates for different scenarios are shown in Figure 8. To interpret these plots, it is useful to step back to findings from point models of erosion rates. Several authors have investigated the effect of time varying erosion rates on CRN concentrations (Bierman and Stieg, 1996; Small et al., 1997, Heimsath 2006; Schaller and Ehlers 2006), and their results are useful in understanding CRN concentrations in the context of basin-wide changes in concentrations.

Schaller and Ehlers (2006) explored periodic forcing of erosion rates and found that the time series of apparent erosion rates was damped compared to the time series of erosion rates. This damping was a function of the mean erosion rate: slower erosion rates featured more damping. In addition, the time series of apparent erosion rates lagged behind that of the actual erosion rates, and the phase shift was a function of the periodicity of the erosion rate variation, with longer period changes in erosion rates resulting in greater phase shift between the apparent and actual erosion rate.

Another important feature of CRN concentrations in the face of time varying erosion rates was identified by Heimsath (2006): when erosion rates slow significantly, but then speed up once again, there is not enough time to accumulate the nuclides necessary for the apparent erosion rate to reflect the short period of slow erosion rates. This effect can lead to asymmetric damping where the fastest erosion rates are reflected in cosmogenic concentrations but the slowest erosion rates are not (Heimsath, 2006). With these results in mind, we can now examine how apparent erosion rates determined from basin-wide
CRNs are affected by variations in forcing factors such as climate and tectonics. I consider only the simplified case of landscapes without significant erosion from mass wasting processes, which can further cloud interpretation of basin averaged CRN concentrations (e.g., Niemi et al., 2005; Yanites et al., 2009; West et al., 2014)

**Top-down and bottom-up transience**

Arguably the most striking feature of the time series of apparent erosion rates is the difference between simulations featuring transient uplift rates and those featuring transient $K$ and $D$ (Figures 8 and 9). Panels a, b, g and j in Figure 8 depict landscapes in which the uplift rate varies in time, whereas in the other panels uplift is held constant while $K$ and $D$ vary. Where uplift rates vary, the erosion rates and the apparent erosion rates determined from $^{10}$Be concentrations are significantly damped relative to changes in uplift rates. Changes in $K$ and $D$ can result in large variations in erosion compared to changes in uplift: in Figure 8 uplift has an amplitude of 100% of the mean uplift, whereas $K$ and $D$ only vary with an amplitude that is 30% of their mean values, yet the variations in erosion rates are much greater for $K$ and $D$.

In landscapes with changing erosion rates at base level, signals propagate upstream and upslope (e.g., Whipple and Tucker, 1999). These signals then move up the channel network at a rate controlled by drainage area and the fluvial erodibility coefficient (e.g., Whipple and Tucker, 1999; Royden and Perron, 2013) and then spread to hillslopes (e.g., Mudd and Furbish, 2007; Reinhardt et al., 2007; Prince and Spotila, 2013).
these signals propagate upslope, they can be thought of as “bottom-up” drivers of landscape transience (e.g., Bishop, 2007).

On the other hand, if erodibility coefficients or sediment transport coefficients change, we might reasonably expect the entire landscape to act in concert. For example, both theory (e.g., Whipple and Tucker, 1999) and field studies (e.g., Moon et al., 2011; Ferrier et al., 2013) suggest that precipitation plays a role in determining the erodibility coefficient ($K$) of bedrock channels. As a result of climate change, precipitation may change over an entire landscape. Similarly, the hillslope sediment transport coefficient is thought to be a function of landscape properties, such as vegetation cover, that respond to climate (e.g., Hanks, 2000; Anderson 2002; Dunne et al., 2010; Hurst et al., 2013b; Pelletier et al., 2013; Schlunegger and Norton, 2013; Acosta et al., 2015; Andersen et al., 2015). Such changes may be widespread: for example an entire landscape may shift from forest to grassland (e.g., Roering et al., 2004; Hughes et al., 2009) or diffusive processes may transition from highly efficient frost-related processes to less efficient bioturbation-driven processes (e.g., Hales and Roering, 2009); fluvial processes may then be affected by top down controls on sediment flux (e.g., Wobus et al., 2010). Because such changes act over an entire landscape, these landscapes can be considered to have a “top-down” control on transient erosion. This top down forcing is distributed over the entire catchment, and thus the erosion rate closely follows the changes in sediment transport or erodibility coefficients, mirroring the results presented by Braun et al. (2015).
Bottom up forcing

