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On the Performance of QAM OFDM-Based FSO Links with Nonlinear Clipping Effect over Gamma-Gamma Modelled Turbulence Channels

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Abstract

The free space optical (FSO) communication systems have attracted significant research and commercial interest in the last few years due to their low installation and operational cost along with their very high performance characteristics. However, for terrestrial FSO links the optical signal propagates through the atmosphere which exhibits time varying behavior that implies variations of links' performance. In this work, we estimate the performance metrics for terrestrial FSO links which are using the orthogonal frequency division multiplexing (OFDM) technique with a quadrature amplitude modulation (QAM) scheme over turbulence channels. More specifically, we investigate the influence of the nonlinear clipping effect of the OFDM scheme, along

with the atmospheric turbulence modeled using the gamma-gamma distribution. Both effects significantly influence the performance of the link and here we derive closed form mathematical expressions for the estimation of the average signal to noise ratio (SNR), the outage probability and the average bit error rate (BER) that vital for FSO system performance characterization. Finally, using these expressions, we present the corresponding numerical results for common parameter values of the FSO links and we investigate the accuracy of our expressions for marginal cases with nearly negligible turbulence effect.

I. Introduction

Free space optical FSO communication systems have attracted significant research and commercial interest in the last few years because they can achieve very high performance characteristics with relatively low installation and operational cost [1]-[6]. The main disadvantage of the FSO links is caused by the fact that the optical signal propagates through the atmosphere which shows varying characteristics that affect their performance and reliability significantly. An important phenomenon that impairs the system's performance is the atmospheric turbulence induced scintillation effect [1]-[8]. Due to this effect, the irradiance of the optical signal at the receiver's side does not remain invariable but fluctuates rapidly with respect to time and space. In order to model this irradiance fluctuation, many statistical distributions have been used, depending on the atmospheric turbulence strength [7]-[14]. One model which has been proven to accurately predict weak to strong atmospheric turbulence conditions is the gamma-gamma distribution, [6], [15]-[17].

A multiplexing scheme commonly studied in optical communication systems is the OFDM. It offers the capability to transmit the data in parallel subcarriers that are orthogonal to each other.This technique is an effective way of achieving multicarrier data transmission using multiple relatively narrow band subcarriers. Each subcarrier is modulated by the information signal using an advanced modulation scheme such as the QAM scheme [18]-[24]. Due to the subcarriers' orthogonality, the intersymbol interference (ISI) is minimal and the different subcarriers can thus be separated relatively easily at the receiver [5], [6].

Beside the influence of the propagation channel in optical communication systems, signal distortions/clippings can occur due to other various reasons [18], [21].These include limited dynamic range and nonlinearities in the optical front-ends. Furthermore, with optical-OFDM (O-OFDM), the signal can have very large peak-to-average power ratio (PAPR) [20]-[22] resulting in further clipping at the optical front-end. O-OFDM follows Gaussian distribution due to the fact that many signals are added together so that central limit theorem can be applied. In fact, it has been demonstrated that even for as low as 64 carriers the Gaussian distribution assumption holds [21]. The Gaussianity of the non-distorted signal means that Bussgang theorem can be applied to analyze the clipping, [21], and other nonlinear distortion effects, [18]. More specifically, clipping of the signal can be represented with an attenuated data signal and additional uncorrelated non-Gaussian noise. The clipping-induced attenuation and the additional noise power can be computed by applying Bussgang theorem as reported in Ref. [18].

The need to further increase the transmission rate and mitigate inter-symbol interference (ISI) has contributed to the study of OFDM for FSO [18]-[24]. The effect of atmospheric turbulence on such OFDM-FSO links has been studied and reported in Refs [6], [20], [23], [24]. However, OFDM has innate challenges that significantly affect its performance. These include clipping noise and susceptibility to nonlinearity effects; the limitations imposed on the OFDM FSO system by these have been reported in Refs [18], [21], [22].

Although each individual impairment i.e. turbulence and clipping/nonlinearity noise, has been studied in isolation, it is desirable to have a unified analytical framework that combines these effects and accurately evaluate the system performance. This void in OFDM FSO research is addressed in this paper. Specifically, we present accurate closed form analytical expressions for evaluating the performance of OFDM FSO in the presence of turbulence and clipping/nonlinearity noise. Due to their significance in the design of a communication system, the performance evaluations metrics considered in this work are average SNR, average BER and outage probability.

The remainder of this work is organized as follows: in section II, we present the system and the channel model with the gamma-gamma distribution and its parameters, in section III we derive closed form mathematical expressions for the estimation of the average SNR, the outage probability and the average BER metrics of the wireless optical link while in section IV we present the numerical results for various clipping levels and turbulence strengths obtained from the derived mathematical expressions and the corresponding numerical simulations. Finally, section V contains the concluding remarks of this work.

