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Optimal setting of time-and-level-of-use prices for an electricity supplier

Miguel F. Anjos[a,b], Luce Brotcorne[c], Juan A. Gomez-Herrera[a,*]

[a] GERAD and Department of Mathematics and Industrial Engineering, Polytechnique Montreal, C.P. 6079, Succ. Centre-Ville, Montreal, Quebec, H3C 3A7, Canada
[b] School of Mathematics, University of Edinburgh, James Clerk Maxwell Building, King's Buildings, Peter Guthrie Tait Road, Edinburgh EH9 3FD, Scotland, United Kingdom
[c] Inria Lille-Nord Europe, Team Inocs, 40 Avenue Halley, 59650 Villeneuve d'Ascq, France

Abstract

This paper presents a novel price setting optimization problem for an electricity supplier in the smart grid. In this framework the supplier provides electricity to a residential load aggregator using Time-and-Level-of-Use prices (TLOU). TLOU is an energy pricing structure recently introduced in the literature, where the prices vary depending on the time and the level of consumption. This problem is formulated as a bilevel optimization problem, in which the supplier sets the prices that maximize the profit in a demand response context, anticipating the reaction of a residential load aggregator that minimizes total cost. These decisions are made in a competitive environment, while explicitly considering the aggregator's load shifting preferences and the level of consumption, and ensuring a user-friendly price structure. The optimization problem is reformulated as a single-level problem to be solved using off-the-shelf solvers. We present computational experiments to validate the performance of TLOU, and provide insights on the relationship between the user's demand flexibility, the capacity profile and the resulting structure of prices. We show that the supplier's economical benefit is increased up to 10 % through the implementation of this type of demand response program, while providing savings of up to 6% for the consumers.

Keywords: Demand-Response, Price-Setting, Bilevel Optimization, TLOU, Smart Buildings, Smart Grid.

1. Notation

Sets:

$t \in T$: Time frames
$i \in I$: Generation levels

Parameters:

π_i^G : Generation cost of level i (¢/kWh)
π_t^R : Ramping cost in time frame t (¢/kWh)
K_i : Generation capacity of resource i (kWh)
$\bar{Z}$: Ramping allowed at no cost (kWh)
Ψ : Maximum number of changes in the energy prices
Γ_t : Shifting cost in time frame t (¢/kWh)
D_t : Net demand in time frame t (kWh)
C : Power capacity limit (kW)
Ω_t : Maximum additional energy consumption in time frame t (kWh)

Variables:

π_t^H : Higher energy price in time frame t(¢/kWh)
π_t^L : Lower energy price in time frame t (¢/kWh)
x_{it} : Energy produced by generation level i at time frame t (kWh)
z_t : Energy ramping penalized in time frame t (kWh)
α_t : $\begin{cases} 1 & \text{if there is a change in the energy price between periods } t \text{ and } t+1 \\ 0 & \text{Otherwise} \end{cases}$
y_t^H : Energy consumption at higher price in time frame t (kWh)
y_t^L : Energy consumption at lower price in time frame t (kWh)
w_t^+ : Over-consumption with respect to energy demand D_t in time frame t (kWh)
w_t^- : Under-consumption with respect to energy demand D_t in time frame t (kWh)
v_t : Energy bought to the supplier competitor in time frame t (kWh)

Auxiliary Variables:

λ_t^a, λ^b : Dual variables associated to equality constraints
$\mu_t^{(a,\ldots,g)}$: Dual variables associated to inequality constraints
$\rho_t^{(a,\ldots,g)}$: Binary variables to linearize the complementary slackness condition

2. Introduction

Electricity is a vital resource for modern societies. We benefit from it thanks to the power systems that generate,

*Corresponding author
Email address: juan.gomez@polymtl.ca (Juan A. Gomez-Herrera)

transport and deliver electricity on real-time basis. Ensuring this real-time balance between supply and demand is one of the most important tasks of the system's operators since it affects the operational costs, the user satisfaction, the reliability of the grid. In this context, demand response (DR) is one of the most effective ways to facilitate the aforementioned balance.

According to [1], DR is defined as "changes in electric usage by demand-side resources from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized ". The potential benefits of DR increase in systems with large penetration of energy renewable sources, since a more flexible demand facilitates the integration of intermittent and non controllable generation. Additionally, the continuous development of smart grids allows a multidirectional communication among the multiple players in the power grid. This exchange of information enables better decisions and the opportunity to mobilize DR resources in a more effective way.

Typically, the DR programs can be classified in two groups: incentive-based programs and price-based programs [2]. In the incentive-based programs (IBP), the consumers commit to provide consumption reduction over a specified period of time in exchange of a financial incentive. In the price-based programs, the energy supplier or the system operator defines a set of varying energy prices. These prices reflect the generation marginal cost and therefore the load in the system during the day. Additionally, since there are different energy prices during the day, the end-users are encouraged to shift load in order to profit from the periods of time when the prices are lower.

Several DR pricing schemes are defined in the literature. The survey presented in [3] contains a comprehensive review of DR pricing programs and how these programs are integrated with different optimization-based approaches ([4], [5]).

Some of those price-based programs are currently available in different jurisdictions around the world [6]. One of the most common is Time-Of-Use (TOU), that divides a day into several time windows and fixes energy prices for each time window. In a similar way, Critical Peak Pricing (CPP) programs identify a time window during the day in which a critical situation can arise in the system. A significantly higher price is charged during the aforementioned time window if the critical situation occurs.

Authors in [7] proposed a TOU framework in which different prices are determined for different segments of the population with specific characteristics. The goal is to better incentivize the end-users to obtain the expected response and avoid rebound peaks.

Recently, the authors in [8] and [9] presented a new price scheme called Time-and-Level-of-Use. TLOU offers a similar time-windows-structure to that of the traditional TOU, including power capacity limits that enable multiple energy prices in each time window depending on the consumption level. In TLOU, the consumption below the capacity limit is charged at a cheaper price and the consumption above the capacity limit is charged at a more expensive price. The benefits for consumers of participating in a TLOU program are described in [8] and [9] . Additionally, [8] presents potential benefits on the generation side such as a more homogeneous consumption, reduction of rebound peaks, and the development of backup electricity services business models.

The levels of consumption allow TLOU to consider a *power perspective* that encourages load shifting in a more effective way while offering energy tariffs. We could think of TLOU as a generalization of TOU that incorporates features of power-oriented programs such as demand charges.

