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# Are volcanic seismic $b$ -values high, and if so when?

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## 1 Abstract

The Gutenberg-Richter exponent  $b$  is a measure of the relative proportion of large and small earthquakes. It is commonly used to infer material properties such as heterogeneity, or mechanical properties such as the state of stress from earthquake populations. It is 'well known' that the  $b$ -value tends to be high or very high for volcanic earthquake populations relative to  $b=1$  for those of tectonic earthquakes, and that  $b$  varies significantly with time during periods of unrest. We first review the supporting evidence from of 34 case studies, and identify weaknesses in this argument due predominantly to small sample size, the narrow bandwidth of magnitude scales available, variability in the methods used to assess the minimum or cut-off magnitude  $M_c$ , and to infer  $b$ . Informed by this, we use synthetic realisations to quantify the effect of choice of the cut-off magnitude on maximum likelihood estimates of  $b$ , and suggest a new work flow for this choice. We present the first quantitative estimate of the error in  $b$  introduced by uncertainties in estimating  $M_c$ , as a function of the number of events and the  $b$ -value itself. This error can significantly exceed the commonly-quoted statistical error in the estimated  $b$ -value, especially for the case that the underlying  $b$ -value is high. We apply the new methods to data sets from recent periods of unrest in El Hierro and Mount Etna. For El Hierro we confirm significantly high  $b$ -values of 1.5-2.5 prior to the 10 October 2011 eruption. For Mount Etna the  $b$ -values are indistinguishable from  $b=1$  within error, except during the flank eruptions at Mount Etna in 2001-2003, when  $1.5 < b < 2.0$ . For the time period analysed, they are rarely lower than  $b=1$ . Our results confirm that these volcano-tectonic earthquake populations can have systematically high  $b$ -values, especially when associated with eruptions. At other times they can be indistinguishable from those of tectonic earthquakes within the total error. The results have significant implications for operational forecasting informed by  $b$ -value variability, in particular in assessing the significance of  $b$ -value variations identified by sample sizes with fewer than 200 events above the completeness threshold.

Keywords:  $b$ -value; volcano; seismology; completeness magnitude

## 31 2 Introduction

32 Volcanic earthquakes provide insight into physical processes acting at volcanoes, such as the  
33 mechanisms of deformation of the volcanic edifice and magma accumulation, and statistical analysis  
34 of earthquake catalogues are a key component of eruption forecasting methods (McNutt, 1996).  
35 Increased rates of earthquakes are a primary indicator of volcanic unrest, and changing locations of  
36 earthquake hypocentres can be used to map magma migration (Wiemer and Wyss, 2002). The  
37 frequency-magnitude distribution (FMD) of volcanic earthquakes can provide insight into the state of  
38 stress or material properties, and are a key component of most studies of volcanic seismicity.

39 Where the catalogue is completely reported, the FMD, commonly takes the form of a Gutenberg-  
40 Richter (GR) relation (Gutenberg and Richter, 1954):

$$\log(N) = a - bM, \quad (1)$$

41 where  $N$  is the total number of earthquakes of magnitude equal to or greater than  $M$ , and  $a$  and  $b$  are  
42 real, positive constants characteristic of the specific catalogue. The parameter  $a$  is the logarithm of  
43 the number of earthquakes with  $M \geq 0$ , and is thus a measure of the seismicity rate of the region. The  
44  $b$ -value represents the relative proportion of large and small events in the catalogue. It is best  
45 calculated or inferred using the maximum likelihood method (Aki, 1965), now used almost universally  
46 in earthquake seismology (Mignan and Woessner, 2012). Other methods such as a least squares fit of  
47 the data to equation 1 are known to produce a biased estimate (Naylor et al., 2010). In addition, if the  
48 bandwidth of data is narrow, or equivalently the sample is small, then it is easy to overestimate the  
49 underlying  $b$ -value (Main, 2000). Finally, the  $b$ -value may also be biased due to incorrect identification  
50 of the threshold for complete reporting, denoted  $M_c$  here (Mignan and Woessner, 2012). These and  
51 other sources of bias introduce an epistemic error to any inference from the data. In principle this  
52 should be accounted for in addition to the aleatory uncertainties inferred from the random error  
53 associated with measurement or statistical fluctuation in the data, but it is often neglected in studies  
54 of volcanic earthquake populations.

55 The Gutenberg-Richter form of the distribution holds, at least for small and intermediate events across  
56 a remarkable range of sizes and loading conditions, from laboratory experiments to volcanic and  
57 tectonic earthquakes (Main, 1996). In controlled laboratory tests, seismic  $b$ -values commonly change  
58 systematically with respect to a variety of controlling factors. These include the degree of material  
59 heterogeneity (Mogi, 1962), the level of applied stress (Scholz, 1968), the degree of stress  
60 concentration, i.e. the stress intensity normalised to the fracture toughness (Meredith and Atkinson,  
61 1983), the chemical reactivity of the pore fluid (Meredith and Atkinson, 1983), and the pore fluid  
62 pressure (Sammonds et al., 1992). In nature other factors that affect the  $b$ -value systematically include  
63 the earthquake focal mechanism (Schorlemmer et al., 2005), the depth (Mori and Abercrombie, 1997),

64 and the degree of coupling or strain partition between seismic and aseismic deformation at plate  
65 boundaries (Mazzotti et al., 2011).

66 The  $b$ -value for tectonic earthquakes, using best practice and large regional or global data sets, is  
67 commonly reported as taking values near unity (Frolich and Davis, 1993). In contrast the reported  $b$ -  
68 values from published studies of earthquake populations associated with volcanic unrest are  
69 commonly reported as being significantly higher than this, allowing for the random error expected for  
70 a  $b$ -value of unity (described in more detail below). The main question we address here is whether this  
71 difference is real or, at least to some extent, an artefact of the known sources of bias described above.

72 To examine this question we first use synthetic data to explore the effect of various factors on the  
73 estimated  $b$ -value, denoted  $\tilde{b}$ , and the underlying  $b$ -value, henceforth denoted  $b$ . Uncertainties in  $\tilde{b}$   
74 at one standard deviation, denoted  $\sigma_{\tilde{b}}$ , are estimated using the method of Shi & Bolt (1982), which  
75 correctly reflects the (approximately) Poisson ‘counting errors’ expected from sampling a whole  
76 number of events (Greenhough and Main, 2008). The advantage of using synthetic data is that we can  
77 distinguish between the random error  $\sigma_{\tilde{b}}$ , and the systematic error or bias  $\tilde{b} - b$ , or equivalently to  
78 errors of precision and accuracy respectively. We show how both depend intrinsically on the sample  
79 size. First we determine an optimum method of estimating the cut-off magnitude of complete  
80 reporting of events,  $M_c$ , for catalogues of different sizes, and then propose a formal workflow for the  
81 estimation of  $M_c$ . The proposed workflow is then applied to two volcanic seismic catalogues at Mount  
82 Etna and El Hierro as important examples of recently-active volcanic systems to address the questions:  
83 (a) are the  $b$ -values higher than 1? And (b) do they vary with time significantly outside the estimated  
84 margins of error? For these examples,  $b$  is remarkably stationary and similar to ( $\sim 1$ ) or only somewhat  
85 larger (1-1.5) than to those of tectonic earthquakes, except for specific transients where the  $b$ -value  
86 can be significantly greater than background at 95% confidence. The results presented here will  
87 provide greater confidence in identifying statistically-significant variations in  $b$ -value, and in identifying  
88 physical causes for this variability.

### 89 3 Review and synthesis of previous studies

90 In this section we extend the review of McNutt (2005), who summarised reported  $b$ -values and  
91 associated parameters such as source depth from 13 different volcanoes around the world. This review  
92 includes  $b$ -values as high as 3 in one case (McNutt, 2005). In Table 1 we extend this study to 21  
93 volcanoes, and include a wider range of associated parameters, including: the number of events; the  
94 range of magnitudes used in the analysed catalogues; the methods used to calculate the completeness  
95 magnitude and fit the  $b$ -value; and the range of  $b$ -values reported in each study, including a typical  
96 value. Multiple studies use several methods for analysing  $b$ -value variations and thus the results are

97 reported separately in Table 1, giving 34 separate results for comparison in this new synthesis.  
98 Information on all the different fields of data could not be found in all cases, e.g. how the threshold  
99 magnitude was estimated, resulting in some blank entries in Table 1.

100 The maximum reported  $b$ -values range between 1.4 and 3.5, with a peak at  $b=1.7$  (Figure 1c). From  
101 Figures 1b there is no clear dependence on the magnitude and  $b$ -value. Bonnet et al. (2001) also found  
102 there was no direct dependence of the scaling exponent for fracture length on the scale of observation  
103 and that no significant trends could be determined in the type of faulting (Bonnet et al., 2001).

104 Figure 1 shows the distribution of  $b$ -values compared to the other variables in the study. There are no  
105 clear trends with depth (Figure 1a) or magnitude range or size (Figure 1b). However, there is a weak  
106 decreasing trend in the  $b$ -value as the number of events in the sample,  $N$ , increases (Figure 1c). The  
107 data only spans from 10 to 300 events covering just over one magnitude unit, with over half, (16 of  
108 25) of the studies using catalogues with either 50 or 100 events. One further study (Ibanez et al., 2012)  
109 containing 7000 events reports a relatively high  $b$ -value of 1.57 that does not follow this trend.  
110 However, this study - and many others cited in Table 1 - use the Least Squares method to fit  $b$  or to  
111 check the results of the maximum likelihood estimation, introducing a known source of potential bias  
112 outlined in the introduction.

113 In summary this review has highlighted a significant variability in the reported values of  $b$ , and a  
114 significant variability in the methods of analysis used in the different studies. Typical  $b$ -values are  
115 usually in the range 1-1.2. They are never (for this list) less than one, and are occasionally very high  
116 (up to 3.5). The variability is much larger than any systematic trends, except that the  $b$ -value tends to  
117 decrease with increasing sample size. In this paper we use synthetically-generated data to address  
118 some of the most important origins of this variability, in particular the choice of threshold magnitude  
119 and the sample size.

## 120 **4 Methods for analysis of Frequency-Magnitude Distributions**

121 A variety of statistical methods have been used to model FMD's and to quantify whether those models  
122 are consistent with the observed data. Most methods involve modelling the proportion of the  
123 distribution above the completeness magnitude. Therefore there is a strong inter-dependence  
124 between estimates of the completeness magnitude and values of parameters of prospective FMD  
125 models. In this section we summarise the current methods used to address this problem.