Consider the bottom-up scenario (that with varying uplift, $U$). If uplift forcing is periodic, one finds some parts of the landscape responding to rapid erosion forcing and others responding to slow erosion forcing (Figure 9a-d). For example, in Figure 9b, a period of rapid uplift introduces a wave of fast erosion (as seen in the red area near the boundaries) that has propagated upslope in Figure 9c. The remnant of a previous cycle of fast erosion in Figure 9c exists along the ridgeline. Because the landscape contains both rapidly eroding and slowly eroding areas the erosion rate averaged over the entire landscape is averaged to a value that is very close to the mean uplift rate (Figure 8a,d,g, and j). This averaging is reflected in the apparent erosion rates calculated from basin averaged CRN concentrations. It is important to note that this spatial averaging is reliant on the fact that uplift is periodic: rapid erosion rates from one cycle are counterbalanced by slow erosion rates from another. This is not the case if there is a step change or monotonic change in the rate of base level fall (e.g., Rheinhardt et al., 2007; Willenbring et al., 2014).

Top down forcing

Now consider landscapes with top down control on varying erosion. The simulations are set so that the amplitude is 30 % of the mean value, so for example if the mean value of $D$ is 0.005 m$^2$ yr$^{-1}$, then $D$ varies between 0.0035 and 0.0065 m$^2$ yr$^{-1}$. In these landscapes there still is some spatial variation in erosion rates but the dominant behavior is that when transport coefficients are at their maximum values the erosion rate across the landscape is higher than the time-averaged mean erosion rate (Figure 9f), and conversely low values of the $K$ and $D$ coefficients lead to low erosion rates across the landscape (Figure 9h).
Between peaks or and troughs in $K$ and $D$, one can see spatial variations in the erosion rate (Figures 9e, g). This is because these landscapes are more sensitive to changes in $D$ than $K$ (compare, for example, Figure 8b and c, where only small variability in erosion rates from variations in $K$ are apparent). Because hillslope adjust rapidly, their slopes adjacent to channels adjust more quickly to changing $D$, but the channels must accommodate these changing side slopes (e.g., in Equation 13 the erosion rate in the channel depends on all adjacent pixels). At positions in the landscape with large drainage areas, hillslopes have almost no role because erosion is controlled by channel incision. In contrast, near the tips of the drainage network the effect of side slopes is more substantial since the drainage area along the channel is low and hillslope sediment fluxes are of a similar magnitude to removal of mass by the channel. This could potentially be an artefact of the relative values of $K$ and $D$ in the simulations; higher values of $K$ would reduce this effect. However preliminary simulations featuring $K$ values high enough for channels and hillslopes to respond on similar timescales resulted in unrealistic drainage densities. Landscape dissection is a function of the relative magnitude of $K$ and $D$ (Tucker and Bras, 1998; Perron et al., 2009) and to arrive at landscapes with qualitatively reasonable hillslope lengths (i.e. on the order of tens of meters) I was unable to generate landscapes with channels that responded faster than hillslopes. Relatively fast hillslope response has been documented in natural landscapes (Reinhardt et al., 2007; Hurst et al., 2012) but an exhaustive exploration of $K$ and $D$ to see if this is an expected feature of landscapes is beyond the scope of this study.
Because erosion rates respond to changing $K$ and $D$ values across the landscape, the erosion rate and the concentration of basin averaged CRNs reflects closely the variation in these parameters (Figures 8b, e, h and k). As predicted in one dimensional simulations, there is greater lag between the basin averaged erosion rate and the apparent erosion rate calculated from CRN concentrations in landscape forced with higher frequency oscillations (compare figures 8b and e).

In summary, bottom up changes tend to lead to strong spatial variations in erosion rates, but the average erosion rates from basins affected by bottom up forcing remain relatively constant. This is reflected in CRN concentrations. On the other hand, top down forcing results in a more spatially heterogeneous erosion pattern that features strong temporal variation.

