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Quantitative modelling of residential smart grids

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Abstract. Generation of electricity has traditionally taken place at a small number of power stations but with advances in generating technology, small-scale generation of energy from wind and sun is now possible at individual buildings. Additionally, the integration of information technology into the generation and consumption process provides the notion of smart grid. Formal modelling of these systems allows for an understanding of their dynamic behaviour without building or interacting with actual systems. This paper reports on using a quantitative process algebra HYPE to model a residential smart grid (microgrid) for a spatially-extensive suburb of houses where energy is generated by wind power at each house and where excess energy can be shared with neighbours and between neighbourhoods. Both demand and wind availability are modelled stochastically, and the goal of the modelling is to understand the behaviour of the system under different redistribution policies that use local knowledge with spatial heterogeneity in wind availability.

Keywords: smart grid, microgrid, renewable energy, process algebra, quantitative modelling, stochastic hybrid, collective adaptive system

1 Introduction

The way in which electricity is generated is changing. Until recently, there were a few large producers and many consumers (domestic, commercial and industrial). As it becomes cheaper and easier to install equipment that allows one to generate electricity from sun and wind power on individual buildings, more consumers are becoming generators of energy. Furthermore, the introduction of information technology allows for exchange of information. This paper investigates the possibilities that these changes bring to residential areas consisting of standalone or semi-detached houses. Currently, some countries allow excess renewable energy to be fed back into the grid but this is not the only option and it may be possible to share directly between households. If we consider a large suburb consisting of groups of houses, each supplied by one transformer from the grid (called *neighbourhoods* in this paper) then questions arise about the best way in which to share this energy between neighbourhoods. Location then becomes important and spatial differences in renewable energy can be investigated. A novel formal model of this scenario is developed in this paper and evaluated through simulation.

A smart grid is an example of a collective adaptive system because it consists of different components that interact and it must adapt to changes in the environment in which it operates. Collective adaptive systems are becoming more common in everyday life, and they are often invisible to users and people affected by them. Hence it is crucial that formal methods are used to reason about their behaviour so that we can obtain a good understanding of how they work and how they may fail. Formal methods can be used to reason about functional properties of systems (such as liveness and correctness with respect to a specification or logical property) as well as nonfunctional properties such as performance. This paper focusses on the quantitative behaviour of smart grids taking into account spatial distribution of the system.

Stochastic HYPE is a quantitative process algebra developed to model systems which include continuous evolution of variables, stochastic behaviour and instantaneous jumps [3, 11]. It has been used to model various systems, artificial and biological and combinations of both [2, 8, 9, 12]. In the case of modelling smart grids, continuous modelling is required for calculating the energy consumption from changing energy rates, stochastic modelling is required for natural phenomena such as wind and instantaneous behaviour occurs when policies change due to events (that themselves may be instantaneous such as a change in the price of electricity). The expressiveness of stochastic HYPE allows for different approaches to modelling spatial aspects. It can model both continuous space and logical space (which is the approach taken here). For the current research, analysis of the model is done by simulation, specifically by considering the averages of variable trajectories over multiple simulation runs [2].

The paper is structured as follows. First, residential smart grids are described, and their behaviour quantified in terms of energy flows. Policies for distributing surplus renewable energy between neighbourhoods are described. Next, stochastic HYPE is introduced and the basic model is presented together with the model parameters. Results of simulation are presented and discussed. Related work is assessed and the paper finishes with conclusions and future work.

2 Smart grids

As mentioned above, energy generation is changing and a number of recent factors have led to this change and will cause greater changes in the future. Amongst these factors are concerns about energy scarcity due to finite quantities of fossil fuels (sustainability), public distrust of nuclear power, desire for sustainability, availability of equipment for small scale generation from renewable resources, and integration of information in the electricity network infrastructure which aids decision making in production and consumption. Thus, some who were historically only the consumers have become producers as well (sometimes referred to "prosumers"). Producers wish to produce energy to cover demand and no more. Consumers on the other hand, want to pay a reasonable cost for their energy. Information can be used by producer and consumer alike to achieve their goals. For example, smart meters allow consumers to understand consumption, and producers can vary prices to shape demand.

An example residential smart grid consists of 4 to 7 houses served by a single transformer that steps down grid power to domestic voltage. Each house also has photovoltaic cells and a wind turbine [18]. Additionally, each house may have a plug-in electric hybrid vehicle (PHEV) which has a battery which is used both to power the vehicle and to store excess renewable energy. Various types of information can be used. For example, information about the current energy price transmitted from the grid can be used to determine how to use the renewable energy being generated [18], and a limit of the number of vehicles charging from the grid during peak times can be enforced [17].

The specific scenario envisaged in this paper involves small groups of houses (each referred to as a neighbourhood) served by a transformer as described above. There is a wind turbine on each house and no local storage. The focus is on energy sharing (as opposed to reselling) within and between these groups of houses. In such a scenario, sharing energy in a fair way between houses within a neighbourhood where the houses have the same turbine and similar wind speeds is straightforward, as there is an easy argument for fairness under an assumption that demand from each house is similar[1]. In terms of infrastructure, distance between houses in a neighbourhood is assumed to be similar and hence no spatial aspects are introduced within a neighbourhood.

Space is introduced when considering sharing between neighbourhoods. This is more complex, distances vary, and assuming infrastructure for sharing energy that is distinct from the grid, it does not make sense to connect each neighbourhood directly to every other neighbourhood but rather to directly connect neighbourhoods. This paper explores different energy sharing policies between neighbourhoods that use knowledge about the local conditions such demand or wind strength.