### 126 **4.1 Gutenberg-Richter parameters**

127 There is a well-established literature that describes the merits of different statistical methodologies  
128 for FMD analysis. Methods involving regression on cumulative frequencies, or using least-squares

129 regression, are known to give biased estimates of the  $b$ -value (Naylor et al., 2010) as they are known  
 130 to give disproportionate weighting to higher magnitude events (Ghosh et al., 2008). The maximum  
 131 likelihood technique has become standard in seismic hazard analysis (Mignan and Woessner, 2012).  
 132 The data are assumed to be exponentially distributed (as in eq. 1) and the maximum possible  
 133 magnitude is assumed to be at infinity (Aki, 1965). Physically, earthquakes must have a finite maximum  
 134 size dependent on the size and strain limits within the Earth, but  $M_{max}$  is not well constrained by global  
 135 data (Main et al., 2008; Holschneider et al., 2014). The maximum likelihood method weights each  
 136 event equally and correctly allows for error structure of the data: in frequency data in the form of a  
 137 Poisson distribution (Naylor et al., 2010). Formally, the maximum likelihood estimate of the  $b$ -value is:

$$\tilde{b} = \frac{\log_{10} e}{\bar{M} - (M_c - \Delta M/2)} \quad (2)$$

138 where  $\tilde{b}$  is the estimate of the  $b$ -value,  $\bar{M}$  is the mean magnitude,  $M_c$  is the completeness magnitude,  
 139 and  $\Delta M$  is the magnitude bin size of the histogram (Aki, 1965). Aki also showed the uncertainty on  
 140 this estimate at one standard deviation (67% confidence) can be approximated to:

$$\sigma_{\tilde{b}} = \frac{\tilde{b}}{\sqrt{N_c}} \quad (3)$$

141 Where  $N_c$  is the number of events in the complete part of the catalogue, or 1.96 times this value at  
 142 95% confidence.

143 A summary study by Marzocchi & Sandri, (2003), tested two further improvements on this estimation  
 144 of  $b$  using binned magnitudes, equation (4) (Bender, 1983), and an improved uncertainty estimate (eq.  
 145 5) (Shi and Bolt, 1982; Marzocchi and Sandri, 2003):

$$\tilde{b} = \frac{1}{\ln 10[\hat{\mu} - (M_c - \Delta M)]} \quad (4)$$

$$\sigma_{\tilde{b}} = 2.30\tilde{b}^2 \sqrt{\frac{\sum_{i=1}^N (M_i - \hat{\mu})^2}{N_c(N_c - 1)}}$$

146 where  $\hat{\mu}$  is the average magnitude of the sample, and  $\Delta M$  is the binned magnitude width. The  $b$ -value  
 147 is relatively insensitive to the upper magnitude cut-off, so assuming an infinite cut-off in deriving  
 148 equations (3) and (5) does not introduce a significant bias. However, in both cases the quoted error is  
 149 formally conditional on the choice of  $M_c$ , which in practice must be estimated. This introduces an  
 150 implicit source of bias that can be positive or negative. In this paper we will demonstrate that this  
 151 additional source of uncertainty is comparable to or can greatly exceed the estimates from equations  
 152 (3) or (5).

## 153 4.2 Calculating the completeness magnitude

154 Most studies apply a lower threshold or cut-off magnitude,  $M_c$ , above which the catalogue can be  
155 regarded as completely recorded (Wiemer and Wyss, 2000).  $M_c$  is the lowest magnitude at which 100  
156 per cent of earthquakes in a space-time volume are detected (Rydelek and Sacks, 1989; Woessner and  
157 Wiemer, 2005; Mignan and Woessner, 2012). Earthquakes with smaller magnitudes are less likely to  
158 be completely reported when their amplitude becomes smaller than that of the ambient noise. This  
159 introduces a high-pass filter to the FMD, which could in principle be modelled and fitted to the data.  
160 However, this is rarely (if ever) done explicitly. In practice most studies assume  $M_c$  is the magnitude  
161 at which the log(cumulative frequency)-magnitude curve departs from a linear trend of eq. 1. There  
162 are three main techniques commonly used to estimate this magnitude, namely the Maximum  
163 Curvature (MaxC) method, the Goodness-of-Fit test (GFT) (Wiemer and Wyss, 2000) and  $b$ -value  
164 stability (BVS) method (Cao and Gao, 2002).

165 The MaxC method calculates the highest value of the first derivative of the cumulative frequency-  
166 magnitude curve. In practice this matches the frequency-magnitude bin with the highest number of  
167 events (Figure 2a). The main limitation of this method is that it will systematically underestimate  $M_c$   
168 unless there is a sharp transition between the incomplete and complete portion of the catalogue, as  
169 illustrated in Figure 2a.

170 The GFT method calculates  $M_c$  by comparing the observed FMD with a synthetic one. The best-fit  
171 distribution is calculated for trial cut-off magnitudes using the maximum-likelihood estimates of  $a$ - and  
172  $b$ -values of the observed dataset. The residuals between the data and the best fit distribution are then  
173 calculated as a function of cut-off magnitude (Figure 2b). The completeness threshold,  $M_c$ , is selected  
174 as being the first magnitude above which the residual between the synthetic straight line fit model  
175 and observed data falls within a 95% confidence window. If 95% confidence cannot be obtained then  
176 a 90% confidence window can be used as a compromise. This method tends to give systematically low  
177 values for  $M_c$  although not as low as the MaxC method (Wiemer and Wyss, 2000).

178 The BVS method simply evaluates the estimated  $b$ -value as a function of the cut-off magnitude. The  
179 assumption here is that  $\tilde{b}$  will initially increase as the cut-off magnitude increases, until the cut-off  
180 magnitude equals  $M_c$  after which  $\tilde{b}$  will stabilise. The inferred  $b$ -value is deemed to have stabilised  
181 once the average  $\tilde{b}$  for the five successive cut-off magnitudes falls within error of the selected cut-off  
182 magnitude (Figure 2c). The BVS method tends to have high  $M_c$  values relative to other methods  
183 (Woessner and Wiemer, 2005) and consequently higher  $\tilde{b}$  values.

## 184 5 Results for Synthetic catalogues

### 185 5.1 Generating synthetic catalogues

186 We now evaluate which of the three methods for calculating the  $M_c$  is the most accurate and reliable,  
187 by generating synthetic catalogues with known  $M_c$  and  $b$ -value, but different forms of the cut off  
188 below  $M_c$ . As a benchmark check we first generated synthetic data to determine  $\tilde{b}$  and  $\sigma_{\tilde{b}}$  for  $b=1$  and  
189  $b=2$  as a function of the complete sample size  $N_c$ , conditioned on an exact value for  $M_c$ . This provided  
190 a good match to Fig. 1a,b of Marzocchi and Sandri (2003). However, in reality  $M_c$  is not known  
191 independently a priori. Ideally we would hope the incremental FMD would have a sharp and easily  
192 distinguishable peak at  $M_c$ , defining the lower limit of the complete catalogue (Figure 3a). In reality  
193 the peak of the distribution is often curved and much broader due to the complexity of the signal to  
194 noise ratio at the recording stations, and of locating and calculating magnitudes for small events, so  
195 defining  $M_c$  can be much more challenging (Figure 3b). This introduces an additional source of  
196 uncertainty that is the prime focus of the current paper.

197 To test each of the three methods, we use two end-member scenarios. The first has a sharp peak  
198 (Figure 3a) and the second a broader peak (Figure 3b). Both catalogues have  $M_c$  set to 1.0. The  
199 complete part of both catalogues was created by randomly generating individual events from an ideal  
200 parent Gutenberg-Richter law distribution with a  $b$ -value of 1.0. For the sharp-peaked distribution the  
201 incomplete part of the catalogue was generated using a filter with a linear slope of 3, for values below  
202  $M_c=1.0$  decaying to zero probability at  $M=0$ . For the broad-peaked distribution a GR distribution was  
203 used to generate events all the way down to  $M=0$ . The probability function shown in Figure 3c was  
204 then applied as a filter to remove events below the known threshold  $M_c=1.0$ , until the required  
205 number of events were left in the complete catalogue.

206 To examine the role of catalogue size, catalogues were generated with a complete size of 50, 100, 200,  
207 500, 1000 and 5000 events. Finally the  $b$ -value was varied from a typical tectonic value of 1.0 to a  
208 significantly high  $b$ -value of 2.0, to test whether each method can reliably calculate  $M_c$  and inferred  $b$ -  
209 values for the case that the underlying  $b$ -value is high.

210 For each catalogue size,  $b$ -value, and distribution shape; 100 catalogue were iteratively generated, and  
211 the estimated  $M_c$  and  $b$ -values determined using the different methods described in section 4. A bin  
212 size  $\Delta M$  of  $0.1M$  is used throughout. Figure 3 shows both the average catalogue (solid line) and the  
213 spread of the outcomes associated with the finite sample size (dashed lines).



## 214 5.2 Synthetic Results

### 215 5.2.1. Sharp-peaked distribution

216 In this case the simulations of Figure 4 demonstrate that the MaxC method performs the best in terms  
217 of calculating  $M_c$ , closely followed by the BVS method. The GFT performs adequately for  $N_c=5000$  but  
218 fails when  $N_c=50$  as for over 90% of the catalogues  $b$  is not even calculated correctly within  $\pm 1.0$  of the  
219 known value. When  $b=1$  and  $N_c=5000$ , MaxC and BVS both correctly lead to a correct calculation of  $b$   
220 with  $<0.01$  error.

### 221 5.2.2. Broad-peaked distribution

222 Figure 5 shows histograms of the best estimates of  $M_c$  for the three methods, for different catalogue  
223 sizes and  $b$ -values, for the case of the broad-peaked distribution. When  $N_c=50$  for both  $b=1$  and  $b=2$ ,  
224 MaxC and BVS both systematically underestimate  $M_c$ , because very few events have a greater  
225 magnitude than  $M_c=1.0$  (Figure 6). Both MaxC and BVS methods give results with some scatter,  
226 centred on  $b=1$ , but several iterations had significantly higher  $b$ -values of 2 or above. Both methods  
227 perform poorly when  $b=2$ , as there too few events in the catalogue, with median values of  $\tilde{b}\approx 1.5$ . The  
228 GFT over-estimates  $M_c$  when  $b=1$  but appears to give a reasonable estimate when  $b=2$ . However, the  
229 95% confidence is only reached when  $M_c$  is very close to the maximum magnitude and thus the  
230 complete catalogue size is very small. This results in the inferred  $b$ -values being very high for both  $b=1$   
231 and  $b=2$ .

232 When  $N_c=5000$  it becomes apparent that MaxC is not a good method for broad-peaked distributions.  
233 For  $b=1$ ,  $M_c$  is heavily underestimated, with a median value of  $M_c=0.4$ , and resulting  $\tilde{b}$ -values all less  
234 than  $b=1$ . These underestimates are amplified when  $b=2$  with median values of  $M_c=0.4$  and  $\tilde{b}\approx 1.3$ . The  
235 GFT performs much better for both  $b=1$  and  $b=2$  however it gives a conservative estimate for both.  
236 The BVS method performs the best for a broad-peaked distribution, giving only a slightly conservative  
237 estimate of  $M_c$  with a median value of  $M_c=0.9$  for  $b=1$  and  $b=2$ . The BVS method returns the correct  
238  $\tilde{b}=1.0$  in over 80 iterations. The median value for  $b=2$  is also approximately correct, however there is  
239 a very broad range of results with a slight skew towards values higher than  $b=2.0$ . This is a very large  
240 catalogue and the BVS method is clearly the best when  $b=2$ . Our results show that it is intrinsically  
241 more difficult to calculate high  $b$ -values, however it is possible to find an estimate with a correct  
242 median value with the BVS method, albeit with a large spread in  $\tilde{b}$ .

### 243 5.2.3. Comparison of method performance

244 For a sharp-peaked distribution the MaxC method correctly calculates  $M_c$  the highest proportion of  
245 times for both high and low  $b$ -values. This outcome is not surprising as the MaxC method finds the  
246 magnitude bin with the highest number of events that, trivially, is the  $M_c$  set by the parent distribution.