Although TLOU was initially developed for the residential and commercial energy sectors, authors in [10] have demonstrated its benefits for industrial customers.

The definition of energy prices is a key factor in the successful implementation of a DR priced-based program. If the prices are not sufficiently attractive users will not respond in an effective way and the system will not benefit from the DR program. On the other hand, if the prices are very appealing, the users can shift more load than required by the grid creating rebound peaks that can worsen the initial state of the system. This effect can specially occur in the context of smart buildings in which multiple end-users make optimal decisions locally. The survey presented in [3] highlights the importance and the potential of game theory approaches in the proper configuration of pricing programs.

In this regard, bilevel optimization is an effective way to define energy prices ([11], [12], [13]). Bilevel optimization is a paradigm in which one optimization problem is embedded within another. The outer optimization problem is commonly referred to as the leader's problem and the embedded optimization problem is commonly referred to as the follower's problem. The leader makes optimal decisions considering the follower's optimal reaction to the leader's decisions. This anticipation process can be specially useful to define DR pricing problems. Additionally, this type of problems are typically difficult to solve due it computationally complexity [14].

Bilevel optimization has been used in the energy price setting context by several works in the literature [13]. We consolidate and compare the most relevant works in Table 1. We identify key features such as the decision makers and their objectives, the solution approach used, and the resulting structure of prices.

We see in Table 1 different entities playing the role of leader, having profit as their main interest. When the end-users are the follower, it is common to consider some type of trade-off between cost and comfort to better take into account behavioral aspects and assess the impact of the designed price structures. As for the solution methods, reformulation seems to be very effective. Different algorithms using reformulation and/or decomposition meth-

	Leader		Follower		Solution	Resulting
Reference	Entity	Objective	Entity	Objective	method	prices
[15]	DRA	Profit	Market	Cost	RE	IBP
[16]	WG	Profit	DRA	Revenue	RE	Market
[17]	WG	Profit	Market	Social welfare	RE	Market
[18], [19], [20]	ES	Profit	End-users	C&C	RE	TOU
[21]	ES	Profit	DRA	Profit	RE	TOU
[22]	ES	cost	End-users	Cost	RE	TOU
[23]	System Operator	Welfare	ES	Revenue	DM	TOU
[24], [25], [26]	ES	Profit	End-users	C&C	DM	TOU
[27]	ES	Profit	DRA & End-users	C&C	DM & RE	TOU
[28]	ES	Revenue	End-users	Cost	DM	TLOU
This paper	ES	Profit	End-users	C&C	RE	TLOU

Table 1: Relevant literature review compared to the framework presented in this article. (ES: energy supplier, WG: wind generator, DRA: demand response aggregator, C&C: cost and comfort, RE: reformulation, DM:Decomposition methods.)

ods are used when multiple leaders or multiple followers are considered. The resulting prices range from day-ahead market prices to more traditional TOU structures. These energy prices depend on the hour of the day and frequently neglect the user's consumption level. Recently, authors in [28] presented the first bilevel optimization approach to set TLOU prices by profiting from the specific structure of the follower level problem. Although that paper provides significant insights about TLOU definition, it does not consider, load shifting provided by the end-users, generation costs and supplier's competitors.

In this regard, the objective of this article is to fill the need for a means to determine TLOU prices for an energy supplier that seeks to maximize profit, encouraging the users' load shifting in a more effective way, and avoiding undesired effects such as rebound peaks.

To achieve this objective we propose a novel framework that explicitly considers the users' shifting capabilities, the electricity generation perspective, and the level of consumption, in a competitive environment. Additionally we consider determining the structure of prices as part of the problem, which guarantees a user-friendly scheme that facilitates social acceptance. Our paper is the first one, to the best of our knowledge, to include these aspects in the definition of a price-based DR program.

This paper is structured as follows. The proposed approach is defined in Section 3, the computational experiments and results analysis are presented in Section 4, and the conclusion is given in Section 5.

3. Optimization problem

TLOU is a DR price-based program where energy prices vary depending on the time frame t and the amount of energy consumed. If the energy consumed in t is lower than the defined power capacity limit C_t the user pays the basic price π_t^L. If the user overshoots C_t he is charged π_t^H for the fraction of energy consumed beyond the limit.

The general operations of the electricity supplier are presented in Figure 1. The supplier is a generic entity that delivers electricity. It could be a public utility, a private generator or an energy retailer that buys electricity from energy generators via bilateral contracts. The supplier sets the energy prices in order to serve a population of smart homes represented by a residential load aggregator (RLA). The supplier makes decisions in a competitive environment in which the competitor offers a pre-defined energy flat rate. In other words, the RLA has the possibility to buy energy v_t from a competitor at a price Υ. This feature becomes specially significant due to the inelastic behavior of the electricity demand [29]. The competitor plays an important role since it represents a reference cost for the RLA and a boundary for the potential supplier's profit. We assume that the competitor offers a flat rate so that this does not encourage any shifting and therefore we can clearly illustrate the shifting effect of TLOU.

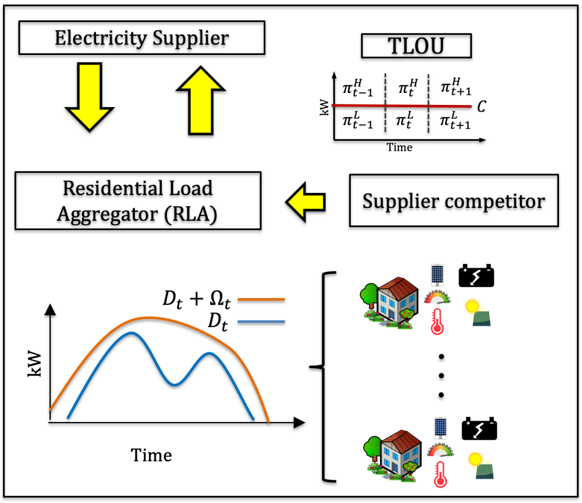

Figure 1: Supplier's operation

The RLA is an entity that aggregates and coordinates

DR actions of multiple residential end-users. Authors in [8] present a collaborative approach in which the RLA coordinates load shifting from a group of smart homes that have been offered TLOU prices.