**Basin-scale topographic indicators of transience**

Having examined techniques for determining landscape transience on a hillslope scale, following on from CRN tracing of landscape transience, I now briefly discuss basin and regional scale tracing of landscape transience. Perhaps the most widespread method for looking for differing erosion rates of wide areas is to quantify how steep channels are. Over a century ago, G.K. Gilbert (1877) recognized that topographic gradients drive erosion; in his seminal 1877 report on the geology of the Henry Mountains, he said “we have already seen that erosion is favoured by declivity. If declivity is great the agents of erosion are powerful; where it is small they are weak; where there is no declivity they are powerless.” Topographic gradient is still considered one of the driving factors of erosion.
in channels, along with substrate composition, sediment supply and discharge. The latter
varies systematically with drainage area so even if substrate and sediment supply are
equal the channel gradient must be normalized for discharge if one is to compare the
erosive potential of one channel to another.

It has been suggested that in eroding landscapes featuring bedrock rivers, channel
erosion can be described by the stream power law $E = KA^mS^n$ (e.g., Howard and Kerby,
1983; Whipple and Tucker 1999). The equation can be rearranged as $S = (E/K)^{1/n} A^{m/n}$.
The term $(E/K)^{1/n}$ is frequently recast as the steepness index, $k_s$ (which is equal to $SA^{m/n}$
according to the equation), or the normalised steepness index $k_{sn}$ if the ratio $m/n$ is set
to a fixed reference value (Wobus et al., 2006). The steepness index can be calculated
by plotting the logarithm of drainage area against the logarithm of slope (both of these
quantities are easily extracted from digital elevation models): if slope-area data is plotted
in log-log space, the gradient of a regression line will be $-m/n$ and the intercept where
log($A$) = 0 will be $k_s$. Normalized steepness index can be calculated numerically as $k_{sn} =
SA^{m/n}$ for a fixed value of $m/n$, frequently set to 0.45 (Wobus et al., 2006). Even if the
stream power law is an imperfect description of channel incision (c.f., Lague, 2014), one
can still calculate $k_{sn}$ to compare the relative steepness of channels from a purely
geometric perspective. The channel steepness index has been used widely to detect
regions of anomalously steep channels; these channels can indicate, for example, areas
with relatively high tectonic uplift rates (for reviews, see Wobus et al., 2006 and Kirby and
Whipple, 2012).
Another method to investigate the relative steepness of a channel is to use the channel elevations themselves, rescaled either by integrating drainage area as a function of flow distance (e.g., Royden et al., 2000; Perron and Royden, 2013), or by rescaling flow distance by the $m/n$ ratio (e.g., Smith et al., 2000; Pritchard et al., 2009). A number of authors have used transformed river profiles to calculate erosion histories using inverse modelling (e.g., Roberts and White, 2010; Fox et al., 2014; Goren et al., 2014; Fox et al., 2015; Glotzbach, 2015; Rudge et al., 2015), and these authors have provided valuable constraints on the transient uplift histories of Southern Africa, Australia, Taiwan, the Andes and individual mountain ranges in California.

One potential pitfall of inversion studies is that if the slope exponent, $n$, does not equal unity, then dynamic information about changing uplift or erosion rates are not entirely preserved by channels (Royden and Perron, 2013). Royden and Perron (2013) demonstrated that for $n > 1$, channel segments generated by periods of faster uplift will consume those generated by slower uplift, whereas for $n < 1$ channel segments generated by slow uplift will consume those generated by rapid uplift; in both cases information about the past is lost. There is evidence that the slope exponent, however, is often not unity (e.g., Snyder et al., 2003; Ouimet et al., 2009; DiBiase et al., 2010; Whittaker and Boulton 2012; Lague 2013; Croissant and Braun, 2014). Even if information is lost, however, one may use statistical methods to look for channel reaches with varying
channel steepness, in order to identify reaches that may be eroding at different rates to their neighbors without assuming a historical forcing (e.g., Mudd et al., 2014).