247 The BVS method performs almost as well as the MaxC method for low  $b$ -values, but with higher  $b$ -  
248 values the method returns too high estimates of  $M_c$ . However, as long as for larger catalogue sizes the  
249 BVS method continues to return good estimates of the  $b$ -value. The GFT method does not work with  
250 small catalogues as the 95% confidence threshold is only reached when the  $M_c$  is very close to the  
251 maximum magnitude event, therefore there are a minimal number of earthquakes left in the  
252 catalogue, and thus the uncertainty is very large. For larger catalogues GFT performs much better.  
253 However for both  $b=1$  and  $b=2$ , using the GFT-calculated value of  $M_c$  results in fewer correct  
254 calculations of  $\tilde{b}$  than the MaxC and BVS methods. Therefore we consider it to be the least-well  
255 performing method. For  $b=2$  the steeper slope of the complete catalogue leads to a larger spread of  
256 calculated  $\tilde{b}$ -values for all three methods than for  $b=1$ . This is due to the random scattering of data  
257 due to sampling which has a greater influence on the FMD at high  $b$  compared to low  $b$ -values, and is  
258 not inherently linked to any of the methodologies.

259 Figure 7 and Figure 8 compare the performance of the different methods for the case of a broad-  
260 peaked distribution, using the mean and standard deviations of  $\tilde{b}$  calculated from the data in Figure 6.  
261 For both  $b$ -values the GFT method does not reliably calculate  $M_c$ , resulting in a biased estimate of the  
262  $b$ -value. For  $N_c \leq 500$  the correct  $b$ -value is calculated within the statistical error, but the distribution is  
263 heavily skewed towards high  $b$ -values, meaning that this method performs sub-optimally for these  
264 small catalogue sizes. However for larger catalogues ( $N_c=1000$  &  $5000$ ) the GFT method does calculate  
265 accurate  $b$ -value estimates for both  $b=1$  and  $b=2$ . The MaxC method returns a systematically-low  
266 estimate of  $M_c$  for all catalogue sizes, resulting in under-estimates of the  $b$ -value for both  $b=1$  and  
267  $b=2$ . We conclude that it is not an appropriate method for calculating  $M_c$  for a broad-peaked  
268 distribution.

269 The estimates of  $M_c$  returned by the BVS method increase in accuracy with catalogue size. For  $N_c \geq 200$   
270 the BVS method correctly calculates  $M_c$  within the 95% confidence limits for both  $b=1$  (Figure 7) and  
271  $b=2$  (Figure 8). When  $b=1$  and the catalogue size is  $N_c \geq 200$ , the 95% confidence spread around the  
272 true  $b$ -value is very small,  $\pm 0.25$ . Using the BVS method with smaller catalogue sizes can result in  $b$ -  
273 value estimates as high as 2 even with  $b=1$  (Figure 7). This observation suggests that care must be  
274 taken to not over-interpret high  $b$ -values calculated for small catalogue sizes. For  $b=2$ , the standard  
275 deviation of results is independent of catalogue size at about  $\pm 0.75$ . However, the median and mean  
276 of the  $\tilde{b}$ -value estimates tend towards the parent  $b=2$  as catalogue size increases. Again for  $N_c \geq 200$   
277 for  $b=2$  the BVS method estimates  $\tilde{b}$  to within 95% confidence.

278 In terms of defining a threshold minimum complete catalogue size, when  $N_c \geq 500$  our results show  
279 both  $b=1$  and  $b=2$  can be estimated accurately and precisely (Figure 7). For  $N_c=100$  the statistical error  
280 in estimating  $b=1$  is large, indicating a lack of precision, and for  $b=2$  the average and median values are

281 significantly below 2, indicating a residual bias. However, a threshold of 500 for completely-reported  
282 events is a relatively large number for many practical applications. From the results in Figure 7, a  
283 pragmatic choice of  $N_c=200$  is an acceptable threshold for a trade-off between accuracy, precision,  
284 and realistic catalogue size.

### 285 **5.3 A proposed workflow for the calculation of $M_c$**

286 Informed by this analysis, we propose a workflow for analysing the FMD of volcanic earthquake  
287 catalogues (Figure 9). As discussed above, we considered that the minimum catalogue size for reliable  
288 estimation of the  $b$ -value is  $N_c=200$ .

289 First,  $M_c$  is estimated using each of the MaxC, GFT and BVS methods. If all three  $M_c$  estimates agree  
290 within  $\pm 0.1$ , the FMD can be modelled by a sharp-peaked distribution, and so the MaxC estimate of  
291  $M_c$  should be used. If the  $b$ -value calculated using this  $M_c$  has an error of  $\leq \pm 0.25$  it should be  
292 considered to be reliable. An error of  $> \pm 0.25$  makes it difficult to interpret the  $b$ -value and may indicate  
293 an unreliable estimate of  $M_c$ .

294 If the three estimates of  $M_c$  vary by  $\geq 0.1$ , or the  $b$ -value calculated from the MaxC estimate of  $M_c$  is  
295  $\geq 0.25$ , we recommend that the BVS method should be used. If the resulting  $b$ -value has an error of  
296  $\leq 0.25$  it should be considered to be reliable. If this is not the case, the GFT analysis should be used. If  
297 a  $b$ -value with an error of  $\leq 0.25$  cannot be obtained using any of the 3 methods, we argue that the  
298 catalogue is too small for reliable FMD analysis. If the complete catalogue has over 5000 events and  
299 the  $b$ -value uncertainty is still too high, it is likely that the FMD is not consistent with an underlying  
300 Gutenberg-Richter distribution.

301 For the analysis of variations in FMDs, a large volcanic earthquake catalogue can be split on the basis  
302 of spatial or temporal windows, and this workflow applied to each sub-catalogue in turn. However,  
303 the same minimum complete catalogue size and reliability criteria rules apply to sub catalogues too.

### 304 **5.4 Error introduced from the completeness magnitude**

305 We now use the workflow of Figure 9 to consider the relative effect of  $M_c$  estimation for catalogues  
306 of different size on the accuracy and precision of the estimate of  $\tilde{b}$  for the case of a broad-peaked  
307 distribution. Figure 10 shows a histogram of the  $\tilde{b}$  for 100 catalogue realizations with  $b=2$ , along with  
308 examples of its standard deviation  $\sigma_{\tilde{b}}$  estimated from equation 5.  $\tilde{b}$  is beyond 1 standard deviation of  
309  $b$  in more than 1/3 of the cases, indicating a significant epistemic error in the estimation. We show in  
310 this section that this is due to the bias  $\tilde{b} - b$  in the finite-sized sample. The error due to calculating  $M_c$   
311 for individual realisations is illustrated as a blue bar at one standard deviation in Figure 9. The median  
312  $\tilde{b}$  is close to the true value (the central blue dot is near the vertical dashed line), so the residual bias  
313 due to estimating  $M_c$  is near zero for a large population of trials. However, the standard deviation in

314 the error due to  $Mc$  is much larger than the average statistical error for similar  $b$ -values (the black  
315 error bars).

316 To quantify this error in the general case, we ran many simulations for different values of  $b$  and  $N_c$ ,  
317 with the results shown in Figure 11. Figure 11a shows the average statistical error from equation (5),  
318 Figure 11b the average error in  $\tilde{b}$  due to propagating uncertainties in estimating  $Mc$  as illustrated by  
319 the blue horizontal error bar in Figure 10, and Figure 11c the ratio of the two. The ratio was calculated  
320 5 times for each of 15 catalogue sizes between 50-5,000 events and  $b$ -values of 0.5, 1.0, 1.5, 2.0 & 3.0,  
321 with the average value indicated by the colour scheme in Figure 11. The ratio varies between 1.2 and  
322 a factor 14 or so for the range studied, implying that the sample bias error is always greater than, and  
323 often much greater than the estimated statistical uncertainty in  $\tilde{b}$  from equation (5). This finding  
324 means that the statistical error commonly used on its own to quantify the  $\tilde{b}$ -value uncertainty is not  
325 an adequate description of the total error, though it approaches the total error for large numbers of  
326 events and low underlying  $b$ -values. In Figure 11c the ratio can reach an order of magnitude for  $b > 2$   
327 and event numbers above 1000. This is because the statistical error  $\sigma_{\tilde{b}}$  is very small when  $N_c$  is large.  
328 However the sample bias also increases with  $N_c$  for high  $b$ . This somewhat counter-intuitive result is  
329 because the magnitude range over which  $Mc$  can be calculated is much smaller at low  $N_c$  than at high  
330  $N_c$ , so the uncertainty is bounded to a greater degree at low  $N_c$ , and hence reduces at low  $N_c$ . The  
331 template of Figure 11c can be used empirically to determine a more appropriate error for  $b$ -value  
332 estimation.

### 333 **5.5 Application to volcanic catalogues**

334 We apply our proposed workflow to earthquake catalogues for Mount Etna volcano, Sicily (Murru et  
335 al., 1999; Murru et al., 2005; Murru et al., 2007) and El Hierro volcano, Canary Islands (Ibanez et al.,  
336 2012; López et al., 2012; Becerril et al., 2013; Marti et al., 2013; García et al., 2014) to test the reliability  
337 of any previously reported variations in  $b$ -values. This is simply to compare results from the proposed  
338 workflow to previous volcanic  $b$ -value's and not to make any interpretation about the behaviour of  
339 the volcanos.

340 We analyse the Instituto Geográfico Nacional (IGN) earthquake catalogue for El Hierro between July  
341 2011 and December 2013, a period associated with significant seismic activity associated with magma  
342 emplacement, and including a submarine eruption that began on 10<sup>th</sup> October 2011 (Ibanez et al.,  
343 2012; López et al., 2012). The catalogue contains over 20,000 events, and so it is possible to subdivide  
344 it into several phases to analyse  $b$ -value variations. Figure 12 shows how each phase is defined by  
345 changes in event rate, with the first three phases following the scheme of Ibanez et al. (2012). The  
346 start of each phase is defined as midnight at the start of the selected day, however, if necessary the  
347 resolution of the boundaries can be increased as most catalogues give event time to the nearest

348 second. All phases have over 200 events at or above  $M_c$ , thus the catalogues should be large enough  
349 to calculate reliable  $\tilde{b}$ -values following the synthetic analysis. At this stage the catalogue is simply  
350 divided temporally, so earthquakes may originate from different portions of the volcanic edifice.  
351 Should this occur, the  $b$  estimate may represent an average between sub-catalogues representative  
352 of different processes or stress conditions.

353 The results of applying our proposed workflow to the El Hierro catalogue are shown in Figure 12. These  
354 show a very high  $b$ -value of  $\tilde{b}=2.39\pm 0.10$  before the onset of the eruption, followed by a fluctuating  $\tilde{b}$ -  
355 value between 1-1.5 for the remainder of the catalogue.  $\tilde{b}$ -value uncertainties are determined using  
356 equation 5. The  $\tilde{b}$ -value is always above 1 within these statistical errors. These results are similar to  
357 those of Ibanez et al. (2012), who reported a  $b$ -value before the eruption of  $2.25\pm 0.05$  followed by  
358 values of  $b=1.34\pm 0.04$  and  $b=1.12\pm 0.01$  for the second and third phases respectively (Ibanez et al.,  
359 2012). However, the Ibanez study used the 90% Goodness-of-fit method to estimate  $M_c$ , and least-  
360 squares regression to estimate  $b$ . The  $M_c$  values they report are significant under-estimates, and this  
361 means that the biased least-squares  $b$ -value estimates are, coincidentally, close to the values reported  
362 here.