We assume that the RLA is able to estimate the aggregated end-users' net demand profile D_t as well as the end-user's shifting limit profile $D_t + \Omega_t$ and transfer this information to the energy supplier. By considering the net demand we decouple the local generation and storage decisions from the activity of the RLA. Gathering the users' information is possible nowadays thanks to the massive adoption of smart meters and the continuous development of the smart grid.

Under these conditions, the supplier wants to determine π_t^L, and π_t^H to maximize profit and generate load shifting knowing that the RLA will minimize its own costs. Although the capacity C_t is an important part of the definition of TLOU, we assume that C_t is given and constant during the day. We discuss this idea more deeply in the Section 3.1.3 , and present some experiments on the selection of C_t in Section 4.

3.1. Shifting parameters

3.1.1. Shifting limit

The parameter Ω_t is defined as the maximum consumption increase that the RLA is able to provide at time t. In practice, the RLA estimates this parameter from the individual shifting flexibility offered by the end-users. Consequently $D_t + \Omega_t$ represents the maximum consumption in each time frame. Note that Ω_t represents an intrinsic bound of the user flexibility and it does not depend on the DR program.

We assume that the energy demand is always satisfied at the end of the day ($\sum_{t \in T}(y_t^H + y_t^L + v_t) = \sum_{t \in T} D_t$). Therefore if the RLA provides load shifting from time frame a ($y_a^L + y_a^H + v_a < D_a$ and $w_a^- > 0$), there exists at least one time frame b where the load is being shifted to ($y_b^L + y_b^H + v_b > D_b$ and $w_a^+ > 0$). The parameter Ω_t thus bounds the over-consumption in certain time frames as well as indirectly the under-consumption along the day. Notice that the end-users do not shift any load if $y_t^H + y_t^L + v_t = D_t, \quad \forall t \in T$.

3.1.2. Shifting penalty

The parameter Γ_t represents economically the discomfort generated from shifting. We define the shifting activity as the increase of consumption with respect to the demand (variable w_t^+). The impact of this increase on the user discomfort depends on when it occurs. According to [30], [31], and [32], there is a low on-peak to off-peak price elasticity energy consumption. In other words, shifting load from on-peak to off-peak hours requires a larger change in price. In our case, this price change is a discount that must account for the discomfort that the users perceive as result of the increase of their consumption during off-peak periods. Therefore, we assume that the discomfort is higher in the periods of time when the known demand is lower (i.e. the users do not want to use many electric devices or appliances).

Following this idea, we define the parameter Γ_t as:

$$\Gamma_t = \frac{\beta_t}{D_t}, \tag{1}$$

where β_t is a weight that accounts for the conversion of the dissatisfaction units to monetary units, ensuring the coherence in the order of magnitude with respect to the other cost parameters in the objective function.

Note that when D_t is equal to zero no consumption is possible, and Γ_t is equal to infinity (replaced by a large enough cost in the optimization steps).

3.1.3. TLOU capacity

The capacity C_t is an essential component of TLOU. The capacity profile has a clear meaning for the user perspective: the level of service that accounts for the main electricity needs. Determining these basic needs is not a trivial task since many factors must be considered, namely the behavior of individual loads, shifting limits, and user preferences. In this regard we can obtain C_t through forecasting and estimation approaches such as in [9], [33] and [34]. In this case, C_t becomes the constant parameter C for our optimization problem.

3.2. Bilevel model

In this section we present the bilevel model for the TLOU price setting problem. The leader (i.e the supplier) defines energy prices taking into account the decisions made by the follower (i.e. the RLA).

3.2.1. Leader problem

The supplier seeks to maximize his profit by charging the users TLOU prices. The objective function (2) is the difference between the income obtained from the energy sold to the users, and the generation and ramping costs. The generation cost is an increasing function that depends on the type of generation: baseline, load following, and peaking plant. The ramping cost accounts for high variations in the energy generation curve beyond the technical limitations of the generation technology. For example when the demand grows rapidly, the supplier could be forced to buy electricity from a third party supplier which generates a penalty π_t^R. In a similar way if the demand decreases at a great rate the supplier will face shut-down costs.

Constraints (3) ensure the balance between supply and demand for each time frame including the option to buy electricity to a third party. Constraints (4) are capacity bounds for each generator level.

Constraints (5) establish the ramping limit. For the sake of simplicity, ramp-up and ramp-down events are assumed to have the same limits and costs. Constraints (6)-(8) ensure that the TLOU structure is user-friendly, allowing up to $\Psi + 1$ different prices along the day for each level of price. The price structure must be simple to facilitate the end-users understanding and the acceptance of this novel DR program. The value of M is assumed to be large enough to account for the maximum allowed difference between two consecutive prices of the same level. In a similar way, constraints (9) guarantee that the price remains constant by at least N time frames. Constraints (6) can be replaced by $\pi_t^L - \pi_{t+1}^L \leq M\alpha_t$ and $\pi_{t+1}^L - \pi_t^L \leq M\alpha_t$ in order to remove the absolute value. Constraints (5) and (7) are linearized in a similar fashion.

$$\max_{(\pi,x,z)} \sum_{t\in T}(\pi_t^H y_t^H + \pi_t^L y_t^L) - \sum_{i\in I}\sum_{t\in T}\pi_i^G x_{it} - \sum_{t\in T}\pi_t^R z_t \quad (2)$$

subject to

$$y_t^H + y_t^L = \sum_{i\in I} x_{it} + z_t, \quad \forall\ t\in T \quad (3)$$

$$x_{it} \leq K_i, \quad \forall\ i \in I, \quad t\in T \quad (4)$$

$$\mid \sum_{i\in I} x_{it} - \sum_{i\in I} x_{i,t-1} \mid \leq \bar{Z}, \quad \forall\ t\in T : t > 1 \quad (5)$$

$$\mid \pi_t^L - \pi_{t+1}^L \mid \leq M\alpha_t, \quad \forall\ t\in T : t <\mid T\mid \quad (6)$$

$$\mid \pi_t^H - \pi_{t+1}^H \mid \leq M\alpha_t, \quad \forall\ t\in T : t <\mid T\mid \quad (7)$$

$$\sum_{t\in T}\alpha_t \leq \Psi \quad (8)$$

$$\sum_{n=t-N+1}^{t} \alpha_n \leq 1, \forall\ t\in [N,\ldots,\mid T\mid] \quad (9)$$

$$\pi_t^L \leq \pi_t^H, \quad \forall\ t\in T \quad (10)$$

$$\pi_t^H, \pi_t^L, x_t, z_t, \geq 0, \quad \alpha_t \in \{0,1\} \quad (11)$$

Constraints (10) ensure that the higher price is always greater or equal to the lower price. Finally, constraints (11) are the nonnegativity constraints and binary variables definition.