Another method of looking for landscape transience is to compare a flow length coordinate, normalized for drainage area, across drainage divides. Royden et al. (2000) suggested a coordinate transformation:

$$\chi = \int_{x_b}^{x} \left( \frac{A_0}{A(x)} \right)^{m/n} \, dx, \quad (14)$$

where $A_0$ [L$^2$] is a reference drainage area, introduced to ensure the integrand in equation (14) is dimensionless, $x$ [L] is the distance along the channel and $x_b$ [L] is the location of local base level. The transformed coordinate, $\chi$, has dimensions of length. If channel erosion can be described by the stream power law, then channel elevation can be related to $\chi$ with (Perron and Royden, 2013; Royden and Perron, 2013):

$$\zeta(x) = \zeta(x_b) + \left( \frac{E}{K A_0 m} \right)^{1/n} \chi, \quad (15)$$
Here the gradient of the $\chi$ profile, $M_\chi$, that is the channel profile cast as elevation, $\zeta$, plotted as a function of $\chi$, will be indicative of the erosion (or uplift rate if balanced by erosion, i.e., tectonic steady state):

$$M_\chi = \left(\frac{E}{KA_0^m}\right)^{1/n}, \quad (16)$$

where the chi gradient, $M_\chi$, is related to the channel steepness index by $M_\chi = A_0^{mn/m} k_s$.

Based on equation (15), Willett et al. (2014) reasoned that $\chi$ is therefore a metric for the steady state elevation of a channel, and therefore in a steady state landscape $\chi$ must be balanced across divides. If it is not, then the side of the divide with lower $\chi$ will push the divide until equilibrium is restored, a process equivalent to the pushing of divides away from more rapidly eroding channels described by Mudd and Furbish (2005). Figure 10 shows an example of the chi coordinate across divides near Sorbas, Spain, the site of a well-documented river capture (e.g., Harvey and Wells, 1987; Stokes et al., 2003). Willett et al. (2014) and Yang et al. (2015) have used this method to identify potentially widespread areas of stream piracy and drainage reorganisation in the Appalachians of the United States and the Three Rivers region of China. Due to the ease of calculating $\chi$. 
from topographic data, this method promises to help identify regions of transient landscape evolution, as long as authors are careful to account for changes in discharge and bedrock erodibility that may complicate comparisons of the $\chi$ across adjacent basins.

Conclusions: strategies for detecting landscape transience in eroding landscapes

In upland, eroding landscapes, detection of transience can be challenging because there are limited depositional archives from which to infer past changes in erosion rates (c.f., Whittaker et al., 2010). However, with judicious use of both topographic and isotopic information, we can gain insight into a landscape’s past. Cosmogenic nuclides are a powerful tool for quantifying erosion rates and soil production from the scale of individual soil profiles to entire basins, yet they have been less frequently used to test hypotheses about the past evolution of land surfaces. I have demonstrated that it is possible to use paired cosmogenic nuclides to detect changes in past erosion rates resulting from block removal or acceleration of erosion rates. However, the two nuclides used in the pair must have significantly different decay rates; the only practical way to detect changing erosion rates is by combining the relatively short half-life of cosmogenic in-situ $^{14}$C with a longer lived nuclide such as $^{10}$Be. Changes in erosion rates or removal of mass is unlikely to be detected if background erosion rates are faster than $\sim 0.1$ mm yr$^{-1}$. When background erosion rates are slower, however, detection is possible. If background erosion rates are $\sim 0.01$ mm yr$^{-1}$, block removal can be detected for up to $\sim 10$ kyr after the event. Accelerations in erosion rate can be detected between $\sim 500$ years up to $\sim 50$ kyr after the
event. Decelerations can be detected after \( \sim 2 \) kyr up to, in extreme cases, 500 kyr after the event.