II. System and Channel Model

In terrestrial optical wireless communication links, the atmospheric turbulence phenomenon causes the scintillation effect which results in irradiance fluctuations at the receiver. Depending on the strength of the fluctuation, a number of statistical distributions exist to model the scintillation effect. Most of these models are very accurate for specific atmospheric turbulence conditions. The gamma-gamma turbulence model considered in this work takes into account both the small and the large scale contributions to scintillation effect [7], [8]. Moreover, it has been proven that this model is suitably accurate for weak to strong turbulence and its parameters can be directly estimated from the link's characteristics [1]-[2], [6]-[8], [15].

The probability density function (PDF) of the gamma-gamma distribution, $f_I(I)$, as a function of the normalized irradiance I is given in [7], as:

$$f_I(I)=\frac{(ab)^{\frac{a+b}{2}}}{\Gamma(a)\Gamma(b)}I^{\frac{a+b}{2}-1}K_{a-b}\left(2\sqrt{abI}\right) \qquad (1)$$

with $K_{a-b}(.)$ being the modified Bessel function of the second kind of order $(a-b)$, $\Gamma(.)$ is the gamma function and the parameters a and b can be estimated from the link's characteristics through the following expressions [7], [8], [15]:

$$a=\left[\exp\left(\frac{0.49\delta^2}{\left(1+0.18d^2+0.56\delta^{12/5}\right)^{7/6}}\right)-1\right]^{-1}$$

$$b=\left[\exp\left(\frac{0.51\delta^2\left(1+0.69\delta^{12/5}\right)^{-5/6}}{\left(1+0.9d^2+0.62d^2\delta^{12/5}\right)^{5/6}}\right)-1\right]^{-1} \tag{2}$$

where $d=\sqrt{kD^2/(4L)}$, $k=2\pi/\lambda$, is the optical wave number, $\lambda$is the operational wavelength of the communication system, L represents the length of the link while D stands for the receiver's aperture diameter. The parameter δ^2 stands for the Rytov variance which for relatively weak turbulence conditions is given as $\delta^2=1.23C_n^2k^{7/6}L^{11/6}$, [7]. C_n^2 is the refractive index structure parameter which depends on the altitude and the atmospheric conditions and is given as $C_n^2=\left(79\times10^{-6}P/T^2\right)^2C_T^2$ where P, T and C_T^2 stand for the atmospheric pressure, temperature and temperature structure constant respectively, [8]. Moreover, it is worth mentioning that the value of the parameterC_n^2 varies from 10^{-17} to 10^{-13} $\mathrm{m}^{-2/3}$ for weak to strong turbulence conditions, [16].

Taking into account the irradiance fluctuations at the receiver's side due to the scintillation effect, the optical channel can be considered as a fading one. Moreover, if the OFDM technique is used, it is assumed as flat fading one over the entire OFDM bandwidth of up to 20 MHz, [21], [24]. Additionally, O-OFDM systems suffer from signal clipping due to the limited dynamic range of the optical end and nonlinearities. Considering the clipping effect, the optical path gain coefficient is given as [8], [21], [25]:

$$\gamma = \frac{S_{PD}\rho_{PD}GI}{E[P_t]\sqrt{r_l}} \tag{3}$$

with $S_{PD} = \pi D^2/4$ and ρ_{PD} stand for the photosensitive area and the responsivity of the photodetector (PD), respectively, while G represents the gain of the transimpedance amplifier, $E[P_t]$ is the average transmitted optical power and r_l, the resistance over which the current of the receiver is measured [21], [25].

From (3) and [21, Eq. 25], the following mathematical expression is derived for the estimation of the instantaneous effective electrical SNR per bit, $\mu(I)$, in the OFDM based FSO link as a function of the undistorted electrical SNR per bit, γ_b [21]:

$$\mu(I) = \frac{K^2\Lambda^2 G_{DC}P_B\gamma_b I^2}{\sigma_{clip}^2\Lambda^2 G_{DC}G_B\gamma_b I^2 + G_B P_B} \tag{4}$$

where $\Lambda = S_{PD}\rho_{PD}G/\left(E[P_t]\sqrt{r_l}\right)$, K quantifies the effective attenuation factor due to nonlinear clipping, G_{DC} denotes the attenuation of the useful electrical signal power of time domain electrical signal due to the biasing of the transmitter front-end in the least signal clipping scenario, G_B is the utilization factor for the double-sided bandwidth B

of the OFDM frame, P_B represents an average electrical power per bit and σ^2_{clip} denotes the clipping noise variance [21].