3.2.2. Follower problem

The RLA aims to minimize the end-user's total cost (12) defined as the sum of the energy cost from the TLOU program, the shifting cost, and the cost of buying energy from the competitor.

Constraints (13) establish the balance between the energy bought from the supplier and the competitor, and the demand. Constraints (14) account for the total demand satisfaction at the end of the day. Constraints (15) limit the energy consumption at the lower level of the TLOU. In a similar way, constraints (16) limit the over-consumption. Finally, constraints (17)-(21) are the nonnegativity constraints.

The follower's problem is embedded in the constraints of the leader problem. When the follower has multiple optimal solutions, we assume that he selects the one favoring the leader. This leads to an optimistic formulation of the bilevel program. In our context it means that the RLA will not act deliberately against the supplier's interests.

$$\min_{(y,w)} \sum_{t\in T}(\pi_t^H y_t^H + \pi_t^L y_t^L) + \sum_{t\in T}\Gamma_t w_t^+ + \sum_{t\in T}\Upsilon v_t \quad (12)$$

subject to

$$y_t^H + y_t^L - w_t^+ + w_t^- + v_t = D_t, \quad \forall\ t\in T : (\lambda_t^a) \quad (13)$$

$$\sum_{t\in T}(y_t^H + y_t^L) + \sum_{t\in T} v_t = \sum_{t\in T} D_t \quad : (\lambda^b) \quad (14)$$

$$y_t^L \leq C, \quad \forall\ t\in T : (\mu_t^a) \quad (15)$$

$$w_t^+ \leq \Omega_t, \quad \forall\ t\in T : (\mu_t^b) \quad (16)$$

$$y_t^H \geq 0, \quad \forall\ t\in T : (\mu_t^c) \quad (17)$$

$$y_t^L \geq 0, \quad \forall\ t\in T : (\mu_t^d) \quad (18)$$

$$w_t^+ \geq 0, \quad \forall\ t\in T : (\mu_t^e) \quad (19)$$

$$w_t^- \geq 0, \quad \forall\ t\in T : (\mu_t^f) \quad (20)$$

$$v_t \geq 0, \quad \forall\ t\in T : (\mu_t^g) \quad (21)$$

3.2.3. MILP Reformulation

The TLOU price setting model is a bilinear bilevel mixed integer problem. For fixed leader variables, the follower problem is convex and can be replaced by its Karush-Kuhn-Tucker conditions [11]. The KKT conditions include the primal feasibility constraints ((13)-(21)), the dual feasibility constraints, the complementary slackness constraints, and the stationary constraints.

The dual feasibility is ensured by Constraints (22) that are the non-negativity for dual variables of the primal inequality constraints.

$$\mu_t^a, \mu_t^b, \mu_t^c, \mu_t^d, \mu_t^e, \mu_t^f, \mu_t^g \geq 0, \quad \forall\ t\in T \quad (22)$$

The complementary slackness is given by the following non-linear equations:

$$(C - y_t^L)\mu_t^a = 0, \quad (\Omega_t - w_t^+)\mu_t^b = 0, \quad y_t^H\mu_t^c = 0,$$
$$y_t^L\mu_t^d = 0, \quad w_t^+\mu_t^e = 0, \quad w_t^-\mu_t^f = 0, \quad v_t\mu_t^g = 0, \quad \forall\ t\in T$$

These equations can be linearized as follows:

$$C(1-\rho_t^a) \geq C - y_t^L, \quad \mu_t^a \leq M\rho_t^a, \quad \forall\ t\in T \quad (23)$$

$$\Omega_t(1-\rho_t^b) \geq \Omega_t - w_t^+, \quad \mu_t^b \leq M\rho_t^b, \quad \forall\ t\in T \quad (24)$$

$$(D_t + \Omega_t - C)(1-\rho_t^c) \geq y_t^H, \quad \mu_t^c \leq M\rho_t^c, \quad \forall\ t\in T \quad (25)$$

$$C(1-\rho_t^d) \geq y_t^L, \quad \mu_t^d \leq M\rho_t^d, \quad \forall\ t\in T \quad (26)$$

$$\Omega_t(1-\rho_t^e) \geq w_t^+, \quad \mu_t^e \leq M\rho_t^e, \quad \forall\ t\in T \quad (27)$$

$$D_t(1-\rho_t^f) \geq w_t^-, \quad \mu_t^f \leq M\rho_t^f, \quad \forall\ t\in T \quad (28)$$

$$(D_t + \Omega_t)(1-\rho_t^g) \geq v_t, \quad \mu_t^g \leq M\rho_t^g, \quad \forall\ t\in T, \quad (29)$$

where M is an arbitrary large constant.

The M is replaced in all the constraints that do not contain dual variables, by upper bounds computed directly from the structure of the problem to have a tighter formulation. For example, the maximum value for the expression $C - y_t^L$ in constraint (23), equals C when $y_t^L = 0$ since we know that $0 \leq y_t^L \leq C$ from constraints (15) and (18). Finally the stationary conditions are:

$$\pi_t^H - \lambda_t^a - \lambda^b - \mu_t^c = 0, \quad \forall \ t \in T \tag{30}$$
$$\pi_t^L - \lambda_t^a - \lambda^b + \mu_t^a - \mu_t^d = 0, \quad \forall \ t \in T \tag{31}$$
$$\Gamma_t + \lambda_t^a + \mu_t^b - \mu_t^e = 0, \quad \forall \ t \in T \tag{32}$$
$$-\lambda_t^a - \mu_t^f = 0, \quad \forall \ t \in T \tag{33}$$
$$\Upsilon - \lambda_t^a - \lambda_t^b - \mu_t^g = 0, \quad \forall \ t \in T \tag{34}$$