When we move to the basin or landscape scale, it becomes extremely difficult, if not impossible, to detect landscape transience from basin-wide CRN concentration measurements, and the interpretation of these concentrations is fraught with danger because the response of apparent erosion rates, determined by inverting CRN concentrations, exhibits markedly different behavior depending on how landscape transience is forced. If oscillating landscape transience is forced by changing base level (e.g., through changing sea level or tectonic uplift where differential motion occurs along a fault), then apparent erosion rates will reflect a mean erosion rate averaged over several uplift cycles rather than the current uplift rate. These apparent erosion rates will reflect the actual, basin averaged, erosion rates, but will bear little resemblance to local erosion rates. On the other hand, if erosion rates are transiently forced by climate that affects either channel erodibility or the hillslope sediment transport coefficient, then apparent erosion rates will track the forcing closely over the entire basin. Thus if one is to identify whether apparent erosion rates reflect recent erosion rates or a long term mean, and are consistent with local erosion rates, one must have some constraint on the nature of the transient forcing.

Topography can complement, or be used in lieu of, CRN data to detect landscape transience. In soil mantled landscapes, the relationship between ridgetop curvature and
hillslope relief can be a powerful indicator of landscape transience. Roering et al. (2007) demonstrated that normalized forms of relief ($R^*$) and hilltop curvature ($E^*$) should lie on a single curve if a hillslope is in steady state. Deviations from this curve, therefore, should indicate landscape transience, as demonstrated by Hurst et al. (2013a). In this contribution I show that one should be able to resolve a doubling of erosion rate using this technique, and that the signal should persist for hundreds to thousands of years in most landscapes.

Moving to the scale of basins, both channel steepness, as measured by the normalised steepness index, $k_{sn}$, or the chi gradient, $M_\chi$, can be indicative of changing erosion rates and landscape transience where other factors, such as sediment supply, or channel substrate, do not vary substantially. The chi coordinate, a coordinate derived by integrating drainage area over channel length, can also be used to identify landscape disequilibrium by looking for variation across drainage divides. Thanks to both geochronologic and topographic tools, geomorphologists now have a variety of tools for examining landscape disequilibrium at scales ranging from single points on the landscape to entire basins. These tools may be used to reconstruct past erosion rates occurring over hundreds to hundreds of thousands of years.
Acknowledgements

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Software and Data Availability

Scripts for plotting the figures and the source code and data for model simulations are available through github. Figures 1, 2, 8 and 9 are plotted with scripts from https://github.com/simon-m-mudd/CRN_model_plotting. For Figures 3-6 see https://github.com/simon-m-mudd/OneD_hillslope. Figure 7 is plotted using scripts from https://github.com/simon-m-mudd/LSDMappingTools. Figure 10 is generated using code available at https://github.com/LSDtopotools/LSDTopoTools_AnalysisDriver.

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Ecosystem Studies Lidar, an emerging remote sensing technology that directly measures
the three-dimensional distribution of plant canopies, can accurately estimate vegetation
structural attributes and should be of particular interest to forest, landscape, and global


Figure Captions

**Figure 1.** Ratio of the apparent erosion rate calculated from $^{14}$C to the apparent erosion rate calculated from $^{10}$Be if a block is removed at some time in the past. The depth of block removal ($d_{br}$) is listed in each panel. The dashed and solid lines in each panel represent different background erosion rates. An erosion rate of 0.0026 g cm$^{-2}$ yr$^{-1}$ is equivalent to an erosion rate of 0.01 mm yr$^{-1}$ if the rock density is 2.6 g cm$^{-3}$. The shaded area denotes a 20% difference between erosion rates predicted by in-situ $^{14}$C and $^{10}$Be; differences below this are likely undetectable.
**Figure 2.** Ratio of the apparent erosion rate calculated from $^{14}$C to the apparent erosion rate calculated from $^{10}$Be if there is a step change in erosion rate at some time in the past. Panel a. shows scenarios in which erosion rates increase, whereas panel b. shows scenarios in which erosion rates decrease. The shaded area denotes a 20% difference between erosion rates predicted by in-situ $^{14}$C and $^{10}$Be; differences below this are likely undetectable.

**Figure 3.** Responses of hillslopes to transient perturbation. Panel a. shows in black the curve of dimensionless relief vs dimensionless erosion rate predicted by Roering et al., (2008). The red and green curves show the effect of an increase of erosion rate of two and one orders of magnitude, respectively; stars represent starting and ending positions on the steady state curve. Increased erosion rates result in hillslopes that plot above the steady state curve. Panel b. shows a dimensionless hillslope profile. The blue profile is the initial condition, the green curve shows the profile after a dimensionless time ($t^*$) of 0.05 (that is, after 0.05 times $L_H^2/D$) following an order of magnitude increase in erosion rate. The dashed grey curve shows a steady state hillslope with the same ridgetop curvature as the green profile, demonstrating that the steady profile has lower relief than the transient profile and illustrating why hillslopes that have increased erosion rates plot above the steady profile in dimensionless relief vs. apparent erosion rate plots.