III. Performance Estimation of the Optical Wireless Link

First, we estimate the average electrical SNR at the receiver by taking into account both the clipping noise of the QAM-O-OFDM signal and the atmospheric turbulence induced fading. More specifically, the average electrical SNR per bit at the receiver, $\bar{\mu}$, is estimated from the corresponding instantaneous electrical SNR, (4), by integrating over I, using the PDF of the gamma-gamma distribution presented in (1), through the following expression:

$$\bar{\mu} = \frac{\Xi (ab)^{\frac{a+b}{2}}}{\Gamma(a)\Gamma(b)} \int_0^\infty \frac{I^{\frac{a+b}{2}+1}}{\Psi I^2 + \Omega} K_{a-b}\left(2\sqrt{abI}\right) dI \qquad (5)$$

with $\Xi = K^2 \Lambda^2 G_{DC} P_B \gamma_b$, $\Psi = \sigma^2_{clip} \Lambda^2 G_{DC} G_B \gamma_b$ and $\Omega = G_B P_B$ .By substituting the appropriate hypergeometric Meijer functions [26], i.e. $\left(\Psi I^2/\Omega + 1\right)^{-1} = G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 0 \\ 0 \end{matrix}\right)$, $K_{a-b}\left(2\sqrt{abI}\right) = \frac{1}{2} G_{0,2}^{2,0}\left(abI \middle| \begin{matrix} - & - \\ \frac{a-b}{2}, & \frac{b-a}{2} \end{matrix}\right)$, in (5) results in the following closed form mathematical expression for the average effective electrical SNR of the OFDM-based FSO with the clipping noise and turbulence:

$$\bar{\mu} = \frac{2^{a+b+1}\Xi}{\pi\Omega(ab)^2\Gamma(a)\Gamma(b)} G_{5,1}^{1,5}\left(\frac{16\Psi}{\Omega(ab)^2}\middle|\begin{matrix}0,-\frac{a+1}{2},-\frac{a}{2},-\frac{b+1}{2},-\frac{b}{2}\\ 0\end{matrix}\right) \qquad (6)$$

Where $G_{p,q}^{m,n}[\cdot]$ represents the Meijer-function, a standard built-in function which can be evaluated with most of the well-known mathematical software packages and it can be expressed by the more general hypergeometric functions, [26].

Next, we estimate the outage probability of the FSO link, P_{out}, as a function of the SNR threshold, μ_{th}, which represents the minimum SNR required for satisfactory performance in the presence of turbulence and nonlinear clipping and is given as [23]:

$$P_{out}(\mu_{th}) = \Pr(\mu \le \mu_{th}) = F_{\mu}(\mu_{th}) \qquad (7)$$

where $F_{\mu}(\mu)$ stands for the cumulative density function (CDF) of the appropriate distribution model and it is estimated by integrating the PDF. Thus, for the gamma gamma PDF of Eq. (1) and using (4), we arrive at the following expression:

$$F_{\mu}(\mu) = \frac{(ab)^{\frac{a+b}{2}}\left(\frac{\Omega\mu}{\Xi-\Psi\mu}\right)^{\frac{a+b}{4}}}{\Gamma(a)\Gamma(b)} G_{1,3}^{2,1}\left(ab\sqrt{\frac{\Omega\mu}{\Xi-\Psi\mu}}\middle|\begin{matrix}1-\frac{a+b}{2}\\ \frac{a-b}{2},\frac{b-a}{2},-\frac{a+b}{2}\end{matrix}\right) \qquad (8)$$

From (7), (8), we derive the following closed form mathematical expression for outage probability estimation of the OFDM based FSO link as a function of μ_{th} and γ_b:

$$P_{out}(\mu_{th}) = \frac{\left(\dfrac{\Omega}{\xi\gamma_b/\mu_{th} - \zeta\gamma_b}\right)^{\frac{a+b}{4}}}{\Gamma(a)\Gamma(b)(ab)^{-\frac{a+b}{2}}} G_{1,3}^{2,1}\left(ab\sqrt{\frac{\Omega}{\xi\gamma_b/\mu_{th} - \zeta\gamma_b}} \middle| \begin{matrix} 1-\frac{a+b}{2} \\ \frac{a-b}{2}, \frac{b-a}{2}, -\frac{a+b}{2} \end{matrix} \right) \tag{9}$$

where $\xi = \Xi\gamma_b^{-1}$ and $\zeta = \Psi\gamma_b^{-1}$.

The next metric of the M-QAM OFDM FSO link that we study in this work is the average BER. In order to estimate this quantity, we first evaluate the instantaneous BER, which is given accurately as in [21], [27]:

$$BER(I) = \lambda_1 Q\left(\lambda_2\sqrt{\mu(I)}\right) + \lambda_3 Q\left(3\lambda_2\sqrt{\mu(I)}\right) \tag{10}$$

where $\lambda_1 = \frac{4\left(\sqrt{M}-1\right)}{\sqrt{M}\log_2(M)}$, $\lambda_2 = \sqrt{\frac{3\log_2(M)}{M-1}}$, $\lambda_3 = \frac{4\left(\sqrt{M}-2\right)}{\sqrt{M}\log_2(M)}$, M stands for the QAM modulation order and $Q(.)$ represents the Q-function.