The bilinear terms in the objective function (2) of the leader's problem can be linearized by using the strong duality theorem for the follower problem as follows:

$$\sum_{t\in T}(\pi_t^H y_t^H + \pi_t^L y_t^L) + \sum_{t\in T}\Gamma_t w_t^+ + \sum_{t\in T}\Upsilon v_t$$
$$= \sum_{t\in T} D_t\lambda_t^a + (\sum_{t\in T} D_t)\lambda^b - \sum_{t\in T} C\mu_t^a - \sum_{t\in T}\Omega_t\mu_t^b \tag{35}$$

By reorganizing equation (35) and replacing in (2), we obtain the linear leader's objective function:

$$\max \quad \Big[\sum_{t\in T} D_t\lambda_t^a + (\sum_{t\in T} D_t)\lambda^b - \sum_{t\in T} C\mu_t^a - \sum_{t\in T}\Omega_t\mu_t^b$$
$$-\sum_{t\in T}\Gamma_t w_t^+ - \sum_{t\in T}\Upsilon v_t\Big] - \sum_{i\in I}\sum_{t\in T}\pi_i^G x_{it} - \sum_{t\in T}\pi_t^R z_t \tag{36}$$

4. Computational experiments

In this section we present and discuss numerical results based on real life data. We first define the parameters and the instances generation process in Section 4.1. Next, we discuss the results of the bilevel problem in Section 4.2.

4.1. Parameters and instances definition

We consider consumption of a population of 1000 households. Their consumption profiles are based on the daily consumption profiles provided by the independent system operator in the province of Ontario, Canada. The resulting demand profile are used in all the experiments and it corresponds to a daily demand of 6.8 MWh. The generation cost (¢/kWh) π_i^G : $\{4, 7, 20\}$ is taken from [35], where three types of generators are available: nuclear, hydro, and gas turbines. For the two first resources the generation capacities are equal to 150 kW. The generation cost (¢/kWh) π_i^G : $\{4, 7, 20\}$ is taken from [35], where three types of generators are available: nuclear, hydro, and gas turbines. For the two first resources the generation capacities are equal to 150 kW.

We design instances to assess the impact of two of the main factors in the definition of TLOU prices, the load shifting flexibility and the capacity profile. The supplier wants to take advantage of the flexibility that the RLA offers, and uses the capacity of TLOU to encourage a more beneficial shifting while protecting itself of undesired scenarios such as rebound peaks.

First, we define two flexibility profiles Ω_t^f where $f \in \{1, 2\}$. This information is presented in Figure 2. We see that Ω_t^1 corresponds to a RLA representing a group of end-users that provide low shifting flexibility, while Ω_t^2 corresponds to RLA representing a group of end-users that provide high shifting flexibility.

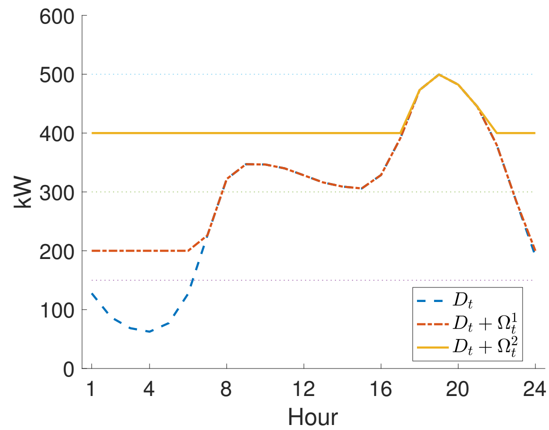

Figure 2: Demand and shifting capabilities

We consider different TLOU capacities $C = \{0, 150, 300\}$. Note that having a TLOU with $C = 0$ is equivalent to have the traditional TOU. These TOU instances provide us with a reference to analyze and compare the performance of TLOU, more specifically the impact of having a capacity profile and two different sets of energy prices. This comparison will be discussed in the next sections.

4.2. Bilevel optimization results

Instances with different shifting flexibility and capacities are denoted by $S^{f,C}$. The optimization models containing 529 continuous variables, 192 binary variables, 1392 constraints, are solved with Gurobi version 8.1 on a computer with 2.3 GHz Intel Core i5 CPU and 8 GB RAM. The average solution time is 0.65 seconds.

Table 2 presents different metrics assessing the shifting effect generated by the definition of energy prices, as well as the economical performance of the RLA and the supplier for each instance. We quantify the shifting effect using two quantities. The first one is the shifted load, which is the percentage of variation of the RLA energy consumption (supplier + competitor) with respect to the original demand. The second one is the Peak to Average Ratio (PtoAR) that accounts for the level of flattening of the energy curve provided by the supplier. The PtoAR for the original demand profile is 1.79.

	Shifting effect		RLA Costs ($)				Supplier ($)		
Instance	RLA Shifted load (%)	PtoAR supplier	Shifting	Energy from supplier	Energy from competitor	Total	Income	Operational cost	Profit
S^0	0.0	n/a	0.0	0.0	100.0	100.0	0.0	0.0	0.0
$S^{1,0}$	9.5	1.14	1.9	87.6	7.7	97.2	87.6	41.6	46.0
$S^{1,150}$	9.5	1.14	1.9	87.6	7.7	97.2	87.6	41.6	46.0
$S^{1,300}$	9.5	1.13	1.9	88.6	7.1	97.6	88.6	42.5	46.1
$S^{2,0}$	1.2	1.26	0.0	83.1	16.6	99.7	83.1	39.1	44.0
$S^{2,150}$	5.9	1.19	1.1	84.6	11.9	97.6	84.6	39.3	45.3
$S^{2,300}$	19.0	1.05	3.4	93.8	0.0	97.2	93.8	45.3	48.5

Table 2: Economical performance and shifting effect by instance.

To quantify the economic impact, we take as reference the case in which the RLA must buy all the electricity from the competitor. In this case, the RLA pays the flat rate offered by the competitor, generating no shifting and the supplier has no participation. This instance is denoted as S^0 and its total cost ($824.0) is set to be equivalent to $100. The economic results of the other instances were normalized accordingly dividing by 824.0. We report in Table 2 the normalized RLA costs and supplier profit.