2.1 Quantifying smart grids

We consider n neighbourhoods where the number of houses in neighbourhood N_i is m_i. Each house H_{ij} has a_{ij} appliances as well as a background energy profile which is deterministic and distinguishes nighttime when residents are asleep (after 11pm and before sunrise), evening (from sunset to 11pm) and daytime (from sunrise to sunset).

For each point in time, the consumption rate (or demand) within a household can be determined and expressed as $ld_{ij}(t) = b(t) + \sum_{k=1}^{a_{ij}} o_{ijk}(t) \cdot app_{ijk}$ where $b(t)$ is the background consumption rate which is assumed to be the same across all houses, o_{ijk} is an indicator of whether the kth appliance is on in house H_{ij} and app_{ijk} is the energy consumption rate of that appliance.

[1] Nevertheless one household could get a greater share of the renewable energy by ensuring their appliance use is at different times to the other households, and there are other similar actions that some people would consider unfair.

We define lr_i as the available renewable energy rate for a household in neighbourhood N_i (assuming that it is the same for every household in a neighbourhood). Three quantities can be calculated for each house.

Use of local renewable energy:	$lru_{ij}(t) = \min(ld_{ij}(t), lr_i(t))$
Local excess demand:	$lxd_{ij}(t) = ld_{ij}(t) - lru_i(t)$
Local excess renewable energy:	$lxr_{ij}(t) = lr_i(t) - lru_{ij}(t)$

Clearly, if lxd_{ij} is nonzero at time t then $lxr_{ij}(t)$ will be zero at that time point, and if $lxr_{ij}(t)$ is nonzero at time t, lxd_{ij} will be zero.

It is unnecessary to work at the level of individual houses as long as there is an assumption that surplus renewable energy is allocated maximally between neighbours, in the sense that all energy is allocated and there is no wastage with in a neighbourhood (one possibility for maximal allocation is proportionally with demand). The neighbourhood calculation of demand and renewables are $nd_i = \sum_{j=1}^{m_i} ld_{ij}$ and $nr_i = m_i \cdot lr_i$ and from this other values can be calculated, similarly to above.

Use of neighbourhood renewable energy:	$nru_i(t) = \min(nd_i(t), nr_i(t))$
Neighbourhood excess demand:	$nxd_i(t) = nd_i(t) - nru_i(t)$
Neighbourhood excess renewable energy:	$nxr_i(t) = nr_i(t) - nru_i(t)$

Owing to the assumption of maximal allocation, we can conclude that $nxd_{ij}(t) > 0 \Rightarrow nxr_{ij}(t) = 0$ and $nxr_{ij}(t) > 0 \Rightarrow nxd_{ij}(t) = 0$. Without description here (it will be covered in the next section) we assume a policy that allocates some of the surplus to each neighbourhood where the allocation is f_i (there may be wastage). Again we assume maximal allocation between the houses which have excess demand within a neighbourhood and we can determine a neighbourhood-level excess demand after this allocation that must be satisfied from the grid.

Use of shared renewable energy:	$nsu_i(t) = \min(nxd_i(t), f_i(t))$
Use of energy from the grid:	$g_i(t) = nxd_i(t) - nsu_i(t)$
Wasted renewable energy:	$w_i(t) = f_i(t) - nsu_i(t)$

Again, nonzero g_i implies zero w_i and *vice versa*. These equations are then the basic mathematical equations for energy, and describe rates at a point in time. To determine actual quantities over time, further calculation must be done. For example, $g(t)$ expresses the rate of grid consumption at time t. To determine the overall consumption of energy, we need to solve the ordinary differential equation (ODE) $dG(t)/dt = g(t)$ to give the quantity $G(t)$, the amount of grid energy consumed up to time t. Furthermore, the calculations do not include explicitly the losses incurred when converting DC current (from wind turbines) to AC current (for appliances and background consumption) and hence the rate of renewable energy obtained from the wind will have these losses deducted. The issue of losses due to conversion are more important in models that include battery storage.

2.2 Policies

As mentioned above, the surplus energy from each neighbourhood nxr_i is distributed to other neighbourhoods, with neighbourhood j receiving the quantity f_j. We consider policies where energy is supplied to all neighbourhoods[2].

There are two groups of calculations needed to determine what each neighbourhood receives, associated with two decision phases. The first decision is to determine how much to supply in each direction. For a 2-dimensional layout, one can envision "waves" of energy being sent in each direction, considering either the four main compass points (von Neumann neighbourhood) or eight compass points (Moore neighbourhood). Each neighbourhood must determine how its excess is divided up and this division results in expressions denoted by tr_{iX} where tr_{iX} represents the transfer from N_i in direction $X \in \mathcal{C}$ where $\mathcal{C}$ is the set of compass points of interest. The value of tr_{iX} is determined from the excess renewable energy from N_i, nxr_i, and some other factors relating to those neighbourhoods immediately adjacent to N_i for the directions of interest. Clearly, we require that the amount allocated be less than the amount available hence we require that $\sum_{X \in \mathcal{C}} tr_{iX} \leqslant nxr_i$ and a choice that is not equality results in immediate wasted renewable energy. Examples of functions that use local knowledge now follow. Let $h(\mathcal{C})$ be the set of all adjacent neighbourhoods of N_i in the directions of interest.