363 We also analyse the Istituto Nazionale di Geofisica e Vulcanologia (INGV) earthquake catalogue for Mt  
364 Etna between January 1999 and December 2014. This catalogue spans several eruptive episodes,  
365 including the 2001 and 2002-03 flank eruptions and more recent paroxysmal activity at the new South  
366 East Crater. The catalogue contains 8000 events, with an event rate that is more stable through time  
367 than the El Hierro catalogue (Figure 12 and Figure 13). We divide the catalogue into 10 sub-phases on  
368 the basis of changes in earthquake rate, with each phase ideally containing between 200-5000 events.

369 Figure 13 shows the  $\tilde{b}$ -values calculated for Mt Etna using our proposed workflow. During the 2001  
370 and 2002-03 flank eruptions the  $\tilde{b}$ -value is 1.5 or greater. However from the end of the 2002-03 flank  
371 eruption, the  $\tilde{b}$ -value appears to have stabilised at  $1.0\pm 0.2$ . Murru et al. (2007) analysed the spatial  
372 distribution of the  $b$ -value at Mt Etna between 1999 and 2005 and found an average of approximately  
373 1.5, with an increase in average  $b$ -value with depth from  $b=1.2$  to  $b=1.9$ .

374 Although the  $\tilde{b}$ -values for Mt Etna from 2004 onwards are close to 1.0 and there is no systematic trend  
375 in values, the  $\tilde{b}$ -values do not encompass  $b=1$  within error for over half of the sub-phases in Figure 13.  
376 As the Shi & Bolt  $\tilde{b}$ -value uncertainty (eq. 5) defines one standard deviation error in the  $\tilde{b}$ -value we  
377 would expect 68% of the calculated  $b$ -values to capture  $b=1$  within error if the underlying  $b$ -value is  
378 stationary. We might then conclude that the hypothesis that  $b=1$  can be rejected at this confidence  
379 level. However, we have shown that the total error, including sample bias, can be significantly  
380 underestimated in Figure 11.

381 Accordingly we now apply the contour plot for the error multiplication values in Figure 11c to estimate  
382 a more realistic total error for our calculated  $b$ -value. For the 2011-13 El Hierro catalogue (Figure 14a)  
383 the high  $b$ -values at the start of the catalogue now have dramatically increased errors, and 3 of the 6  
384 following  $b$ -values that sat between  $1 > \tilde{b} > 1.5$  now lie within 1 standard deviation error around  $b=1.0$ .  
385 Using the Shi & Bolt uncertainty for the 2004-2014 Etna catalogue, the estimated  $\tilde{b}$ -values for only 2  
386 of 10 phases (20%) lie within one standard deviation of  $b=1.0$ . However, once the modified error is  
387 applied to the catalogue (Figure 14b), the estimated  $\tilde{b}$ -value for 6 of the 10 phases (60%) lie within 1  
388 standard deviation of  $b=1.0$ . The high  $b$ -values associated with the 2001 and 2002-03 flank eruptions  
389 also increase in error and could be consistent with  $b$ -value of no more than 1.5. The  $b$ -values for 3 of  
390 the 10 phases do not lie within 2 standard deviations of  $b=1$  using the modified error. Therefore it  
391 would be hard to reject the hypothesis that  $b$  is a constant near unity for these phases, except at  
392 marginal significance.

## 393 6 Conclusions

394 The almost axiomatic inference that  $b$ -values are systematically higher for volcanic earthquakes is  
395 based on data and methodology that are often insufficient to address the question, notably the very  
396 small sizes of the samples used, the methods of parameter estimation and the different methods used  
397 to infer the completeness magnitude  $M_c$ . The Maximum Curvature method is simple, and can be used  
398 when a catalogue has a sharp peak in the discrete data. Otherwise the  $b$ -value stability method is the  
399 most favourable. If that does not generate a  $b$ -value with a standard error  $\leq 0.25$  the Goodness-of-Fit  
400 method can be used as a third option. If a stable value of  $b$  cannot be obtained then the sample size  
401 must be increased in space and/or time. Our results imply a pragmatic minimum of 200 events above  
402  $M_c$  is generally needed. From further simulations, we also recommend a minimum of 500 events when  
403 dealing with raw incomplete catalogues before this workflow can be applied. This logic is captured in  
404 a new workflow for estimating  $M_c$ . Even when this best practice is followed, there can be a significant  
405 residual error from calculating  $M_c$  in a single sample. This is comparable to or much greater than the  
406 statistical error, particularly for higher values of  $b$ . Nevertheless, when this is accounted for we  
407 confirm  $b$ -values for the El Hierro catalogue are generally higher than 1 at a confidence level of 95%,  
408 and may be significantly higher during eruptive phases. For Mount Etna the hypothesis  $b=1$  can be  
409 rejected for only two time intervals, one associated with a flank eruption. We conclude seismic  $b$ -  
410 values can be high for volcanic earthquake populations, especially when associated with eruptive  
411 phases. Otherwise they appear to be very close to those obtained for tectonic earthquakes at the 95%  
412 confidence level.

413

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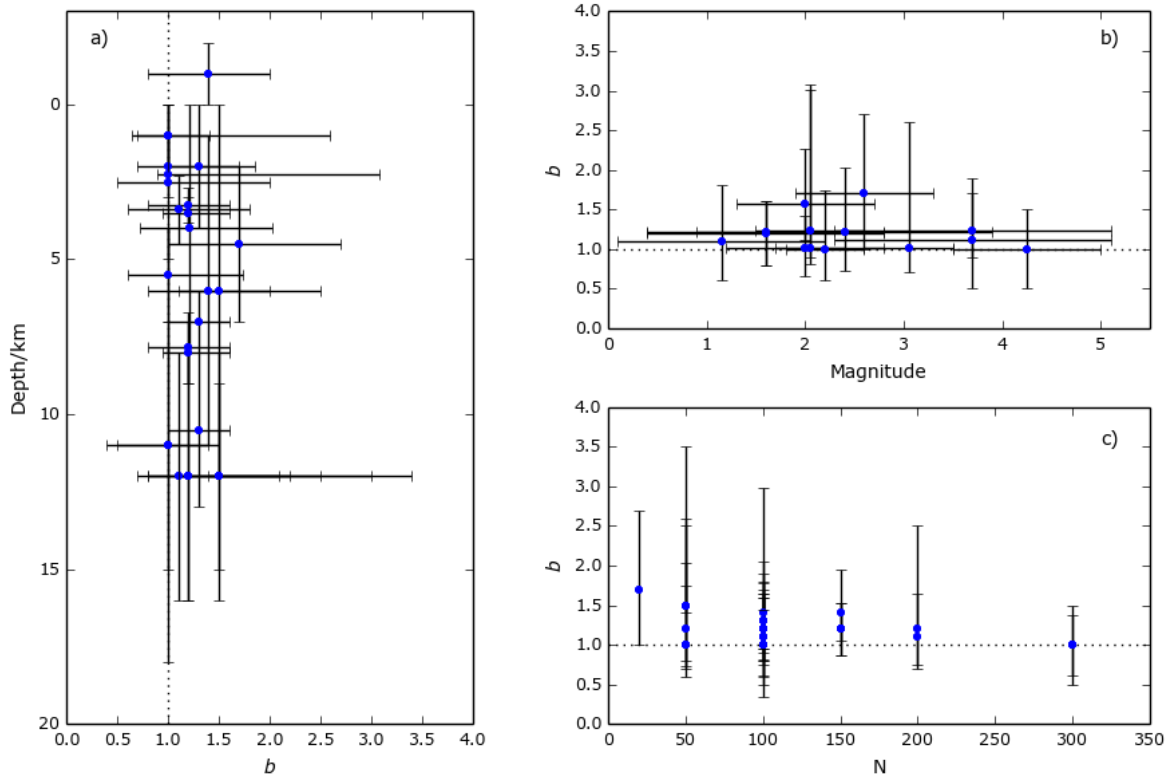
555