By substituting in (10), the approximation $Q(x) \approx \frac{1}{12}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{4}\exp\left(-\frac{2x^2}{3}\right)$ [28], and taking into account that the average BER, *ABER*, of the wireless optical link is estimated by integrating the instantaneous BER over *I*, we obtain the following integral:

$$ABER \approx \kappa_1 \int_0^\infty \exp\left(\frac{\Omega\mu_1}{\Psi I^2+\Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_2 \int_0^\infty \exp\left(\frac{\Omega\mu_2}{\Psi I^2+\Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + $$
$$+ \kappa_3 \int_0^\infty \exp\left(\frac{\Omega\mu_3}{\Psi I^2+\Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_4 \int_0^\infty \exp\left(\frac{\Omega\mu_3}{\Psi I^2+\Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI \tag{11}$$

with $\kappa_1 = (\lambda_1\lambda_4/12)\exp(-\mu_1)$, $\mu_1 = \lambda_2^2\Xi/2\Psi$, $\kappa_2 = (\lambda_1\lambda_4/4)\exp(-\mu_2)$, $\mu_2 = 2\lambda_2^2\Xi/3\Psi$, $\kappa_3 = (\lambda_3\lambda_4/12)\exp(-\mu_3)$, $\mu_3 = 9\lambda_2^2\Xi/2\Psi$, $\kappa_4 = (\lambda_3\lambda_4/4)\exp(-\mu_4)$, $\mu_4 = 6\lambda_2^2\Xi/\Psi$, $\lambda_4 = 2(ab)^{\frac{a+b}{2}}/(\Gamma(a)\Gamma(b))$, $\lambda_5 = (a+b-2)/2$.

By substituting in (11), the exponential function, $\exp(x) = \sum_{n=0}^{+\infty}\frac{x^n}{n!}$, and the quantity $\left(\Psi I^2/\Omega+1\right)^{-n} = \frac{1}{(n-1)!} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega}\middle| \begin{matrix}1-n\\0\end{matrix}\right)$ for $n \geq 1$, we have:

$$ABER \approx \sum_{i=1}^{4}\kappa_i \int_0^\infty I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + $$
$$+ \sum_{n=1}^{+\infty}\left\{\frac{n\sum_{i=1}^{4}\left(\kappa_i\mu_i^n\right)}{(n!)^2}\int_0^{+\infty} I^{\lambda_5} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega}\middle| \begin{matrix}1-n\\0\end{matrix}\right) K_{a-b}\left(2\sqrt{abI}\right) dI\right\} \tag{12}$$

By evaluating the integrals involved in (12), we obtain (13) which accurately predicts the average BER of the M-QAM OFDM based FSO link in the presence of gamma-gamma modelled turbulence and clipping effect:

$$ABER \approx \lambda_4^{-1}\sum_{i=1}^{4}\kappa_i + \\ +\sum_{n=1}^{\infty}\left\{\frac{2^{a+b-3}n\sum_{i=1}^{4}\left(\kappa_i\mu_i^n\right)}{\pi\left(ab\right)^{\frac{a+b}{2}}\left(n!\right)^2}G_{5,1}^{1,5}\left(\frac{16\Psi}{\Omega\left(ab\right)^2}\left|\begin{matrix}1-n,\frac{1-a}{2},\frac{2-a}{2},\frac{1-b}{2},\frac{2-b}{2}\\0\end{matrix}\right.\right)\right\} \tag{13}$$

Although expression (13) contains an infinite number of summands for the index n, it does in reality match the result of the numerical integration of (11) even for small values of n. This will be further discussed in the following section that follows.

IV. Numerical Results

From the derived mathematical expressions (6), (9) and (13), we can now estimate the average electrical SNR per bit for the OFDM-based FSO link under consideration, its outage probability and its average BER. More specifically, we present results for two values for the parameter C_n^2, i.e. 2×10^{-14} $m^{-2/3}$ and 2×10^{-13} $m^{-2/3}$, which correspond to moderate and strong turbulence conditions, respectively, L = 1km and λ = 0.83μm. Additionally, we fix $\sigma_{clip}^2 = 0.002$, $\rho_{PD} = 0.5$, $G_{TIA} = 50000$, $r_{load} = 50\Omega$, $E[x_{time}(k)] = 10$, $G_B = 0.5$, $P_B = 0.5$, $G_{DC} = 0.6$ and $K = 0.5$ [21]. Also, we used two values for the S_{PD}, i.e. 0.002 m^2 and 0.005 m^2 and three QAM levels, 4, 16 and 64. In all cases that we present below, the numerically evaluated results from the above derived closed form mathematical expressions, are accompanied by the corresponding ones obtained with Monte Carlo simulations.