Split equally between adjacent neighbourhoods:
$tr_{iX} = nxr_i/|\mathcal{C}|$ for $X \in \mathcal{C}$

Split proportionally by demand in adjacent neighbourhoods:
$tr_{iX} = nxr_i \cdot nxd_j/(\sum_{N_k \in h(\mathcal{C})} nxd_k)$ for $X \in \mathcal{C}$

Split by relative wind speed in adjacent neighbourhoods:
Let wl_i be the number of adjacent neighbourhoods of N_i that have wind speed less than $wind_i$ then, for $X \in \mathcal{C}$

$$tr_{iX} = \begin{cases} nxr_i/wl_i & wl_i > 0 \wedge wind_j < wind_i \text{ for } N_j \text{ in direction X} \\ 0 & wl_i > 0 \wedge wind_j \geqslant wind_i \text{ for } N_j \text{ in direction X} \\ nxr_i/|\mathcal{C}| & otherwise \end{cases}$$

The second decision is about what energy each neighbourhood receives. To understand this decision, consider energy being supplied from west to east (or left to right). The leftmost neighbourhood can only pass on its surplus (if any). Then for each neighbourhood as one moves eastward, there are two options; either contribute surplus energy to the supply (if there is excess renewable energy) or to consume some portion of the energy that has arrived (if there is excess demand). The maximum amount that can be consumed is determined by the excess demand for that neighbourhood, the amount available (the *available*

[2] One can consider policies where energy is only supplied to adjacent neighbourhoods but this necessarily seems to result in lower use of renewable energy because fewer neighbourhoods can receive excess renewable energy.

energy), any policy restriction on the amount that can be consumed (giving the *allocated* energy) and what is actually consumed (giving the *actual* energy). In the example that appears later, we will associate with each neighbourhood, a percentage which will describe the proportion of the excess demand that can be satisfied and we will illustrate two ways in which this percentage can be chosen.

Expressions of the form tr_{Xi} describe the energy from each relevant direction to each neighbourhood. It is not correct to consider the sum of the tr_{iX} and divide it up because there is directionality and it is necessary to consider the amount available for each neighbourhood, as described above. Assuming a strip of neighbours N_1 to N_n from west to east, N_1 transfers all its excess energy for the east, hence the amount available at N_2 is $av_{X2}(t) = tr_{1X}(t)$. Assuming that the amount allocated for consumption at N_2 is $all_{X2}(t)$ then $tr_{X2}(t) = \min(av_{X2}(t), all_{X2}(t))$ is the actual amount allocated, and the amount available for $N3$ is $av_{X3}(t) = av_{X2} - tr_{X2}(t) + tr_{2X}(t)$ where one or more of $tr_{X2}(t)$ and $tr_{2X}(t)$ are zero since if energy is required there will have been no excess to pass on, and if there is energy to pass on, there can be no demand. The general definitions is then as follows.

$$av_{Xi}(t) = \begin{cases} 0 & i = 1 \\ av_{X(i-1)}(t) - tr_{X(i-1)}(t) + tr_{(i-1)X}(t) & \text{otherwise} \end{cases}$$

$$tr_{Xi}(t) = \begin{cases} av_{Xn}(t) & i = n \\ \min(av_{Xi}(t), all_{Xi}(t)) & \text{otherwise} \end{cases}$$

To keep track of wastage, the last neighbourhood receives all remaining energy regardless of any allocation. For the other directions, the expressions are defined similarly. The total of allocated energy for neighbourhood N_i is defined by $f_i = \sum_{X \in \mathcal{C}} tr_{Xi}$. Different allocation policies to determine all_{Xi} can be used, and the simplest is to allocate the same amount as the demand. A slightly more complex policy is to allocate a proportion of the demand, and other more complex (but not necessarily better) policies can be developed.

3 Stochastic HYPE model

Space limitations prevent an introduction to stochastic HYPE and the reader is referred to [3] and [11]. In this section, a flavour of the model is given and a very brief introduction to the semantics of stochastic HYPE are presented. Subcomponents (see Table 1) define the flows that describe the continuous behaviour of the system and they react to events that may change these flows. Underlined events are instantaneous in nature and occur when a Boolean expression (activation condition or guard) becomes true. Overlined events are stochastic and complete after an exponential duration. Each subcomponent must be able respond to the first event $\underline{\text{init}}$ to ensure that the initial behaviour of the subcomponent is defined. The characteristics of a flow are described by the influence triples. The first element of a triple is the influence name which identifies the

$$
\begin{aligned}
Time &= \underline{\mathrm{init}}{:}(\iota_t, 1, 1).Time\\
App_{ijk} &= \underline{\mathrm{off}}_{ijk}{:}(\iota^a_{ijk}, 0, 0).App_{ijk} + \underline{\mathrm{on}}_{ijk}{:}(\iota^a_{ijk}, app_{ijk}, 1).App_{ijk} + \\
&\quad\ \underline{\mathrm{init}}{:}(\iota^a_{ijk}, 0, 0).App_{ijk}\\
Back_{ij} &= \underline{\mathrm{night}}{:}(\iota^b_{ij}, r_n, 1).Back_{ij} + \underline{\mathrm{evening}}{:}(\iota^b_{ij}, r_e, 1).Back_{ij} + \underline{\mathrm{day}}{:}(\iota^b_{ij}, r_d, 1).Back_{ij} + \\
&\quad\ \underline{\mathrm{init}}{:}(\iota^b_{ij}, r_n, 1).Back_{ij}\\
Grid_i &= \underline{\mathrm{init}}{:}(\iota^g_i, 1, g_i) \qquad Shared_i = \underline{\mathrm{init}}{:}(\iota^s_i, 1, nsu_i) \qquad Waste_i = \underline{\mathrm{init}}{:}(\iota^w_i, 1, w_i)\\
Cost_i &= \underline{\mathrm{init}}{:}(\iota^k_i, GC, g_i) \qquad Renew_i = \underline{\mathrm{init}}{:}(\iota^r_i, 1, nxr_i + nsu_i)
\end{aligned}
$$

$$
\begin{array}{llll}
iv(\iota_t) = T & iv(\iota^w_i) = W & iv(\iota^r_i) = E & iv(\iota^a_{ijk}) = iv(\iota^b_{ij}) = D_i\\
iv(\iota^s_i) = S_i & iv(\iota^r_i) = R_i & iv(\iota^g_i) = G_i & iv(\iota^c_i) = C_i
\end{array}
$$