556

558 Table 1 - Compilation of *b*-values and range of magnitudes for volcanic seismic catalogues

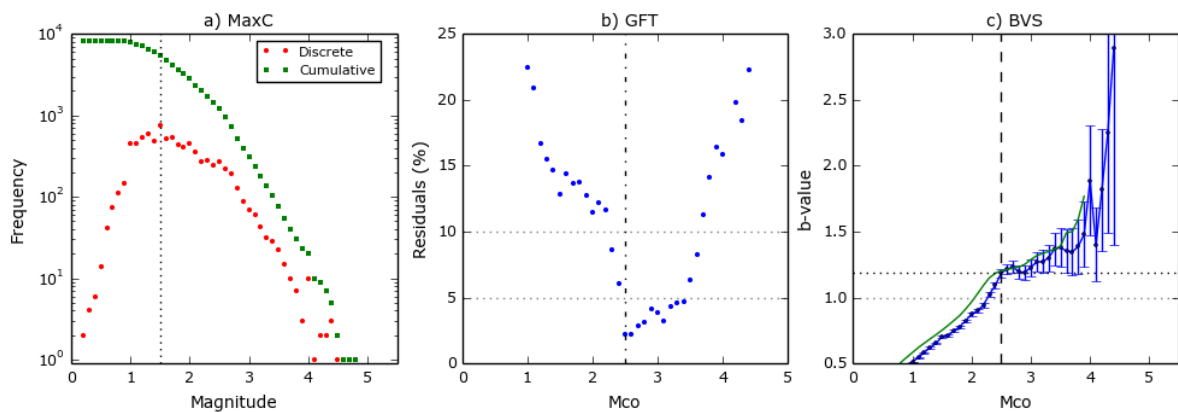
Reference	Volcano	Dates	Depth, km	<i>N</i>	Method <i>Mc</i>	Mag. range	Method <i>b</i>	<i>b</i> <sub>min</sub>	<i>b</i> <sub>typ</sub>	<i>b</i> <sub>max</sub>
(Jacobs and McNutt, 2010)	Augustine	2000 - 2006	-2-0	100	ZMAP	-	MLE	0.8	1.4	2.1
(Jacobs and McNutt, 2010)	Augustine	17/11/05 - 10/12/05	-2-0	~250	ZMAP	-0.1-0.7	MLE	-	-	1.85
M. Wyss (written comm.)	Coso		0.8-3					-	-	1.7
(Ibanez et al., 2012)	El Hierro	19/7/11 - 16/9/11	8-16	7000+	90GFT	1.3-2.7	LS	1.12	1.57	2.25
(Ibanez et al., 2012)	El Hierro	19/07/2011	8-16	200	90GFT	-	LS	0.75	1.25	2.55
(Marti et al., 2013)	El Hierro	14/8/11 - 18/8/11	8-16	-	-	-	MLE	0.8	1.1	2.3
(Ibanez et al., 2012)	El Hierro	19/7/11 - 28/7/11	8-16	-	90GFT	1.5-2.6	LS	0.81	1.2	3.01
(Patane et al., 1992)	Etna	1984	-	200	-	2.8-	MLE	0.8	1.1	1.7
(Patane et al., 1992)	Etna	29/3/1983 - 6/8/1983	-	-	-	2.5-	MLE	0.7	1.0	2.1
(Murru et al., 1999)	Etna	-	9-15	50	MaxC	2.5-	MLE	1.4	1.5	3.5
(Centamore et al., 1999)	Etna	1/1/1990 - 31/12/92	-	100	-	2.3-5.1	LS	0.5	1.2	1.9
(Centamore et al., 1999)	Etna	1/1/1990- 31/12/92	-	100	-	2.3-5.1	MLE	0.9	1.1	1.7
(Murru et al., 2007).	Etna	July - Aug 2001	0-2	50	GFT	2.6-3.5	MLE	0.7	1	2.6
(Murru et al., 2005)	Etna	July - Aug 2001	0-12	50	90GFT	2.6	MLE	0.8	1.5	2.50
(Murru et al., 2007)	Etna	Aug 1999 - Dec 2005	1-3	100	90GFT	2.5	MLE	0.7	1.0	1.86
(Sanchez et al., 2005)	Galeras	Sep 1995 - Jun 2002	0-2	300	-	1.2-2.8	MLE	0.65	1.0	1.4
(Jolly and McNutt, 1999)	Katmai	-	6-8	-	-	-	-	1.0	1.3	1.6
(Wyss et al., 2001)	Kilauea	-	4-7,20	-	-	-	-	-	-	1.9
(Wyss et al., 2001)	Kilauea	1979 - 1997	4-7	50	-	1.8-2.6	MLE & LS	0.6	1.0	1.73
(Wiemer et al., 1998)	Long Valley	1989 - 1998	1-11	150	MaxC	1.3-	MLE	1.1	1.4	2.0
(Jolly and McNutt, 1999)	Mageik	Sep 1996 - April 1997	0-5	-	-	-	WLS	1.0	1.5	2.0
(Bridges and Gao, 2006)	Makushin	July 1996 - April 05	0-8	50	74GFT	0.9-3.9	MLE	0.73	1.21	2.03
(Wiemer et al., 1998)	Mammoth Mtn.	1989 - 1990.5	3-4,7-9	150	MaxC	1.3-	MLE	0.95	1.2	1.6
(Jolly and McNutt, 1999)	Martin/Mageik	Sep 1996 - April 1997	-2-10	-	-	0.7-4.5	WLS	-	-	1.56
(Wiemer and McNutt, 1997)	Mount Spurr	1991 - 1995	2.3-4.5	100	Inspection	0.1-2.2	MLE & LS	0.6	1.1	1.8
(Main, 1987)	Mount St Helens	20 Mar - 18May 1980	na	~300	Inspection	3.5-5	MLE	0.5	1.0	1.5
(Wiemer and McNutt, 1997)	Mount St. Helens	1988 - Jan 1996	2.7-3.8	100	Inspection	0.4-2.8	MLE & LS	0.8	1.2	1.6
(Wyss et al., 1997)	Off-Ito	1982 - 1996	7-15	100	MaxC	1.6-2.5	MLE	0.44	1.0	1.54
M. Wyss (written comm.)	Oshima		4					-	-	1.5
(Sanchez et al., 2004)	Pinatubo	29 June - 19 Aug 1999	0-4,8-13	100	ZMAP	0.73-	MLE	1.0	1.3	1.7
(Novelo-Casanova et al., 2006)	Popocatepetl	Dec 2000 - Jan 2001	2-7	20	Inspection	1.9-3.3	MLE	1.0	1.7	2.70
S. Wiemer (written. comm.)	Redoubt		3-4,6-8					-	-	1.7
(Power et al., 1998)	Soufriere Hills	Aug 1995 - Mar 1996	2.0-2.5	100	-	1.7-2.4	MLE	0.9	1	3.07
(Farrell et al., 2009)	Yellowstone	1984 - 2006	4-18	>10	EMR	1.5-	MLE	0.5	1.0	1.5

Values for *N* are the number of events analysed in each catalogue. These figures are either given or estimated from figures. The methods for calculating the completeness magnitude, *Mc*, are; using ZMAP software; the Goodness-of-Fit method (GFT) with given percentage threshold (e.g. 90Gft is 90% fit); the Maximum Curvature method (MaxC); Inspection is choosing a *Mc* by eye; and using the Entire Magnitude Range method (EMR). The methods for approximating the *b*-value are the Maximum Likelihood Estimation (MLE) and the Least Squares and Weighted Least Squares fit (LS & WLS). The *b*-value ranges in each study are described by the minimum (*b*<sub>min</sub>) and maximum (*b*<sub>max</sub>) quoted values in the study, with a typical value (*b*<sub>typ</sub>) being estimated by eye.



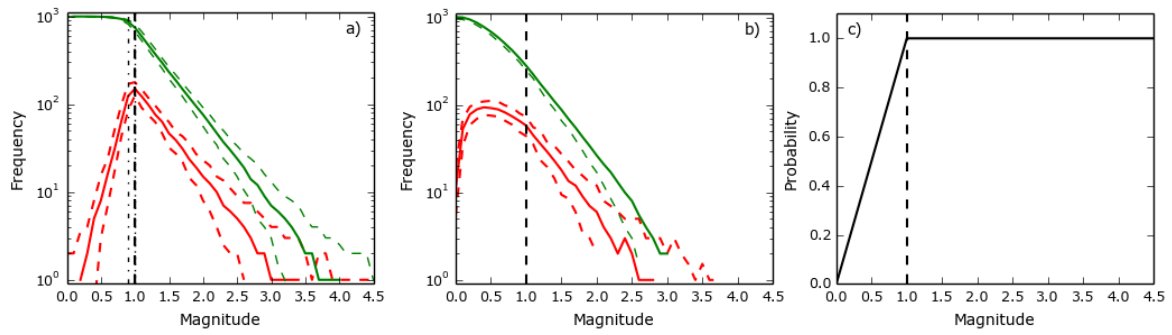
561

562 **Figure 1** – Synthesis of  $b$ -value distributions compared to a) depth, b) Magnitude, and c) the number of events in each  
 563 catalogue,  $N$ . The errors bars show the minimum and maximum values of  $b$  from Table 1, and the range of depth/magnitude  
 564 over which the catalogue was comprised. The blue dots show the typical  $b$ -values. Dotted line marks  $b=1$ .



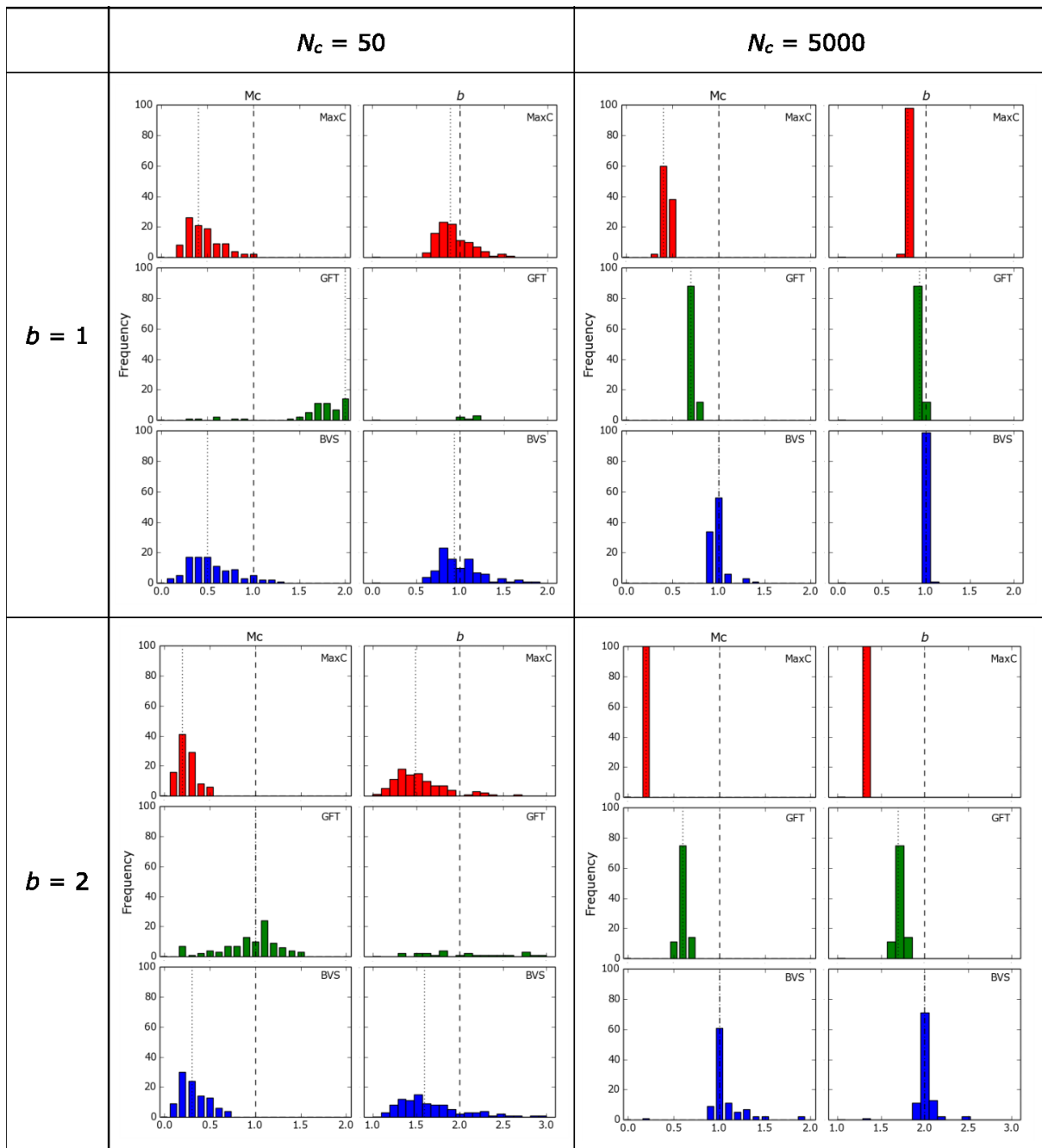
565

566 **Figure 2** – a) Discrete and cumulative frequency-magnitude distributions, demonstrating the Maximum Curvature Method  
 567 (MaxC). The vertical dotted line represents the estimate of  $M_c$  at the highest discrete magnitude bin at ( $M_c=1.5$ ). b) Residuals  
 568 of the Goodness-of-Fit method (GFT) as a function of trial cut-off. Once the residual falls beneath 5% the completeness  
 569 magnitude is selected, in this case  $M_c=2.5$ . c)  $b$ -value stability curve showing the  $b$ -values for each cut-off magnitude. The  
 570 vertical dashed line indicates when successive  $b$ -values (green line) fall within error of the  $b$ -value. Here  $M_c=2.5$ .



571

572 **Figure 3** – a) Example of a sharp-peaked frequency-magnitude distribution. b) Example of a broad-peaked frequency-  
 573 magnitude distribution. Both catalogues have an  $M_c$  of 1.0 and a  $b$ -value of 1.0. Discrete distributions are in reds, cumulative  
 574 distributions are in green. The dashed lines show the 95% confidence intervals representing the scatter in the synthetic data  
 575 c) The probability filter applied to b). Above  $M_c=1.0$  all generated events are kept in the catalogue. Beneath  $M_c=1.0$  there is  
 576 a constantly decreasing probability that that will remain in the catalogue, creating the broad peak in the filtered discrete  
 577 FMD.