A quantity which can be used in order to estimate the influence of the atmospheric turbulence effect at FSO system's performance is the Fried parameter, r_0 [29]. The value of the fraction D/r_0, constitutes an important quantity which shows how strongly the turbulence affects the link's performance. Thus, for a terrestrial FSO link with nearly horizontal propagation path, where the parameter C_n^2 can be assumed to remain invariable for the whole link length, the Fried parameter is given as [29]:

$$r_0 = 3.02\left(Lk^2 C_n^2\right)^{-3/5} \tag{14}$$

By substituting the parameters mentioned in the previous paragraph into (14), for the fraction D/r_0 we obtain the values 1.14 and 4.55 for S_{pd} equal to 2×10^{-3} m^2 and 5×10^{-3} m^2, respectively and weak atmospheric turbulence conditions, i.e. $C_n^2 = 2\times10^{-14}$ m$^{-2/3}$, while for $C_n^2 = 2\times10^{-13}$ m$^{-2/3}$, the corresponding values are 1.81 and 7.2. These outcomes show that the atmospheric turbulence strength and the aperture diameter of the receiver affect significantly the system's performance even for relatively short link lengths. This conclusion will be verified further from the results that follow.

In Figure 1, we present the average electrical SNR per bit for the OFDM-based turbulent FSO link with clipping from (6). It is clear that the value of $\overline{\mu}$, depends strongly on the atmospheric turbulence and the strength of the signal clipping effect.

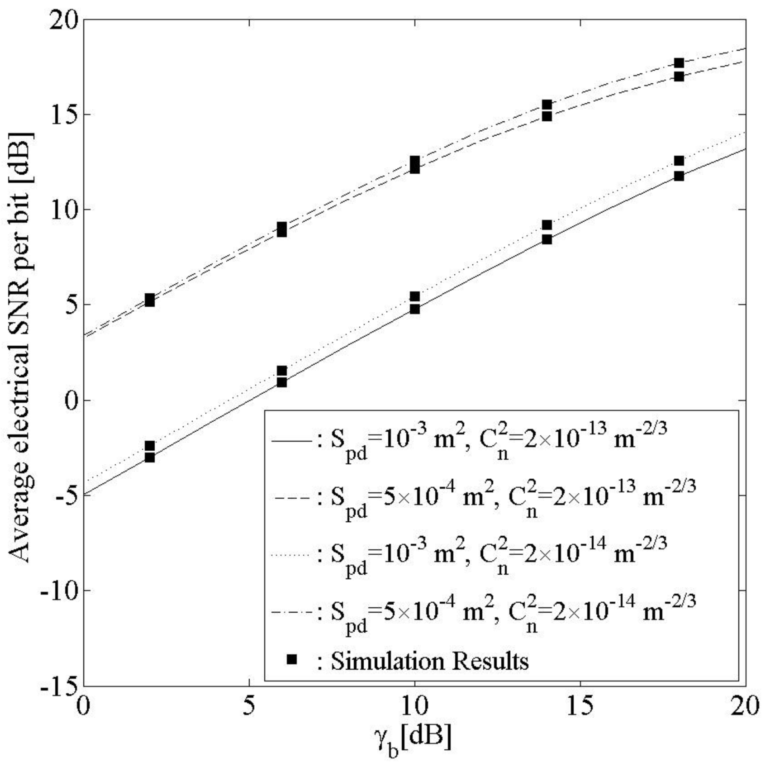

Figure 1: Average electrical SNR per bit, as a function of γ_b, taking into account the signal clipping effect, under the action of moderate or strong turbulence conditions.

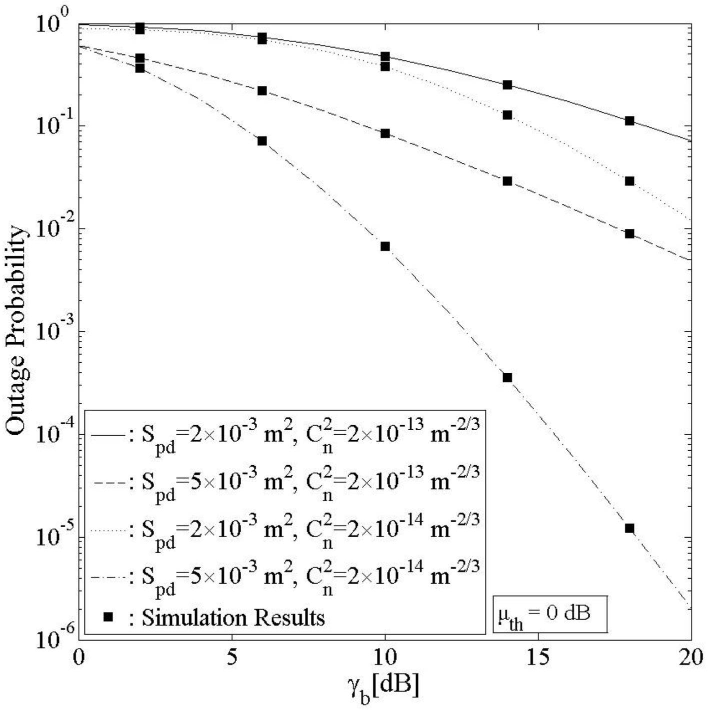

Figure 2: Outage probability as a function of γ_b, for moderate and strong turbulence conditions.