T time
W total wastage
E total renewable energy generated
D_i total demand in neighbourhood N_i
S_i shared renewable energy usage in neighbourhood N_i
R_i total renewable energy usage in neighbourhood N_i
G_i grid energy usage in neighbourhood N_i
C_i cost of grid energy in neighbourhood N_i

$$
\begin{array}{rlrl}
ec(\underline{\mathrm{init}}) &\stackrel{def}{=} (true, (T' = 0) \wedge (W' = 0) \wedge (P' = 0) \wedge \ldots) & & \\
ec(\underline{\mathrm{on}}_{ijk}) &\stackrel{def}{=} (T = T_{on_{ijk}}, T'_{c_{ijk}} = T) & ec(\underline{\mathrm{off}}_{ijk}) &\stackrel{def}{=} (T = T_{c_{ijk}} + T_{d_{ijk}}, true)\\
ec(\underline{\mathrm{day}}) &\stackrel{def}{=} (T\, mod\, 24 = 6, true) & ec(\underline{\mathrm{peak}}_d) &\stackrel{def}{=} (T\, mod\, 24 = 7, GC' = gc_p)\\
ec(\underline{\mathrm{evening}}) &\stackrel{def}{=} (T\, mod\, 24 = 18, true) & ec(\underline{\mathrm{midpeak}}_d) &\stackrel{def}{=} (T\, mod\, 24 = 10, GC' = gc_{mp})\\
ec(\underline{\mathrm{night}}) &\stackrel{def}{=} (T\, mod\, 24 = 22, true) & ec(\underline{\mathrm{peak}}_e) &\stackrel{def}{=} (T\, mod\, 24 = 17, GC' = gc_p)\\
ec(\overline{\mathrm{blow}}) &\stackrel{def}{=} (r_{blow}, WB' = 1) & ec(\underline{\mathrm{midpeak}}_e) &\stackrel{def}{=} (T\, mod\, 24 = 20, GC' = gc_{mp})\\
ec(\overline{\mathrm{noblow}}) &\stackrel{def}{=} (r_{noblow}, WB' = 0) & ec(\underline{\mathrm{offpeak}}) &\stackrel{def}{=} (T\, mod\, 24 = 23, GC' = gc_{op})
\end{array}
$$

$$
\begin{aligned}
CApp_{ijk} &\stackrel{def}{=} \underline{\mathrm{on}}_{ijk}.\underline{\mathrm{off}}_{ijk}.CApp_{ijk}\\
SBack &\stackrel{def}{=} \underline{\mathrm{day}}.\underline{\mathrm{evening}}.\underline{\mathrm{night}}.SBack\\
SWind &\stackrel{def}{=} \overline{\mathrm{blow}}.\overline{\mathrm{noblow}}.SWind\\
SPeak &\stackrel{def}{=} \underline{\mathrm{peak}}_d.\underline{\mathrm{midpeak}}_d.\underline{\mathrm{peak}}_e.\underline{\mathrm{midpeak}}_e.\underline{\mathrm{offpeak}}.SPeak
\end{aligned}
$$

$$
\begin{aligned}
RSG &\stackrel{def}{=} \Sigma \underset{*}{\bowtie} \underline{\mathrm{init}}.Con\\
\Sigma &\stackrel{def}{=} Time \underset{*}{\bowtie} (\ldots \underset{*}{\bowtie} App_{ijk} \underset{*}{\bowtie} \ldots) \underset{*}{\bowtie} (\ldots \underset{*}{\bowtie} Back_{ij} \underset{*}{\bowtie} \ldots) \underset{*}{\bowtie} (\ldots \underset{*}{\bowtie}\\
&\quad (Grid_i \underset{*}{\bowtie} Cost_i \underset{*}{\bowtie} Waste_i \underset{*}{\bowtie} Energy_i \underset{*}{\bowtie} Renew_i \underset{*}{\bowtie} Shared_i) \underset{*}{\bowtie} \ldots)\\
Con &\stackrel{def}{=} (\ldots \underset{*}{\bowtie} CApp_{ijk} \underset{*}{\bowtie} \ldots) \underset{*}{\bowtie} SBack \underset{*}{\bowtie} SWind \underset{*}{\bowtie} SPeak
\end{aligned}
$$

Table 1. Subcomponents, influence mapping, variables, event conditions, controllers/sequencers and the overall system. GC is a variable which records the current cost of energy from the grid.

variable that is affected by the influences in this subcomponent through the mapping iv, as shown in Table 1. The second and third components describe the flow, with the second component representing the strength of the flow as a constant value and the third element describing a function that introduces variables in the flow definition. A feature of stochastic HYPE is that it allows for multiple flows to affect a single variable. In the example, this is illustrated by the fact that multiple influences are mapped to the variable D_i, allowing for multiple appliances and the background level of consumption to determine the ODE that describes the value of D_i over time. The second and third elements are multiplied together in the ODE that is generated for a variable, so their separation in the model is purely a syntactic distinction.

Table 1 also lists event conditions for the model. These consist of an activation condition (in the case of instantaneous events) and a reset of variable values. As is standard in stochastic HYPE models, the $\underline{\text{init}}$ event initialises all variables and has the activation condition *true*. It is the first event that happens because of the structure of the overall system *RSG*.