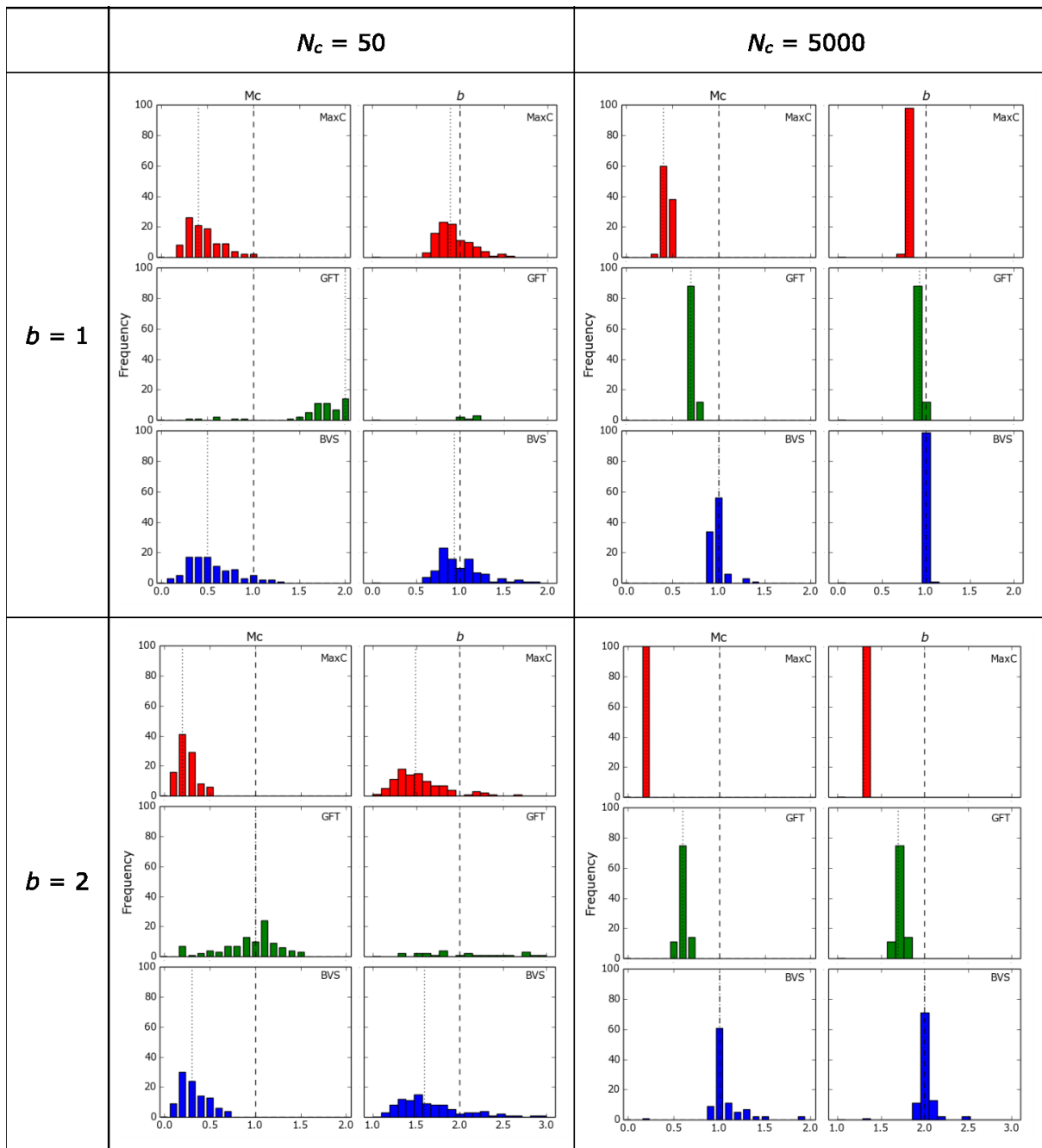


578

579 **Figure 4** – Histograms for the estimated  $Mc$  and  $b$ -value for the MaxC (red), GFT (green), and BVS (blue) methods for different  
 580 catalogue sizes (columns) and  $b$ -values (rows) for the sharp-peaked distribution. The known values of  $Mc=1.0$  and  $b=1.0$  are  
 581 marked with vertical bold dashed lines. The median value calculated by each method is shown by the vertical dotted line.

582

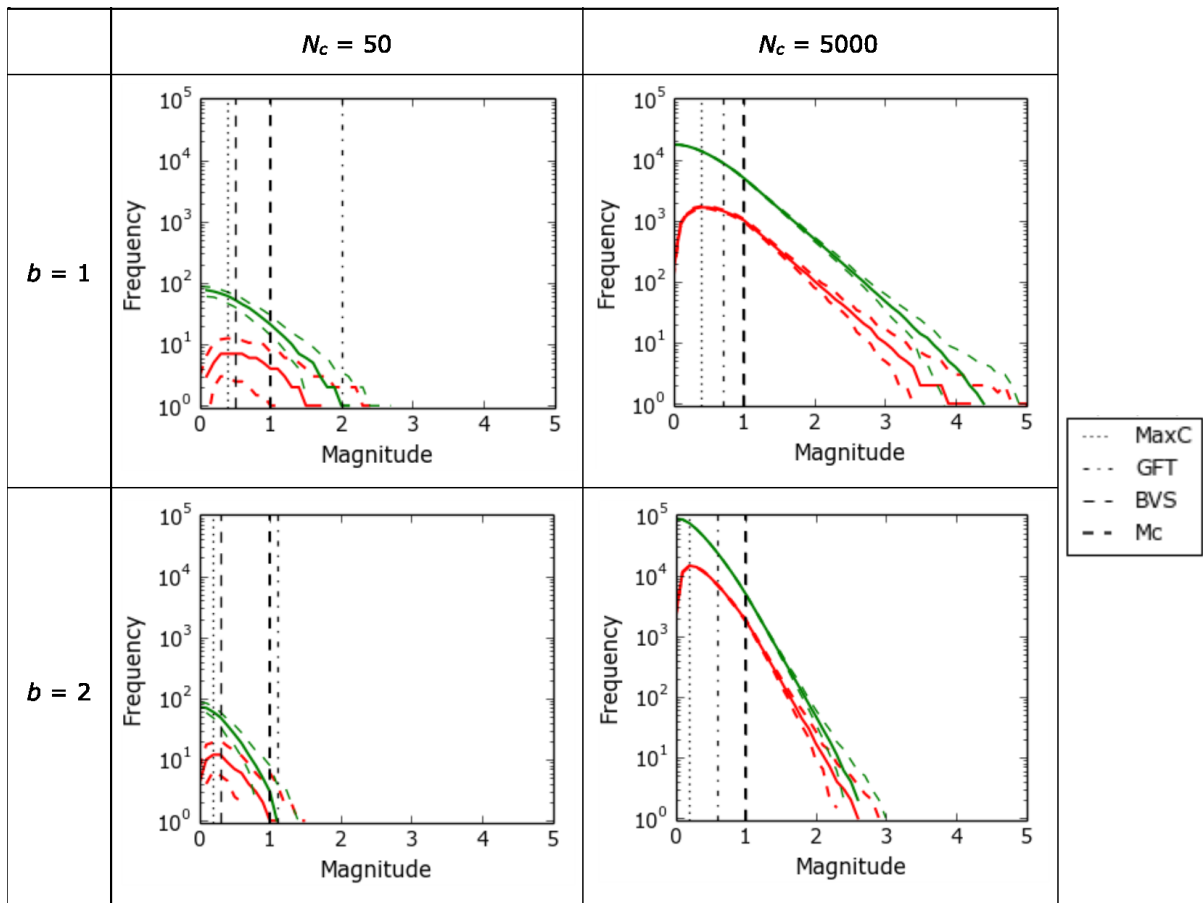
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584

585 **Figure 5** – Histograms as in Figure 4 except for a broad-peaked distribution.