4.2.1. Low shifting flexibility

We first consider the RLA Ω^1 with low shifting flexibility. Increasing the capacity of TLOU does not result in significant changes in the performance of the supplier and the RLA. More precisely, we observe the same TOU structure in instances $S^{1,0}$ and $S^{1,150}$ regardless of the increase of C. Although instance $S^{1,150}$ includes the possibility of having TLOU, the optimal decision is to offer a traditional TOU ($\pi^L_t = \pi^H_t, \quad \forall \quad t \in T$). Figure 3 shows for $S^{1,150}$, the shifting effect during the first hours of the day (times frames 1 to 6), where the energy bought from the supplier ($y^L_t + y^H_t$) increases up to the maximum demand ($D_t + \Omega^1_t$) as reaction to cheaper energy prices. These cheaper prices are offered as compensation during the time frames when the users experience higher shifting cost Γ_t. This shifting is equivalent to 9.5% of demand and leads to a ProAR of 1.14 for both instances.

Let $\pi^L_{(t,t')}$ and $\pi^H_{(t,t')}$ be the optimal energy prices in the time window starting at time frame t until t'. The difference between $\pi^H_{(1,6)}$ and $\pi^H_{(7,24)}$ corresponds to the maximum value of Γ_t in the time frames 1 to 6. This encourages shifting towards this segment of the day since $\pi^H_{(1,6)} + \Gamma_t \leq \pi^H_{(7,24)}, \forall \quad t \in \{1, \dots, 6\}$. This analysis can be easily extended for the other instances.

The supplier's competitor provides energy during the evening peak. This occurs because the supplier can not match the competitor's price when the third generation level is required (dotted line, $> 300kW$). Note that the highest optimal price obtained is equal to the price offered by the competitor (12 ¢/kWh).

For instance $S^{1,300}$, TLOU has a marginal effect on the results (Figure 4).

The supplier price $\pi^L_{(6,9)}$ is slightly cheaper than $\pi^H_{(6,9)}$. In this case $\pi^L_{(6,9)} + \Gamma_t \leq \pi^H_{(6,9)} \quad \forall \quad t \in \{6, \dots, 9\}$.

We observe in Figure 4 a slightly increase of energy bought from the supplier in $t = 8$ with respect to previous instances. This additional shifting allows the supplier to serve a higher proportion of demand. The amount of energy provided by the competitor decreases for this instance. The price $\pi^H_{(1,5)}$ has no effect in the results since $D_t + \Omega^1_t < C, \quad \forall \quad t \in \{1, \dots, 5\}$. This minimum change leads to a shifted load of 9.5% of and a PtoAR of 1.13.

The lack of shifting flexibility of the RLA Ω^1 reduces the potential impact of TLOU, allowing the supplier to obtain the best performance with traditional TOU in two our of three cases. In fact, we see that the prices obtained in the instance $S^{1,300}$ (the only one in which TLOU is selected) is very similar to the TOU of the other two instances. This causes the economical and shifting metrics to be almost identical for the three instances. Nevertheless, even with low flexibility, the bilevel approach finds the most appropriate price structure allowing the supplier to capture most of the demand and the RLA to reduce the energy bill by 4%.

4.2.2. High shifting flexibility

Let us now comment the results for RLA Ω^2 with high shifting flexibility (Figures 5 to 7). The flexibility provided by this RLA allows us to illustrate the benefits of implementing TLOU.

For instance $S^{2,0}$ (Figure 5) a significant fraction of the demand is satisfied by the supplier's competitor. In this case, the high shifting flexibility combined with $C = 0$ results into a very conservative decision by the supplier who offers a very small price incentive in the first time frames. There is an increase in the energy bought from the supplier as result of shifting only in $t = 6$. Any larger incentive will result in a rebound peak reaching the RLA shifting limit and worsening the economical performance of the supplier. In this case the shifted load is only 1.2% and the PtoAR is 1.26.

The equality $\pi^H_{(1,6)} + \Gamma_6 = \pi^H_{(7,24)}$ encourages shifting towards $t = 6$ since Γ_6 is the lowest value in time frames 1 to 6. Therefore, shifting makes no sense for the periods

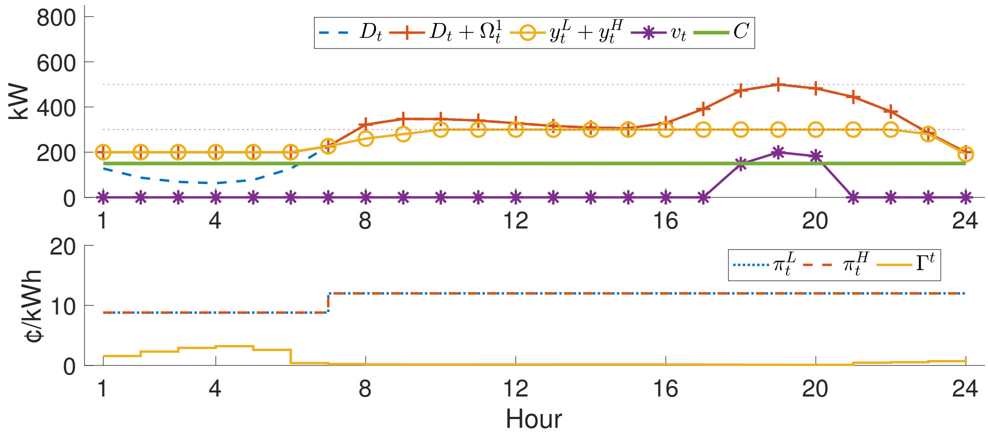

Figure 3: Load shifting and TLOU prices for instance $S^{1,150}$.

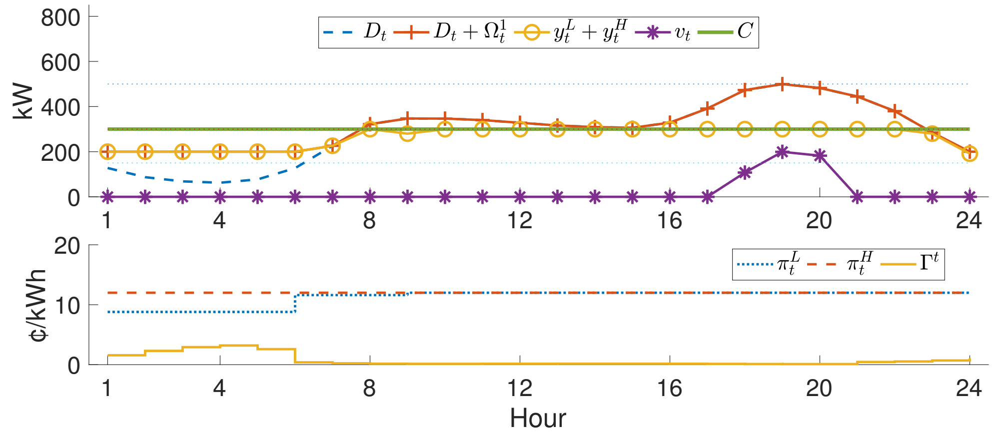

Figure 4: Load shifting and TLOU prices for instance $S^{1,300}$.