Events that turn appliances on and off are required. The value of $T_{on_{ijk}}$ is set at the start of the day using a distribution that describes the probability of that type of appliance starting at a particular hour of the day, and a random number of minutes (uniformly chosen from the interval $[0, 60)$). The duration $T_{d_{ijk}}$ is a fixed value for the type of appliance. There are other events that are dependent on time. There are a number of approaches that could be considered for modelling wind: a constant wind, a wind defined by a stochastic differential equation as in [22] and a stochastic wind that may be present (at a fixed strength) or absent The latter is most appropriate here since goal of the model is to consider energy sharing, hence the renewable energy provided by the wind may be present or absent. This are determined by two stochastic events $\overline{\text{blow}}$ and $\overline{\text{noblow}}$ where the first element of the event condition is the rate. The reset determines the value of the variable *WB* which in turn determines lr_i for each neighbourhood.

We also require some controllers and sequencers and these are given in Table 1 with the full model. Both the *SBack* and *SPeak* are somewhat redundant because of the time-based sequencing in the event conditions, however stochastic HYPE requires that all events should appear in controllers, and this explicitness expresses the intent of the model. However, the other controller definitions are required to ensure the correct alternation of events. The semantics of a stochastic HYPE model are defined via structured operational semantics that define a labelled transition system. In this labelled transition system, a function σ is required to record the current values associated with each influence name, hence the operational semantics are defined over pairs $\langle P, \sigma \rangle$. The labelled transition system can then be mapped to transition-driven stochastic hybrid automata [5], a subset of piecewise deterministic Markov processes [6]. The states of the labelled transition system become the modes of the stochastic hybrid automata, and the ODEs for a mode are defined in terms of the values associated with each influence name in that mode.

To illustrate the ODEs that are obtained from the model above, the ODE that defines the demand for neighbourhood N_i is as follows in the case where there are five houses in N_i, one appliance is on in the third house, two appliances are on in the fifth house, and it is before 07:00 in the morning, then the equation is $dD_i/dt = 5r_n + app_{i31} + app_{i51} + app_{i52}$. For the total waste, we have the ODE $dW/dt = \sum_{i=1}^{n} waste_i$ and for total renewable energy generated, we have $dE/dt = \sum_{i=1}^{n} grid_i$.

The transition-driven stochastic hybrid automaton has the following structure and associated behaviour.

- Modes and their associated ODEs describe how variable values change.
- Continuous evolution of variable values are determined by the current mode.
- Switching between modes occurs when
  1. activation conditions (guards) of instantaneous events become true
  2. durations of stochastic events expire

  with possible jumps in variable values determined by resets.

A trace of an automaton consists of a continuous trajectory for each variable interspersed with non-continuous changes in values. The behaviour of stochastic HYPE models can be explored using the stochastic hybrid simulator described in [2] and this simulator was used for the results reported here.

3.1 Model parameters

Appliances: There are two per house, one washing machine (consumption 0.82 kWh, cycle length 1 hour) and one dishwasher (consumption 2.46 kWh, cycle length 1 and a half hours) [18]. Distributions for the probability of being on in a specific hour are used to determine the starting hour of the appliance [18].
Background consumption: Using the figures from Figure 10 in [25] as a guide, the daytime figure is 0.3 kWh, the evening figure is 0.5 kWh and the nighttime figure is 0.1 kWh.
Wind: It has been argued that on average in the UK, at any specific point, the probability of there being wind sufficiently strong to drive a turbine is 80% [21]. As mentioned above, to explore the issue of sharing renewable energy, it is necessary for the wind to be stochastic, and two exponential distributions are used, one for wind presence and one for wind absence. We consider various possibilities including that from [21]. The average generation capacity of the wind in the UK has been calculated to be somewhere between 25% and 35% [21]. This means a turbine with a rating of x kWh will give that percent of its rated power.
Electricity cost: Here we follow [18], so at peak times, the cost is 0.272 £/kWh, mid-peak cost is 0.194 £/kWh, and off-peak is 0.107 £/kWh.

4 Results

A number of different experiments were considered. The two main wind patterns that were investigated are as follows.

One wind scenario: Seven neighbourhoods in a strip were considered, with N_1 to the west and N_7 to the east. N_1 and N_2 had the full strength of the wind, N_3 and N_4 had half strength wind, and N_5, N_6 and N_7 had quarter strength wind.

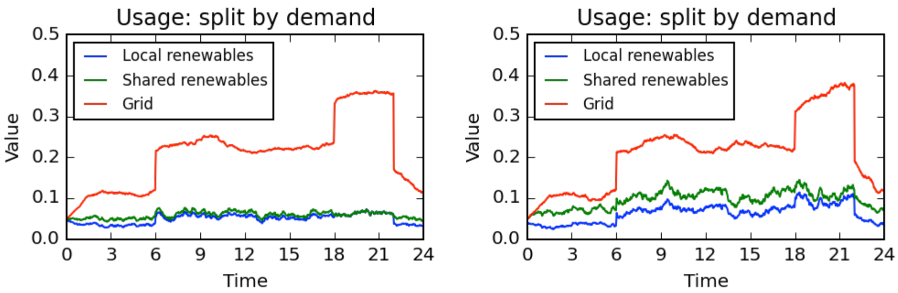

Fig. 1. Stacked graphs of the average instantaneous local renewable usage, shared renewable usage and grid usage over one day (right: one wind scenario, left: two wind scenario).

Two wind scenario: This had the same strip layout with no wind in the central neighbourhood N_4 and one wind at full strength in N_1, half strength in N_2 and a quarter strength in N_3. The second wind was available in N_7 at full strength, N_6 at half strength and N_5 with quarter strength.