**Figure 4.** Detection and duration of transience in hillslope profiles in the case of a step change in erosion rate. Panel a. shows maximum differences between predicted and
measured dimensionless relief ($R^*$) as a function of both initial erosion rate and the ratio between initial and final erosion rates. Perturbations within the shaded area are unlikely to be detected using $E^*$ vs $R^*$. The size of the shaded region will depend on the landscape and is determined by the standard error in $R^*$. Panel b. shows the time to the maximum difference in $R^*$; steeper landscapes (higher $E^*$) have shorter response times because more of the landscape is at critical slope (i.e., topographic gradients approaching $S_c$) and therefore responds very quickly to changes in channel incision rates, consistent with the predictions of Mudd and Furbish (2007). Note the noise at slower erosion rates is an artefact of the adaptive time step of the numerical model.

**Figure 5.** Model predictions of error in estimated transport coefficient ($D$) given a step change in this coefficient. The initial and final values of $D$ are denoted by $D_i$ and $D_f$, respectively. Panels a. and b. show two different background erosion rates. The actual transport coefficient is set within the model. The apparent $D$ is calculated by dividing the apparent erosion rate, as determined by $^{10}$Be in the soil column, by the ridgetop curvature. Shaded areas represent errors of 10% or less. The noise in the data comes from two sources: i) curvature is a numerical approximation and ii) apparent erosion rates are calculated from particles advected toward the surface of the model; the topmost particle is used to calculate apparent erosion rates using equation (5) but slight errors occur because the particle is not always located exactly at the surface of the model.
Figure 6. Hillslope relaxation time \( (4D/(\pi L_H)^2) \); Mudd and Furbish, 2007) plotted as a function of the transport coefficient \( D \) and hillslope length \( L_H \).

Figure 7. Initial landscapes for the transient landscape simulations. Note the difference in color scale between the two figures.

Figure 8. Apparent and actual erosion rates under different transient scenarios. Actual erosion rates are the landscape averaged erosion rates from the previous timestep, whereas the apparent erosion rates are calculated based on simulated concentrations of \(^{10}\)Be emerging from the landscape. The entire landscape is eroding, there is no storage of particles once they are eroded. Variation of forcing parameters has either a 100 kyr period (a-c; g-i) or 40 kyr period (d-f; j-l). For varying \( K \) and \( D \), the parameters are varied with amplitude of 0.3 times the mean value.

Figure 9. Examples of the spatial distribution of erosion rates for different landscape evolution scenarios. All simulations have a period of 100 kyr and a \( K \) value of 0.00005 yr\(^{-1}\). Color scale is the same for all panels. Small plots show the erosion and uplift time series, and the times of the panels are indicated with vertical dashed black lines. Panels a-d show the simulation that is depicted in Figure 8g, and panels e-h show the simulation depicted in Figure 8f.
**Figure 10.** An example of disequilibrium $\chi$ coordinates across drainage divides near a channel capture location. The capture point was identified by Harvey and Wells (1987) near Sorbas, Spain. Catchments with lower $\chi$ values are predicted to be ‘pushing’ the divides toward catchments with higher $\chi$ values.
\[
\varepsilon = 0.026 \text{ g cm}^{-2} \text{ yr}^{-1}
\]

\[
\varepsilon = 0.0026 \text{ g cm}^{-2} \text{ yr}^{-1}
\]

\[
\varepsilon \text{ app from } ^{14}\text{C}/\varepsilon \text{ app from } ^{10}\text{Be}
\]

\[
d_{br} = 50 \text{ g cm}^{-2} \text{ (~20 cm where } \rho = 2.6 \text{ g cm}^{-3})
\]

\[
d_{br} = 150 \text{ g cm}
\]

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