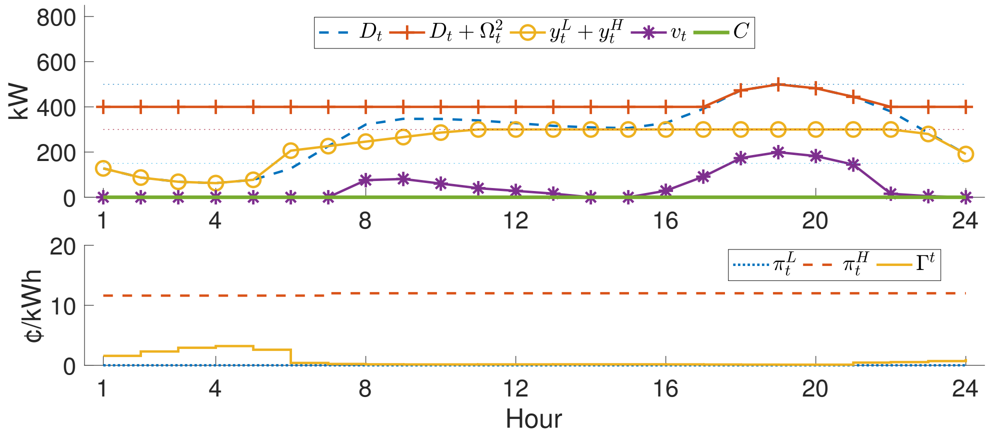

Figure 5: Load shifting and TLOU prices for instance $S^{2,0}$.

1 to 5. This is a strict equality due to the optimistic feature previously explain in Section 3.2.2 . If both options (shifting or not) are similar for the RLA, the model will always favor the leader (i.e. the supplier).

Table 2 shows that $S^{2,0}$ generates the lowest profit for the supplier and the highest total cost for the RLA. In fact, the total user cost is almost equal to the bound defined in the reference case.

The results for instance $S^{2,150}$ (Figure 6) show the effect of TLOU. The supplier offers TLOU prices in time frames 1 to 6. The price $\pi^L_{(1,6)}$ encourages RLA to consume electricity until reaching C. Then $\pi^H_{(1,6)}$ performs as a penalty for consumption higher than C. The RLA only pays π^H_t in $t = 6$.

We observe that $\pi^L_{(1,6)} + \Gamma_t \leq \pi^H_{(7,24)} \forall \quad t \in \{1, \ldots, 6\}$. In a similar way, $\pi^H_{(1,6)} + \Gamma_6 = \pi^H_{(7,24)}$. The later one encour-

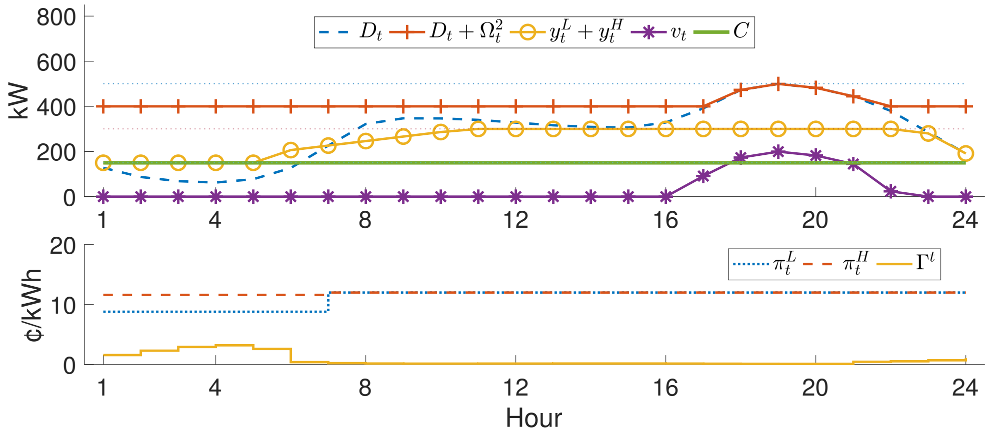

Figure 6: Load shifting and TLOU prices for instance $S^{2,150}$.

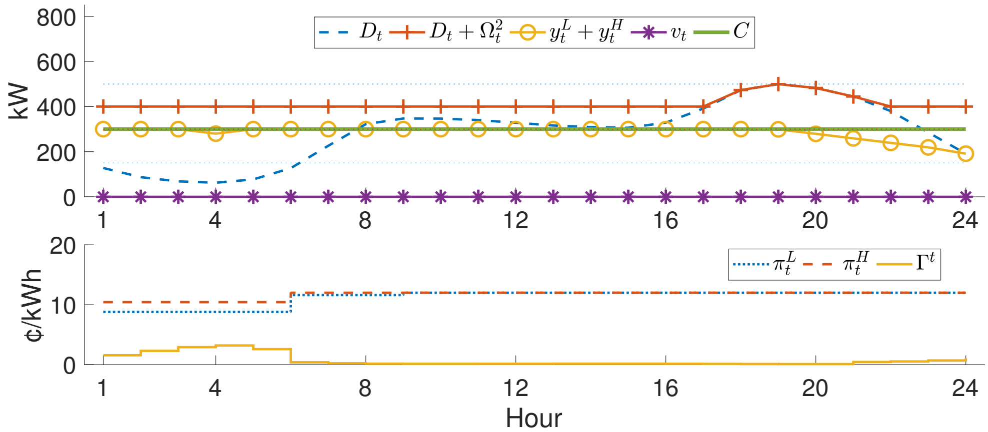

Figure 7: Load shifting and TLOU prices for instance $S^{2,300}$.

ages consumption beyond C only in $t = 6$ during the time window (1,6). This occurs since Γ_6 is the lowest shifting cost in the time window.