A number of policies were investigated and are described by the following abbreviations.

eq100 Split equally in each direction, allowing 100% of demand to be satisfied
wn100 Split by relative wind speed, allowing 100% of demand to be satisfied
dm100 Split proportionally by demand, allowing 100% of demand to be satisfied
dminc Split proportionally by demand, with proportion of demand to be satisfied increasing in the direction of supply
dmwnd Split proportionally by demand, with proportion of demand to be satisfied determined by wind level.
dw100 Split proportionally by demand with a weighting factor to favour higher demand, allowing 100% of demand to be satisfied
da100 Direction of highest demand receives all excess, allowing 100% of demand to be satisfied

Figure 1 shows the instantaneous energy consumption across the day (as a stacked graph). There is a peak in the early evening capturing the higher background level at that time and the higher likelihood of appliance being used. There is more use of shared renewables at night because there is a greater likelihood of excess renewables due to lower consumption then. The right hand graph shows that there is more scope for sharing of renewables in the two wind scenario.

Figure 2 provides heatmaps across the neighbourhoods of the parameter space for wind strength and wind absence (under the dm100 policy in the one wind scenario). The wind strength varies from 0.2 to 1 (and is adjusted by the wind multiplier for the region). The average wind presence rate is 1.2 hours and the average wind absence varies from 0.3 hours (in line with the 80% presence of

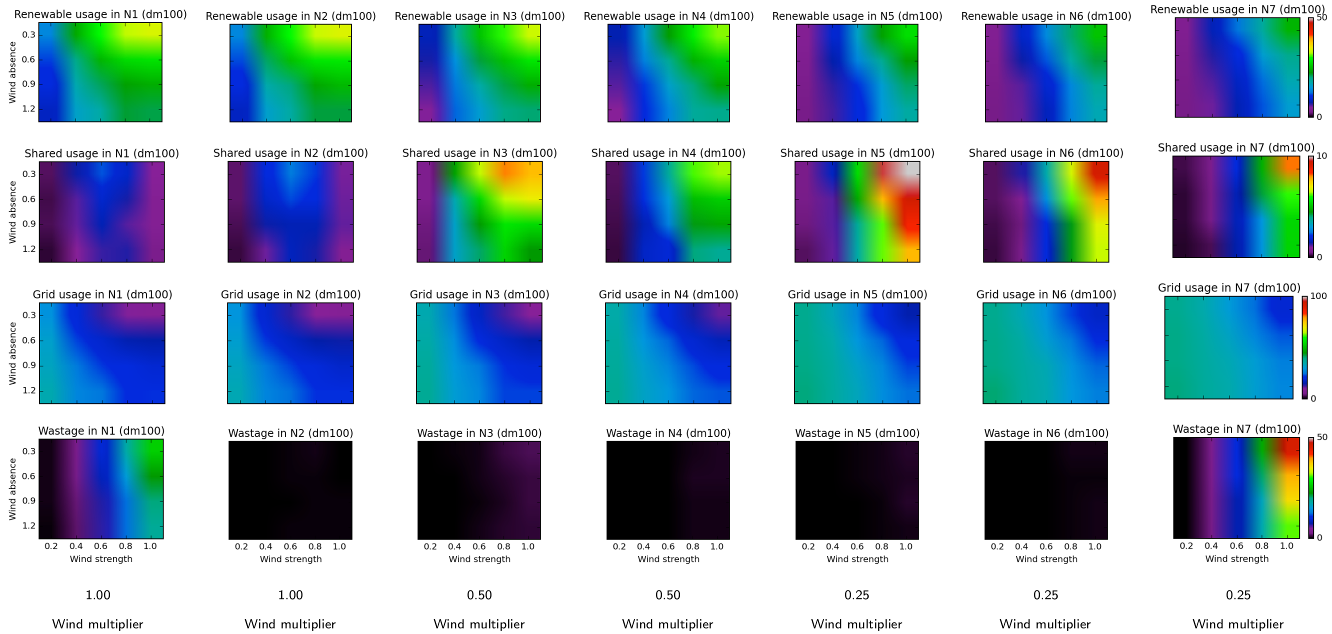

Fig. 2. Heatmaps for renewable usage, shared usage, grid usage and wastage across neighbourhoods for different maximum wind strengths and wind absence rate (one wind scenario over 24 hours) for policy dm100. As the wind decreases over neighbourhoods from left to right, renewable usage decreases and grid usage increases. Shared energy increase is dependent on the wind multiplier of adjacent neighbourhoods. Within a neighbourhood, renewable usage increases and grid usage decreases as the wind absence rate decreases and the wind strength increases. Shared usage shows the same pattern except in N_1 and N_2 where shared usage is very low. Wastage only shows this pattern in N_1 and N_7 as this is where unused renewable energy collects. N_7 has higher wastage because the demand driven policy is likely to allocate more shared energy to the neighbourhoods with lower wind.