In Figs 2 and 3, we present the outage probability, P_{out}, for an FSO link with clipping and gamma-gamma turbulence effects, as a function of γ_b, for two values of the μ_{th}, namely, 0dB and 2dB.These figures show the influence of the receiver's surface area and the strength of the atmospheric turbulence effect as very significant parameters for the system's availability. Thus, if the S_{PD} is large and the turbulence weak, the outage probability becomes very small values even for relatively low values of the undistorted electrical SNR per bit.

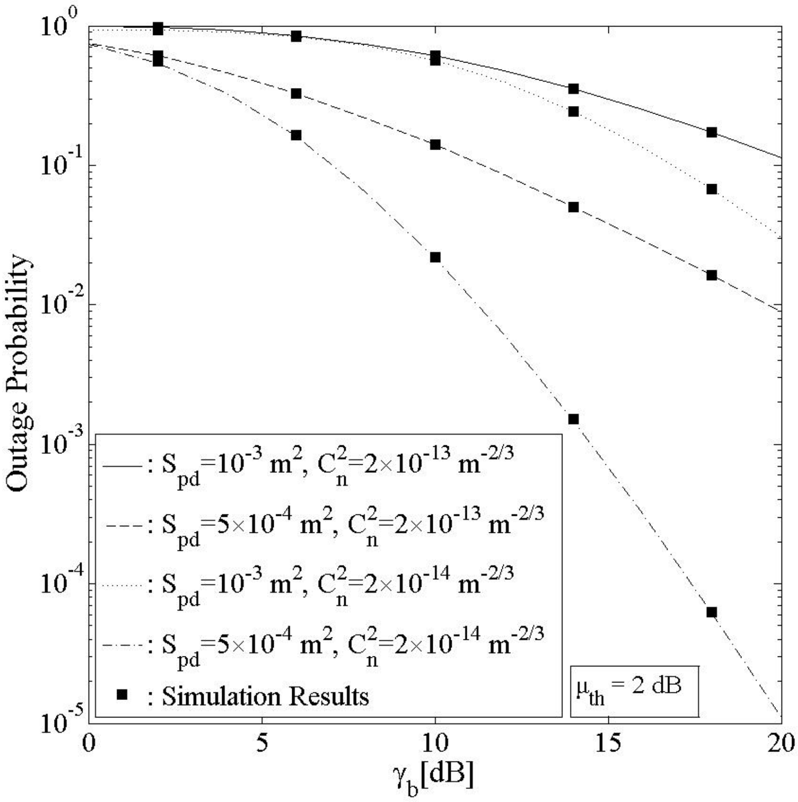

Figure 3: Outage probability as a function of γ_b, for moderate and strong turbulence conditions.

In Figs 4 and 5 we present the average BER as a function of the undistorted electrical SNR per bit for moderate to strong turbulence, various values of *M* and S_{PD}. As in the previous case, the influence of the S_{PD} and C_n^2 parameters is very significant, especially for *M* = 4 and 16.

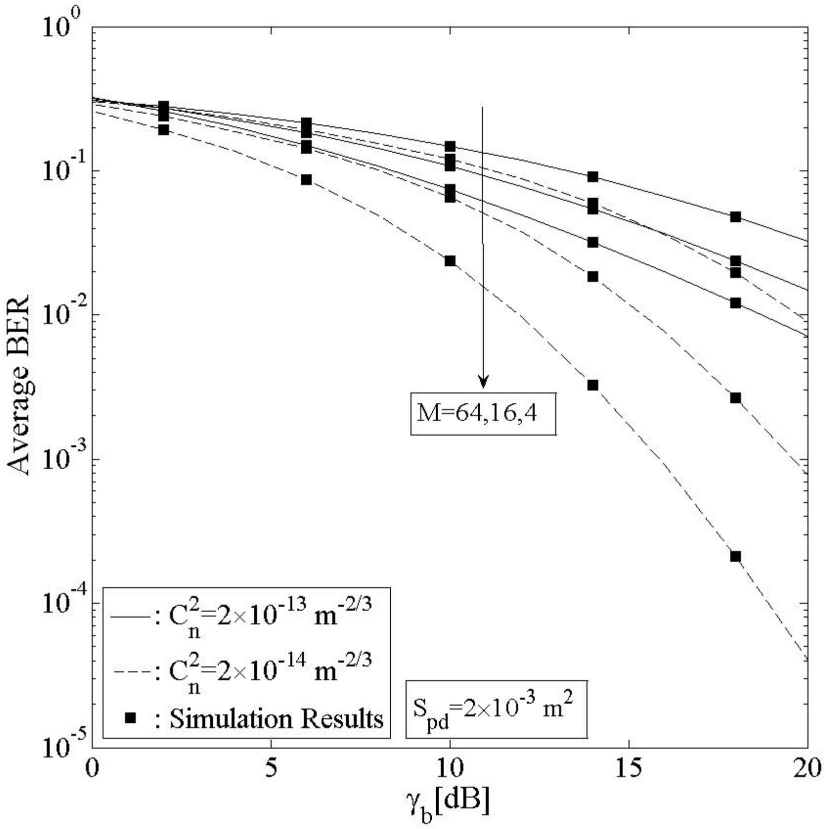

Figure 4: Average BER as a function of γ_b, for moderate and strong turbulence conditions.