As shown in Table 2, for instance $S^{2,150}$ the prices defined for the TLOU by the bilevel approach report benefits for both parties. First, the supplier captures a higher fraction of the demand (competitor only makes \$11.9) which results in a higher profit. Second, the RLA is rewarded in a more effective way (total cost of \$97.6) which leads to a higher shifting (\$1.1). In fact, the load shifted reaches 5.9% which helps to reduce the PtoAR down to 1.19.

Finally Figure 7 puts into highlight the effect of TLOU for instance $S^{2,300}$. In this case $C = 300$ allows the supplier to completely meet the RLA's demand. The supplier offers two different lower prices $\pi^L_{(1,5)}$ and $\pi^L_{(6,9)}$. Once again $\pi^L_{(1,5)} + \Gamma_t \leq \pi^H_{(6,24)} \quad \forall \quad t \in \{1,\dots,5\}$ and $\pi^L_{(6,9)} + \Gamma_t \leq \pi^H_{(6,24)} \quad \forall \quad t \in \{6,\dots,9\}$, which encourage shifting up to C in the first part of the day. On the other hand, $\pi^H_{(1,5)} + \Gamma_t \geq \pi^L_{(6,9)} \quad \forall \quad t \in \{1,\dots,5\}$ and $\pi^H_{(1,5)} + \Gamma_t \geq \pi^H_{(6,24)}$ $\forall \quad t \in \{1,\dots,5\}$ since no consumption in the higher level is desired by the supplier in the time frames 1 to 5. In this case the load shifted increases up to 19% and the PtoAR is reduced down to 1.05 which approaches a flat curve.

For instance $S^{2,300}$ the optimal TLOU prices achieve the highest performance for both supplier and RLA. The supplier obtains profit of \$48.5 which represents an increase of about 10% with respect to instance $S^{2,0}$ (traditional TOU).

The high shifting flexibility of the RLA Ω^2 plays an important role in the definition of the prices and allows us fully utilize the potential of TLOU. As we mentioned before, the supplier makes a very conservative decision in $S^{2,0}$ since any larger incentive will result in rebound peaks and operational over cost. For that instance, the supplier sees its potential revenue reduced due to the lack of a capacity profile. In the case of instance $S^{2,300}$, the supplier uses the capacity profile to manage in a smarter way the potential load shifting and to improve its performance. This is clearly observed in the shape of the curve $y^L_t + y^H_t$ that flattens as the shifted load increases. Additionally, the RLA obtains an energy cost of \$93.8 which represents a reduction of 6.2% with respect to S^0.

4.2.3. Setting the parameter C

In the previous experiments the TLOU were defined to mach directly the generation capacities of the supplier. Figure 8 presents the profits for a RLA $S^{2,C}$ where C

changes from 0 to 500 with a step $\Delta C = 25$.

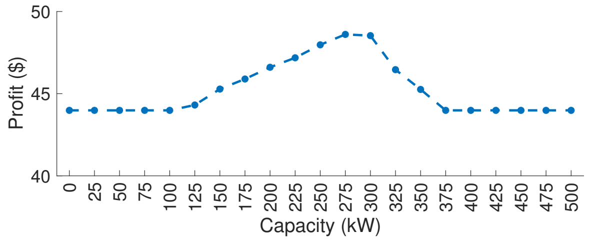

Figure 8: Profit for $S^{2,C}$ with different TLOU capacities.

The maximum profit is reached for $C = 275$. We see the same cost in lower and higher capacities. Having insufficient capacity is equivalent to having a capacity greater than the highest possible demand. In both cases the optimal decision is to implement a TOU or a flat rate.

4.3. Advantages and limitations of the presented approach

As we presented in Section 4.2 the flexibility of the optimization model allows to find the best price structure fitting the needs of the supplier and the characteristics of the RLA. This feature allows the energy supplier to select the appropriate DR pricing program (TLOU, TOU or flat rate) that maximizes its profit. The optimization model can as well be easily integrated with classical demand side management approaches to explicitly integrate the end-users behavior, such as shifting preferences by type of load, battery usage, local generation etc.

One of the main limitations in this approach is that the decisions are made in a static fashion, which is well suited for the definition of energy prices over a given time horizon, but that represents a challenge for scenarios where the prices are computed on a real-time basis. Additionally, the approach strongly depends on the accurate representation of the end-user behavior. Integrating learning and stochastic approaches to model the end-users preferences could improve the accuracy and effectiveness of the proposed approach.

5. Conclusions

In this paper, we have formulated a bilevel optimization problem in which the supplier seeks to set TLOU energy prices to maximize the profit considering the shifting activities of a RLA that minimizes the total cost. The model considers the users' level of consumption and their shifting preferences in the definition of the price structure. In addition, we defined the price setting problem in a competitive environment where the operation of the supplier is conditioned the the prices of the competitor. The proposed model includes also the user-friendly perspective in the design of the price structure to facilitate social acceptance. This bilevel optimization problem is reformulated as a single-level problem and solved using off-the-shelf solvers.

The use of this pricing scheme allows the supplier to improve the profit, encouraging the consumers to shift in a smarter way. This effect is more significant when the RLA is willing to shift more load, since TLOU controls rebound peaks by incentivizing energy consumption up to the defined capacity limit before passing to the second (higher) level of prices. This ends up generating a more effective shifting and therefore a more homogeneous consumption profile.

Numerical results based on real life data are presented to validate and analyze the performance of the implementation of this type of DR program. In instances of a RLA with lower shifting flexibility, TLOU does not report significant impact on the supplier's and the RLA performance. In fact, in some cases the optimal decision is to implement a TOU structure. Even in that case, the proposed bilevel approach determines optimally-designed tariffs for the energy supplier and allows the RLA to reduce the energy bill by 4%.

On the other hand, TLOU shows a significant improvement for both supplier and RLA when the RLA is willing to provide more shifting flexibility. In this case, the supplier benefits from a more efficient operation of the system due to a flatter demand curve and is able to better incentivize the RLA, who in return shifts load in a smarter way. More specifically, the supplier's economical benefit is increased up to 10 % and the RLA operational cost is reduced by 6%.

Future work will explore the definition of TLOU prices under the presence of multiple end-users with different behavior (multiple followers) and the effect of offering personalized TLOU prices. Additional research directions include considering the parameter C as a variable, and the uncertainty in the shifting reaction to prices.

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