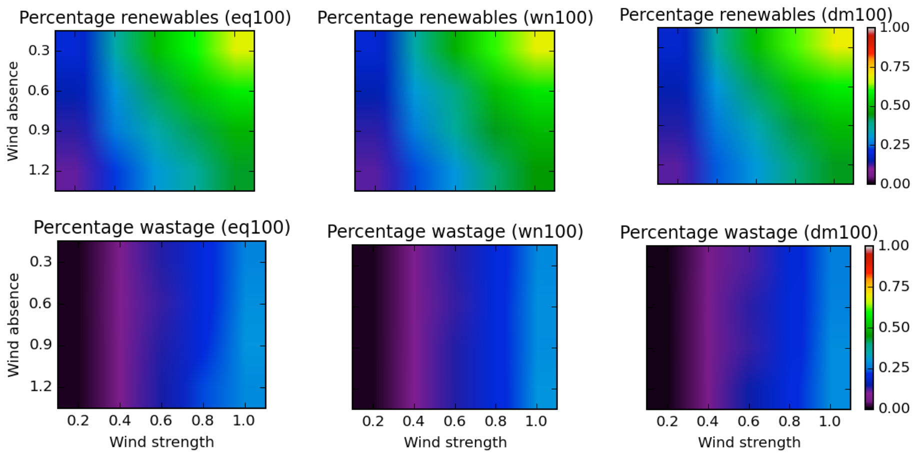

Fig. 3. Heatmaps for percentage of energy used that is renewable (out of total energy consumed) and percentage renewable energy wastage (out of total renewable available) for different maximum wind strengths and wind absence rate (one wind scenario over 24 hours) considering different policies.

[21]) to 1.2 hours (giving a 50% presence). The heat maps show how the figures vary across regions and across parameters. Most wastage occurs at the extreme neighbourhoods since these are supplied with all excess energy not yet allocated.

Heatmaps can also be used to compare policies and Figure 3 shows different heatmaps for three policies in the one wind scenario. Note that percentage wastage appears to only depend on windspeed and be independent of average wind absence. This occurs because wastage only occurs when the wind is present, and hence variations in how long the wind is absence have no effect.

For all policies in the one wind scenario, there are no obvious differences, and thus it appears that using different types of local knowledge have no impact. These policies were also investigated for a four by four grid of neighbourhoods with wind multipliers that decreased from the north-west corner to the south-east corner, and again no major differences were found. This suggests that in the case of a single wind at the strengths and absences investigated, the different policies do not make a major difference in how much energy is supplied to other neighbourhoods and hence no difference is seen. This can be explained by the fact that most wastage occurs at night when there is low background usage and a lower chance of appliance use (as illustrated by Figure 1). Thus when there is high demand, all available renewable energy is used, regardless of policy; and when there is low demand, there is sufficient energy to allocate it all (again regardless of policy). This could be investigated further by reducing the amount of renewable energy to much lower levels so that all is needed and hence differences in policies

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	mean	variance	Grid	W%	R%
da100	1.09	1.16	1.19	1.46	1.18	1.13	1.11	1.19	0.0130	159.3	15.9%	47.4%
wn100	1.11	1.14	1.22	1.37	1.21	1.16	1.16	1.20	0.0064	158.4	16.2%	47.5%
dw100	1.10	1.14	1.22	1.43	1.20	1.13	1.15	1.20	0.0110	158.6	17.1%	47.6%
eq100	1.15	1.13	1.25	1.44	1.22	1.13	1.13	1.21	0.0111	160.9	18.6%	46.7%
dm100	1.13	1.15	1.28	1.47	1.20	1.19	1.13	1.22	0.0129	163.7	16.8%	45.9%
dmdec	1.07	1.21	1.31	1.48	1.29	1.17	1.06	1.23	0.0192	165.0	19.2%	45.3%
dmdwn	1.07	1.30	1.32	1.30	1.32	1.28	1.10	1.24	0.0101	165.6	19.6%	45.2%

Table 2. Cost per day for different policies across neighbourhoods with mean and variance, grid consumption, percentage waste and percentage renewable use (two wind scenario over 24 hours), ordered by average cost.

may be demonstrated. Furthermore, introducing the ability to store renewable energy and use it later might could lead to more significant differences in policies.

Note that all policies described above lead to a greater use of renewable energy and lower grid usage when compared to the situation where no energy is shared between neighbourhoods. On average, the amount of renewable energy consumed (as a percentage of total energy demand) increases to 70% from 55% when neighbourhood sharing is introduced, and the wastage of renewable energy (as a percentage of all renewable energy produced) drops from 57% to 27% through sharing between neighbourhoods.

Next, we consider the two wind scenario and the various policies described above. The results are shown in Table 2. The first nine columns of this table considers the cost of electricity per day. Although the differences are not large, these can accumulate over a year. The results suggest that using a more extreme policy where knowledge of demand is used to allocate all surplus in one direction or the other leads to the lowest average cost per day. However, this policy does not lead to the lowest variance in cost suggesting that is it not as fair as the wind based policy which does have the lowest variance. The policies that allocate less than 100% of demand, dmdec and dmdwn, seem to be poor in terms of average cost, although dmdwn leads to a low variance. Note however, that these results are specific to a particular scenario and hence cannot be generalised without further experimentation.

5 Related work

A recent survey of modelling smart grids considers smart grids as complex systems with emergent behaviour and identifies multi-agent systems as an existing modelling tool [13]. In [20], a multi-agent systems approach is taken to controlling a smart grid and in [15], agents act as elements of the smartgrid. There are limitations to what can be achieved using this method, because the size of models is limited computationally. Pretopology and percolation theory applied to complex systems may be able to successfully model large smart grids [13].

Modelling of smart grids has been done at two main levels: either very detailed, focussing on modelling the electrical components such as wind turbines and inverters in terms of their performance [26, 19, 14], or at the level of a group of houses with various sources of renewable energy [22, 18, 17, 7, 10]. Both of these levels differ from the approach taken here where the focus is on spatial aspects of redistribution of renewable energy.

Fine-grained models of residential consumption have been developed [1, 16, 24, 25]. The goal is to build up realistic profiles of consumption using various data sources about individual human behaviour and from these models estimate demand. To ensure the stochastic HYPE model presented here has a reasonable size, this level of detail has not been used. However, the general profile generated in [25] has been used to provide parameters for the stochastic HYPE model as mentioned in Section 3.1.