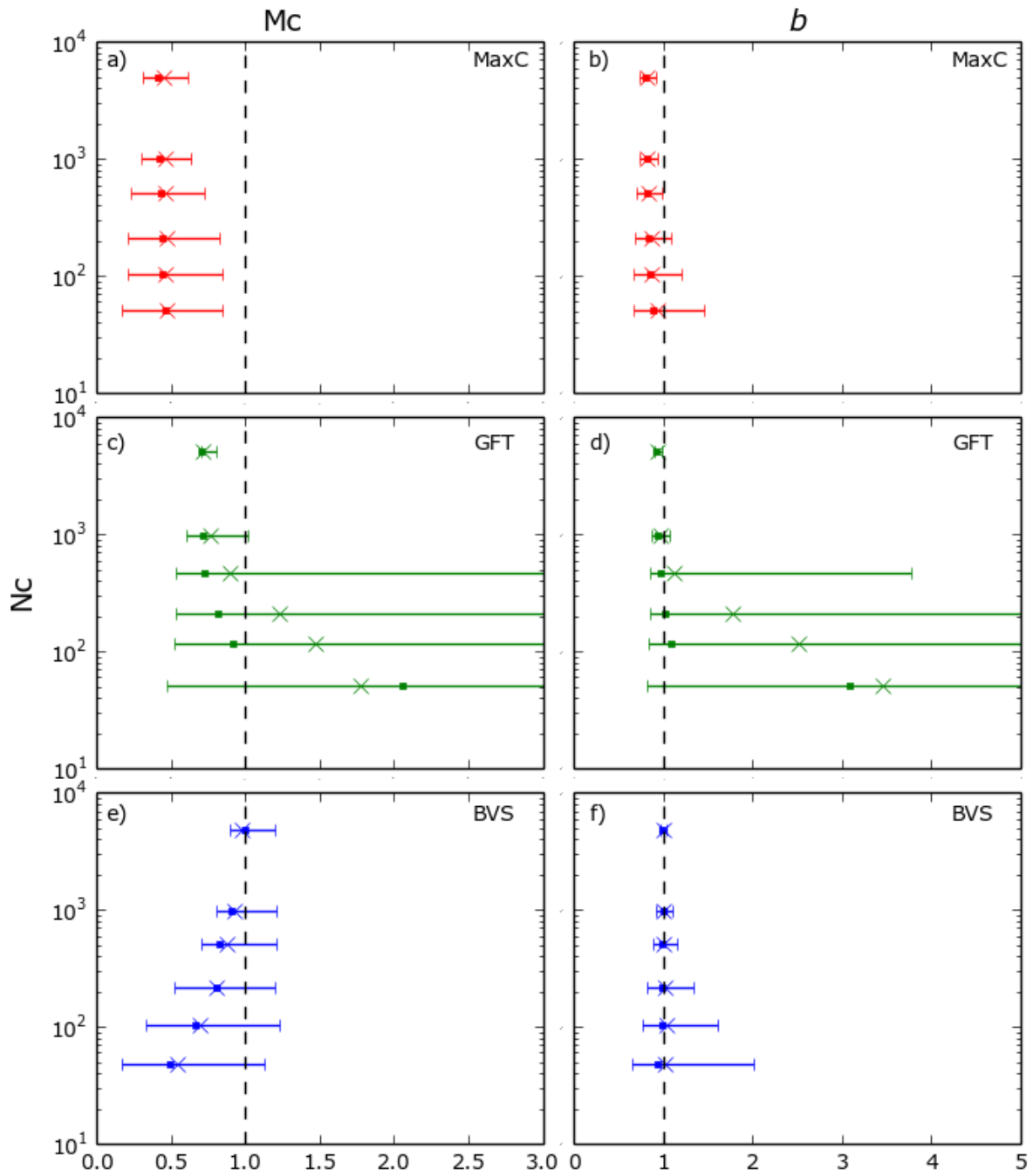




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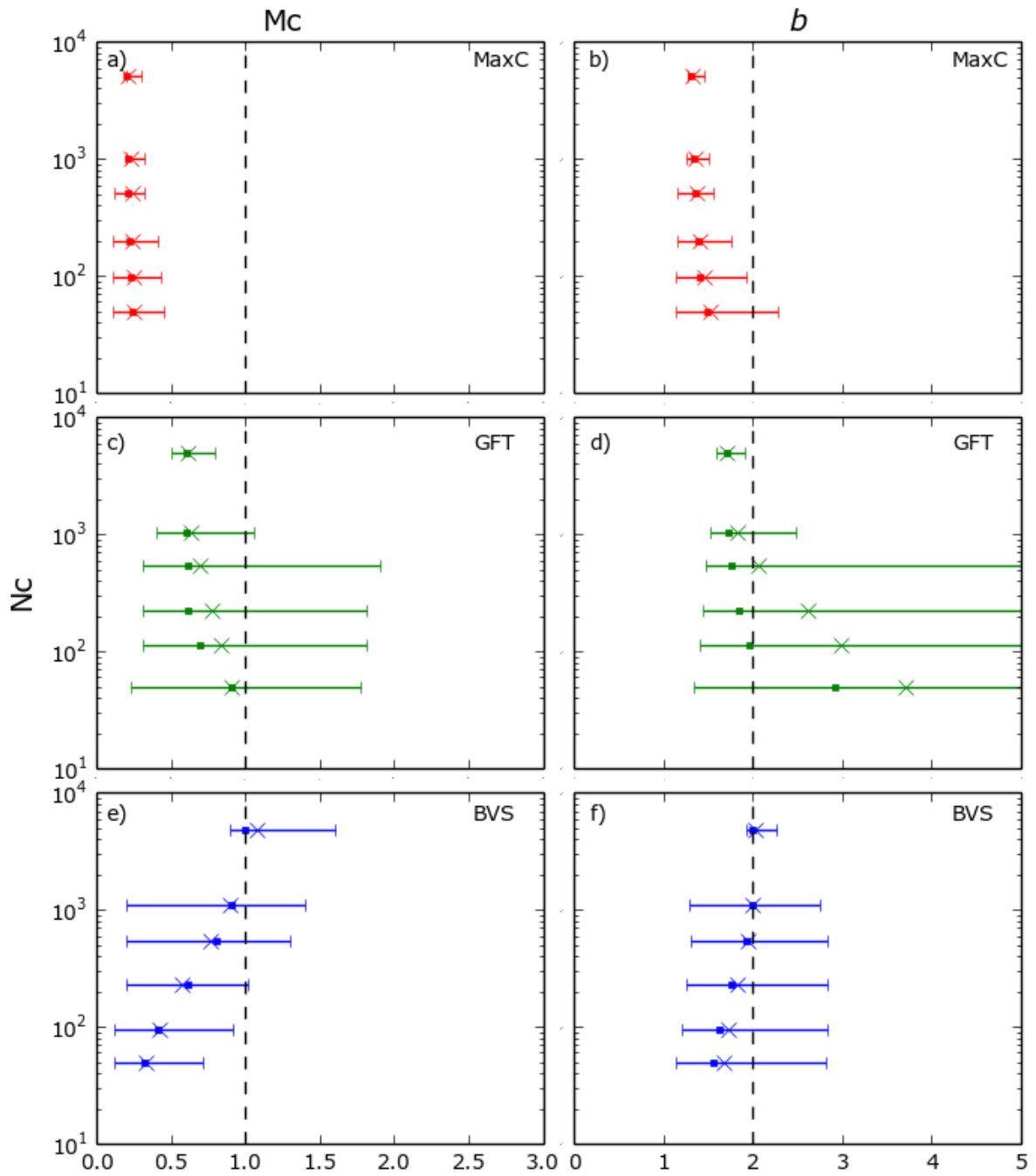
587 **Figure 6** – Frequency-magnitude distributions for  $b=1$  &  $2$ , and  $N_c=50$  &  $5000$  in the case of a broad-peaked distribution. Red  
 588 shows discrete frequency and green cumulative frequency. The solid red and green lines show the average values of the 100  
 589 catalogues. The dashed lines represent a 95% confidence window. The vertical dashed black lines show the known  $Mc$  of the  
 590 catalogue,  $Mc=1.0$ , and the  $Mc$ 's calculated by each method.

591



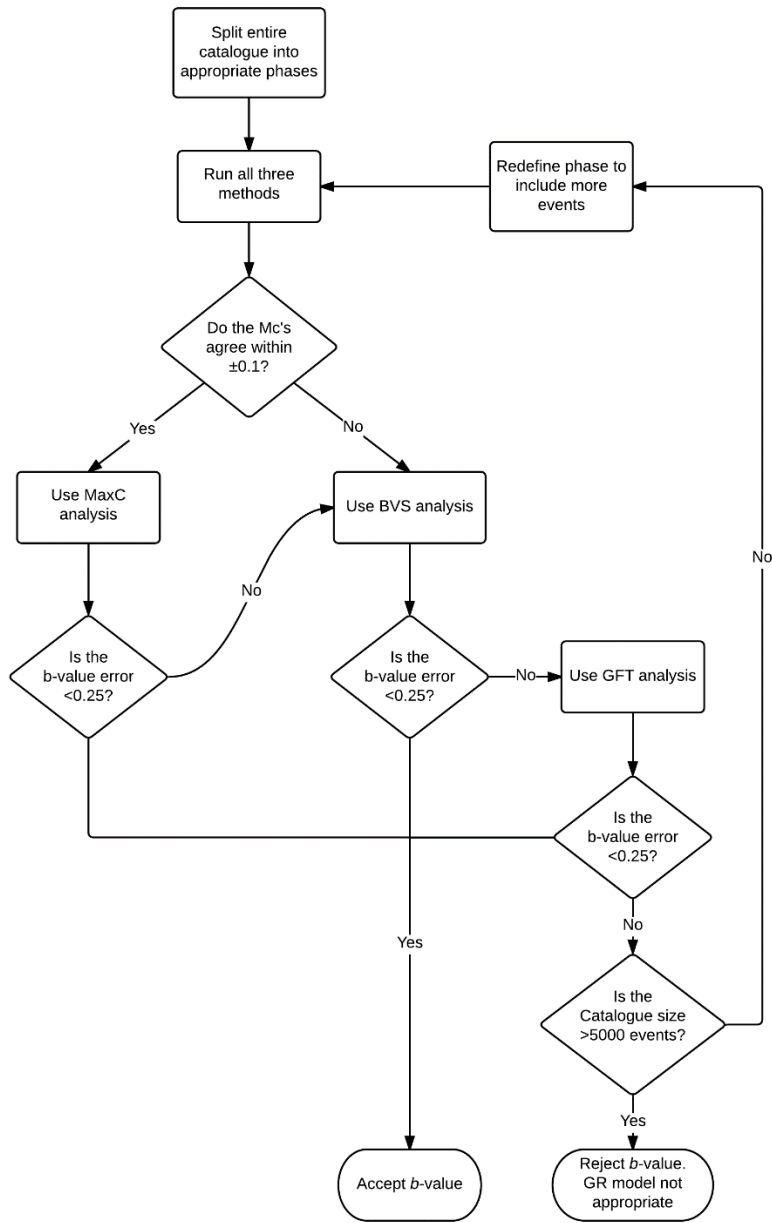
592

593 **Figure 7** - Summary of histograms for broad-peaked distributions in Figure 5 for  $b=1$ . They show the spread of  $Mc$ 's and  $b$ -  
 594 value's against catalogue size,  $N$ , for each of the three methods. Error bars represent a 95% spread of the data, with dots  
 595 representing the median value and x's the average. The known  $Mc=1.0$  and  $b=1.0$  are marked with a vertical dashed line.



596

597 **Figure 8** - Summary graphs as in Figure 7 but for  $b=2$ .