As mentioned above, expression (13), which gives the average BER, includes a summation of infinite number of terms. Thus, a significant issue is to estimate the number of terms which have to be taken into account, at the evaluation of the average BER using (13) with the lower, practical, error. Thus, in order to calculate this relative estimation error, the integral given by (11) has been evaluated numerically and compared the result of (13), for $n = 5$. The relative estimation error of the average BER using (11) and (13), E_R, has been calculated for various parameters values. The results show that even for such a small value of $n = 5$, the relative estimation error did not exceed 0.05%.

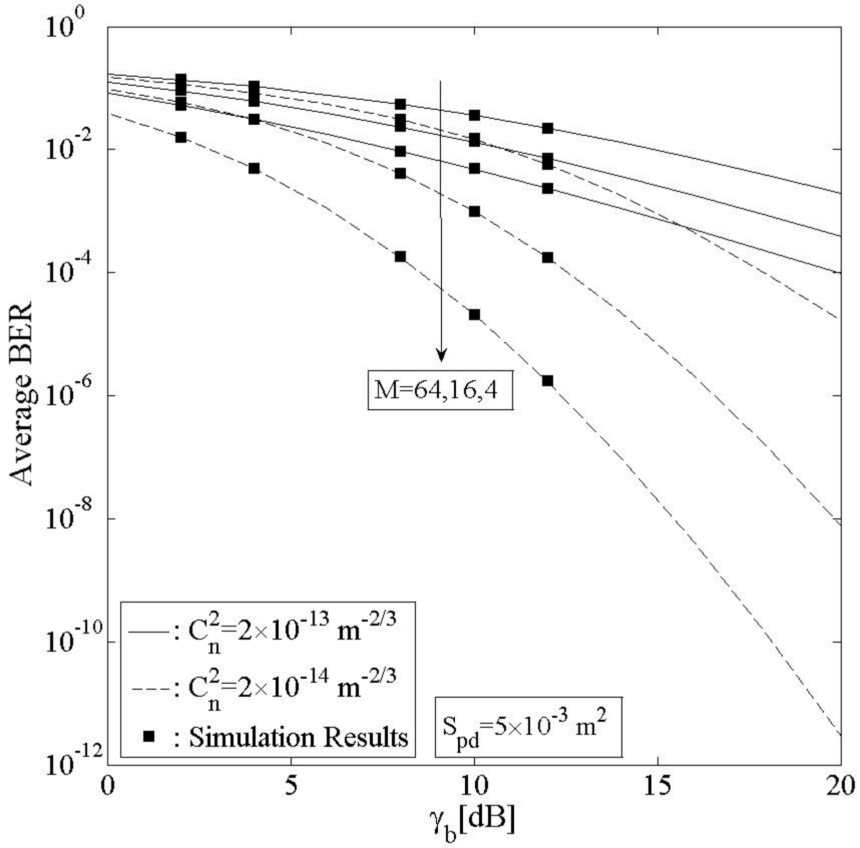

Figure 5: Average BER, as a function of γ_b, for moderate and strong turbulence conditions.

More specifically, in Fig. 6, we present the absolute value of the relative estimation error of the average BER versus the average electrical SNR for the above mentioned fixed parameters and $S_{pd} = 2\times10^{-3}\text{m}^2$, M=4, while C_n^2 takes the values 2×10^{-14} m$^{-2/3}$, 6×10^{-14} m$^{-2/3}$ and 2×10^{-13} m$^{-2/3}$. Thus, $|E_R|$ can be assumed as negligible for the most common cases and its value can be further reduced by increasing n accordingly.