6 Conclusions and further work

To conclude, a stochastic hybrid model has been developed of smart grid generation of power to consider spatial aspects of sharing between neighbourhoods. Different knowledge-based policies for splitting surplus renewable energy appear to have little effect when there is only one wind but the two wind scenario does lead to differences. In general, sharing of energy significantly increases the amount of renewable energy used and reduces costs. Further exploration of wind patterns and the effect on policies is warranted as it is not possible to generalise from a single pattern.

This is ongoing research as part of a project on quantified modelling of collective adaptive systems (see `www.quanticol.eu`). In this paper, we have developed a model at an appropriate level for reasoning about distribution of energy throughout a suburb which can be explored through simulation. However, simulation is an expensive technique, and we wish to develop scalable approximation techniques that allow us to reason about these systems, similar to various fluid and mean-field techniques such as [23, 4].

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References

1. Ardakanian, O., Keshav, S., Rosenberg, C.: Markovian models for home electricity consumption. In: Second ACM SIGCOMM Workshop on Green Networking. pp. 31–36. ACM (2011)
2. Bortolussi, L., Galpin, V., Hillston, J.: Hybrid performance modelling of opportunistic networks. In: QAPL 2012, EPTCS 85. pp. 106–121 (2012)
3. Bortolussi, L., Galpin, V., Hillston, J.: Stochastic HYPE: flow-based modelling of stochastic hybrid systems. CoRR abs/1411.4433 (2014)
4. Bortolussi, L., Hillston, J., Latella, D., Massink, M.: Continuous approximation of collective systems behaviour: a tutorial. Performance Evaluation 70, 317–349 (2013)

5. Bortolussi, L., Policriti, A.: Hybrid dynamics of stochastic programs. Theoretical Computer Science 411, 2052–2077 (2010)
6. Davis, M.: Markov Models and Optimization. Chapman & Hall (1993)
7. Delamare, J., Bitachon, B., Peng, Z., Wang, Y., Haverkort, B., Jongerden, M.: Development of a smart grid simulation environment. In: UKPEW 2014 (2014)
8. Feng, C.: Patch-based hybrid modelling of spatially distributed systems by using stochastic HYPE – ZebraNet as an example. In: QAPL 2014, EPTCS 154 (2014)
9. Galpin, V.: Modelling a circadian clock with HYPE. In: PASTA 2010. pp. 92–98 (2010)
10. Galpin, V.: Modelling residential smart energy schemes. In: FoCAS@SASO14 (2014)
11. Galpin, V., Bortolussi, L., Hillston, J.: HYPE: Hybrid modelling by composition of flows. Formal Aspects of Computing 25, 503–541 (2013)
12. Galpin, V., Hillston, J., Bortolussi, L.: HYPE applied to the modelling of hybrid biological systems. ENTCS 218, 33–51 (2008)
13. Guérard, G., Ben Amor, S., Bui, A.: Survey on smart grid modelling. International Journal of Systems, Control and Communications 4, 262–279 (2012)
14. Kariniotakis, G., Soultanis, N., Tsouchnikas, A., Papathanasiou, S., Hatziargyriou, N.: Dynamic modeling of microgrids. In: International Conference on Future Power Systems. p. 7. IEEE (2005)
15. Kremers, E., Viejo, P., Barambones, O., Gonzalez de Durana, J.: A complex systems modelling approach for decentralised simulation of electrical microgrids. In: ICECCS 2010. pp. 302–311. IEEE (2010)
16. McQueen, D., Hyland, P., Watson, S.: Monte Carlo simulation of residential electricity demand for forecasting maximum demand on distribution networks. IEEE Transactions on Power Systems 19, 1685–1689 (2004)
17. Oviedo, R.M., Fan, Z., Gormus, S., Kulkarni, P.: A residential PHEV load coordination mechanism with renewable sources in smart grids. International Journal of Electrical Power & Energy Systems 55, 511–521 (2014)
18. Oviedo, R., Fan, Z., Gormus, S., Kulkarni, P., Kaleshi, D.: Residential energy demand management in smart grids. In: IEEE PES T&D 2012. pp. 1–8 (2012)
19. Panigrahi, T., Saha, A., Chowdhury, S., Chowdhury, S., Chakraborty, N., Song, Y., Byabortta, S.: A Simulink-based microgrid modelling & operational analysis using distributed generators. In: UPEC'06. pp. 222–226. IEEE (2006)
20. Pipattanasomporn, M., Feroze, H., Rahman, S.: Multi-agent systems in a distributed smart grid: Design and implementation. In: IEEE/PES PSCE'09. pp. 1–8. IEEE (2009)
21. Sinden, G.: Characteristics of the UK wind resource: Long-term patterns and relationship to electricity demand. Energy Policy 35, 112–127 (2007)
22. Střelec, M., Macek, K., Abate, A.: Modeling and simulation of a microgrid as a stochastic hybrid system. In: IEEE PES ISGT Europe. pp. 1–9. IEEE (2012)
23. Tribastone, M., Gilmore, S., Hillston, J.: Scalable Differential Analysis of Process Algebra Models. IEEE Transactions on Software Engineering 38, 205–219 (2012)
24. Widén, J., Wäckelgård, E.: A high-resolution stochastic model of domestic activity patterns and electricity demand. Applied Energy 87, 1880–1892 (2010)
25. Yao, R., Steemers, K.: A method of formulating energy load profile for domestic buildings in the uk. Energy and Buildings 37, 663–671 (2005)
26. Yubing, D., Yulei, G., Qingmin, L., Hui, W.: Modelling and simulation of the microsources within a microgrid. In: ICEMS 2008. pp. 2667–2671. IEEE (2008)