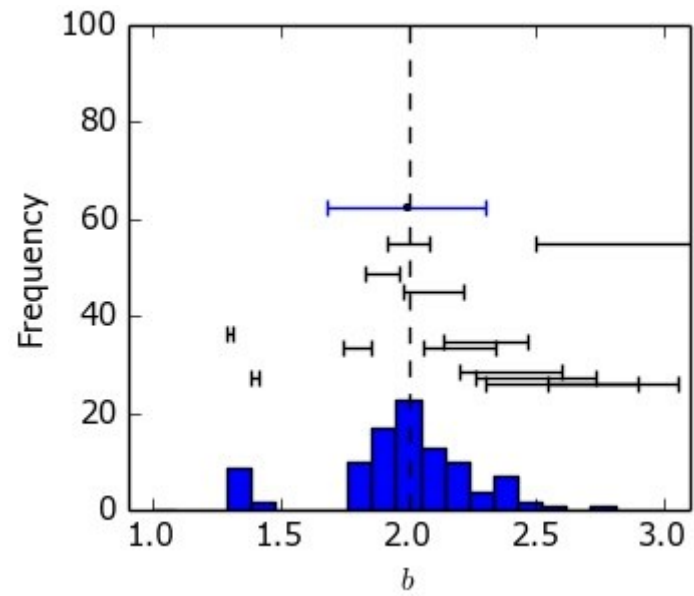


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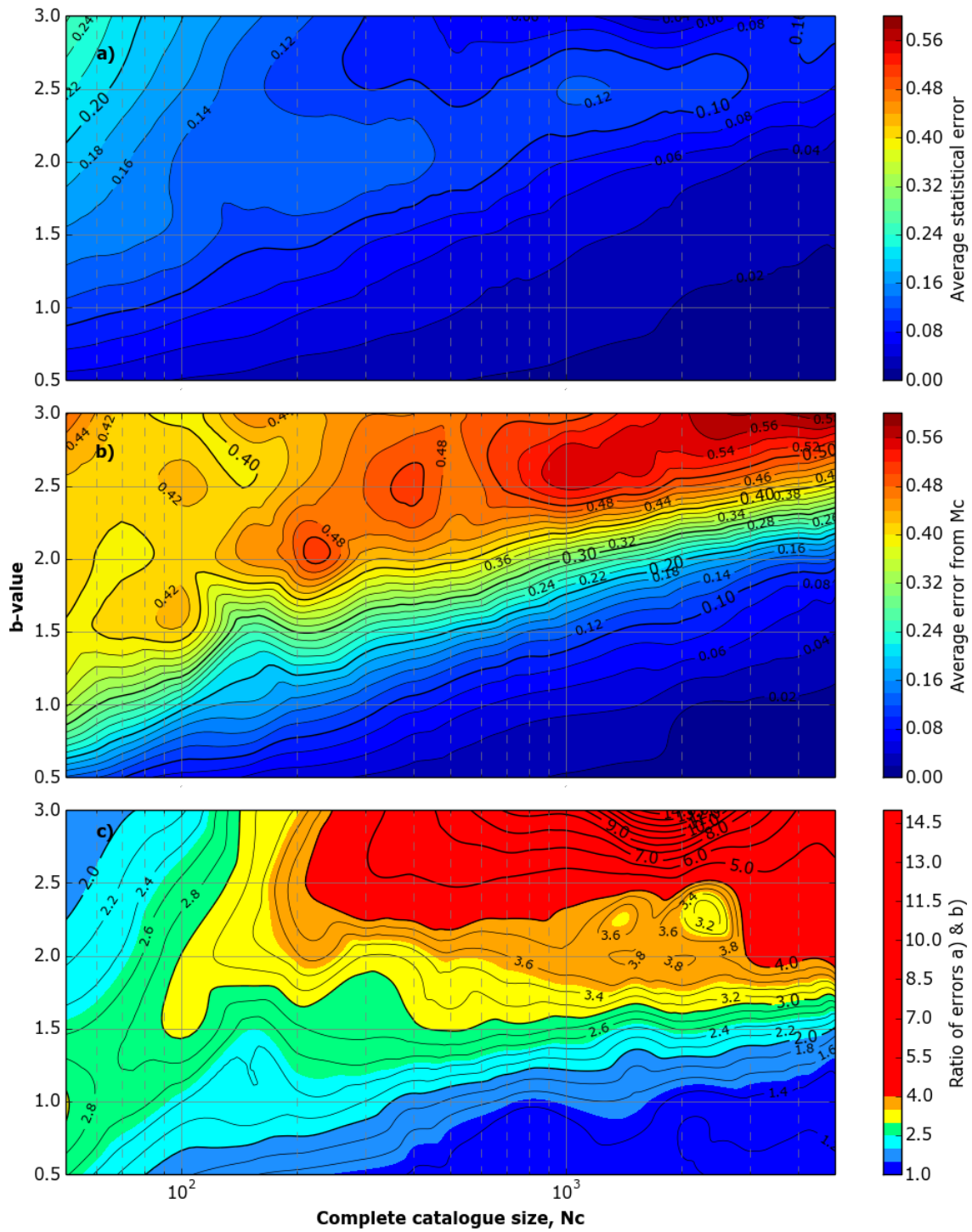
600

Figure 9 - Proposed workflow for best practice based on synthetic analysis.



601

602 **Figure 10** -  $b$ -value frequency plot for 100 synthetic catalogues when  $N_c=1000$  and  $b=2$ . The blue (epistemic) error bar  
 603 represents one standard deviation error in the data centred on the median  $b$ -value. The black error bars show the average  
 604 aleatoric (Shi & Bolt  $b$ -value uncertainty) error for each bin.



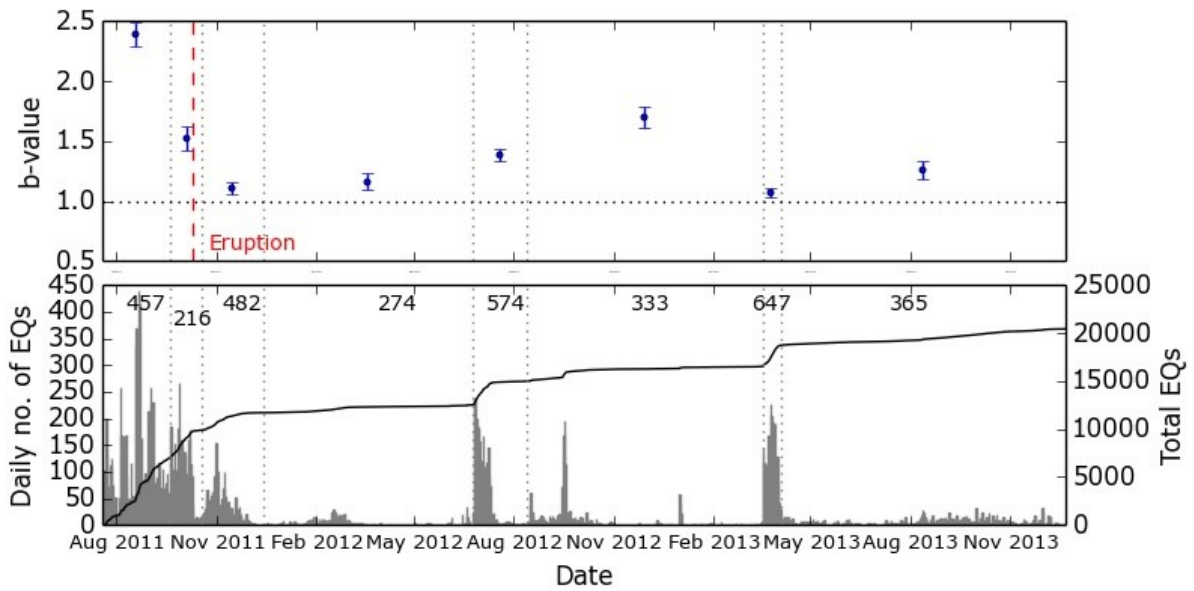
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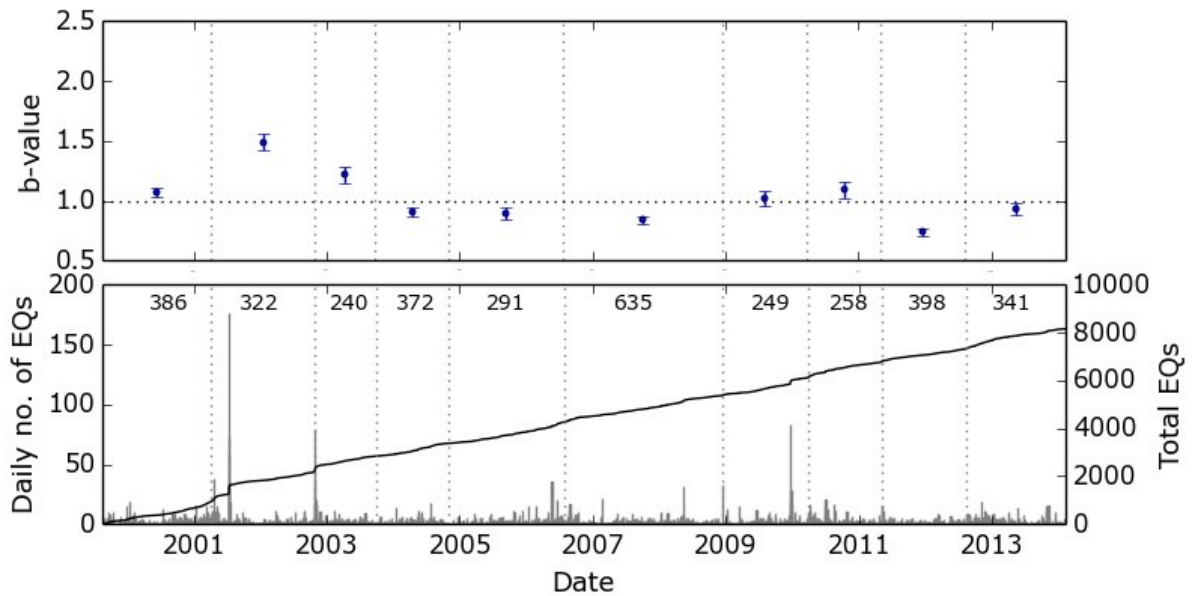
608

**Figure 11** – Contour plots showing a) the statistical error in  $b$ -value estimated from eq. (5) as a function of varying complete catalogue size,  $N_c$ , and  $b$ -value. b) The error in  $b$ -value associated with the uncertainty in calculating  $M_c$ , estimated as in the example given in Fig 10 as a blue horizontal error bar c) The ratio of the error in (b) to the statistical error in (a).



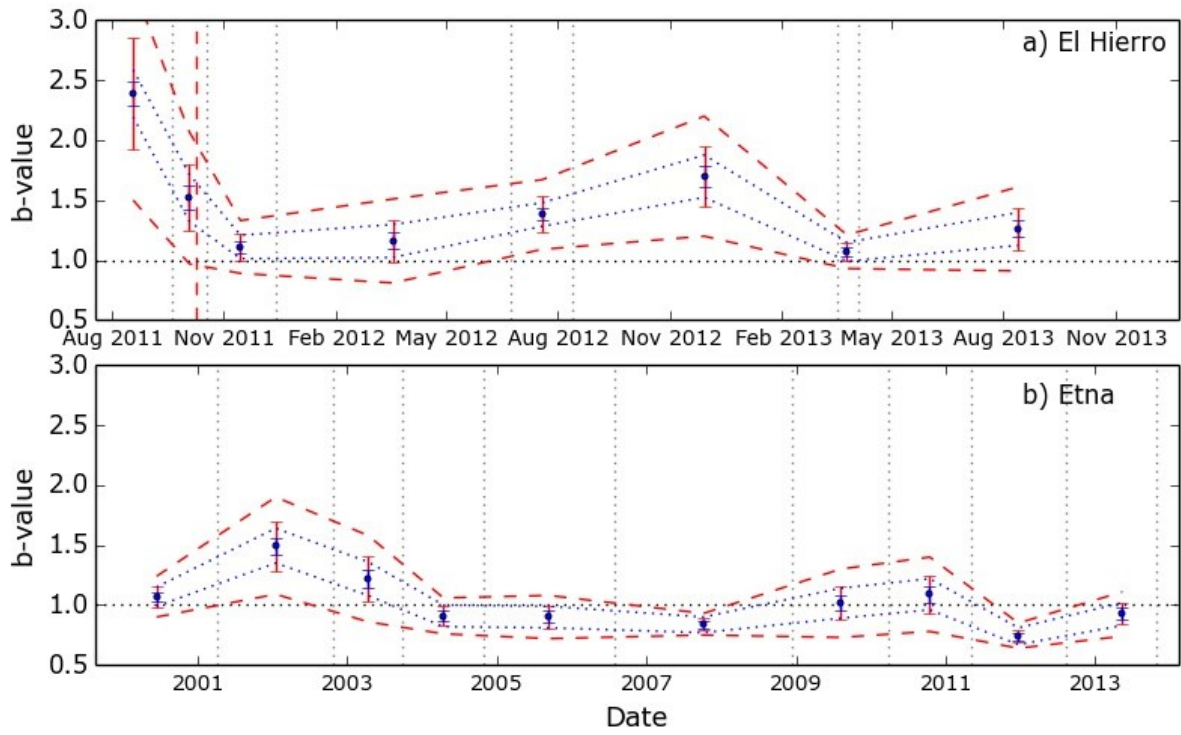
609

610 **Figure 12** – Top: *b*-value variation through time for the July 2011 to December 2013 El Hierro seismic catalogue using the  
 611 proposed workflow. The eruption date is marked by the red dashed line. Bottom: Daily number of events (grey bars) and  
 612 cumulative number of events (black line). The phase divisions are marked by vertical grey dotted lines with the number of  
 613 events in the complete catalogue of each phase noted at the top of the plot.



614

615 **Figure 13** – Plots as in Figure 12 but for the 1999 - 2014 Mount Etna seismic catalogue.



616

617 **Figure 14** - *b*-value variation through time for a) the 2011-13 El Hierro catalogue, and b) the 1999 - 2014 Mount Etna seismic  
 618 catalogue. Sample bias errors in are blue and estimated epistemic error are in grey. One standard deviation error is  
 619 represented by the error bars and the grey dashed and blue dotted line respectively represent the 2 standard deviation error  
 620 envelope.

621