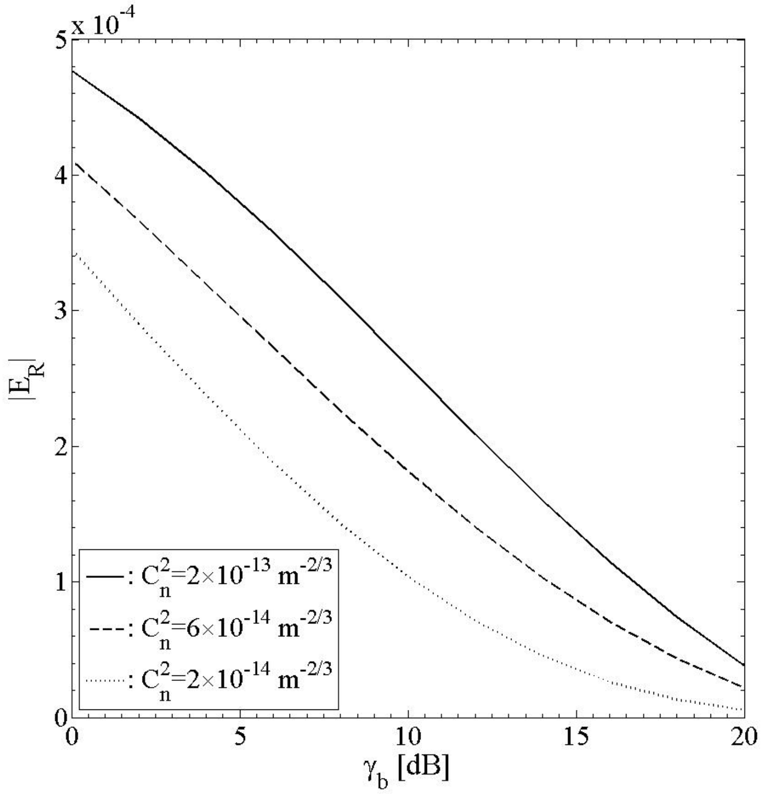

Figure 6: Absolute value of the relative estimation error of the average BER, as a function of the undistorted electrical SNR per bit.

In Figure 7, we present results for average BER evaluated from (13) for the case of extremely weak atmospheric turbulence and the corresponding ones obtained from (10) for the case of no turbulence. More specifically, in Fig. 7 it is shown the average BER, for $S_{pd} = 5\times10^{-3}$ m^2, $M = 4$ and $C_n^2 = 5\times10^{-13}$ m$^{-2/3}$, 5×10^{-14} m$^{-2/3}$ and 5×10^{-15} m$^{-2/3}$. These results verify numerically the fact that the smaller values of the refractive index structure parameter correspond to very weak influence of the atmospheric turbulence effect and the average BER curve tends to coincide with the limiting case without turbulence.

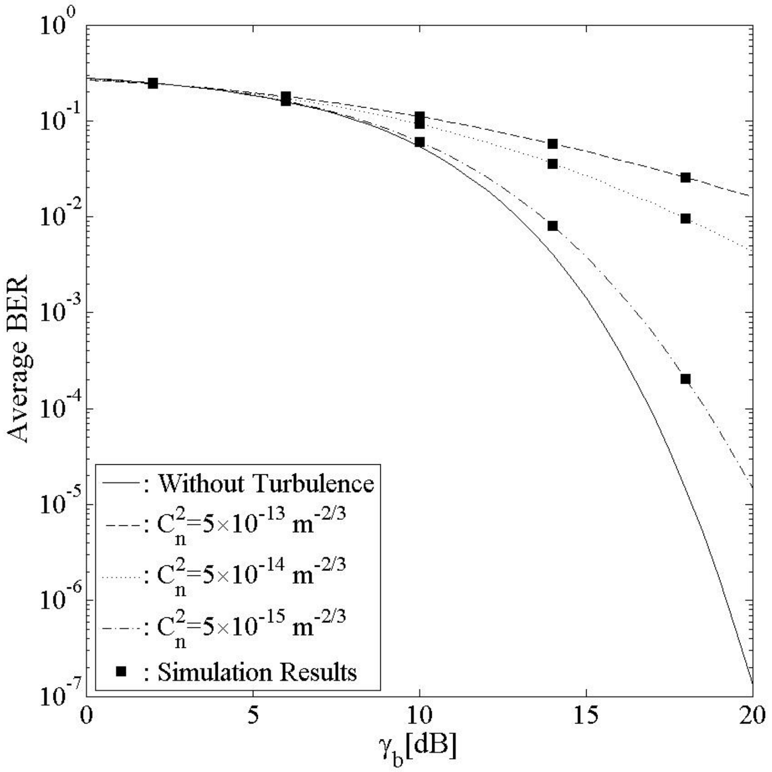

Figure 7: Average BER, as a function of γ_b, for various cases of turbulence strength with S_{pd}= 5×10^{-3} m^2, M=4 and in the limiting case with turbulence's absence.

The results presented above, have been obtained using realistic values for link's parameters. Thus, the obtained outcomes show the dependence of the performance of the specific optical wireless communication system from the strength of the clipping noise and the atmospheric turbulence effect. However, taking into account the large number of parameters which affect significantly the system's operation and have been mentioned above, the presented numerical results can be used, mainly, as a qualitatively measure for the dependence of system's performance on each specific factor of the link. Thus, the derived mathematical expressions, i.e. expression (6), (9), (13), can be used for performance estimation with any parameter value needed for the designing and implementation of any particular QAM OFDM-FSO that suffer from clipping noise and atmospheric turbulence.

V. Conclusions

We have derived analytically closed form mathematical expressions for the key performance metrics for an OFDM FSO channel which explicitly take into account the turbulence modelled with the gamma-gamma distribution and nonlinear distortions caused by the clipping effect. In particular, we have derived expressions for the average SNR, average BER and outage probability, as a function of the physical parameters of the OFDM FSO link. Finally, we presented results obtained from the derived expressions and the numerical simulations in order to express the dependence of these key metrics on turbulence, constellation size and the PD's active area which can be used for design of OFDM FSO links.

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