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The Effects of a Problem-Based Learning Intervention on Primary Students’ Performance on Greatest Common Factor and Least Common Multiple and on their Attitudes towards Mathematics

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Abstract
Greatest common factor (GCF) and least common multiple (LCM) have been two of the most difficult topics for primary students to grasp, especially when they are required to apply the concepts to solving real-life problems. Reported here is a quasi-experimental study of a comparison between a PBL approach and a traditional lecture method. The study recruited 55 Taiwanese sixth-grade students (ages 11–12), and examined the effects of PBL on their performance on GCF and LCM and on their attitudes towards mathematics. A comparison of the groups on pre- and post-tests showed that the treatment group had a higher mean score on the post-test than the control group. Results of the students’ responses to pre- and post-questionnaires indicated that there were statistically significant differences between the groups on the post-questionnaire, implying that PBL has a positive impact on students’ attitudes towards mathematics.

Keywords: Greatest common factor (GCF); least common multiple (LCM); problem-based learning (PBL); primary students; attitudes towards mathematics

Introduction
Research shows that greatest common factor (GCF) and least common multiple (LCM) are closely related to fraction operations (Lee, Choi and McAninch, 2012) and play an essential

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role in students’ understanding of the multiplicative structure of numbers. In addition, the concepts of GCF and LCM are important components of the secondary school mathematics curricula in the domains of prime factorisation and quadratic factoring. However, for many students, learning GCF and LCM results in a narrow step-by-step focus on the algorithms employed, with little attention being paid to the underlying concepts (Foster, 2012).

One of the goals of mathematics instruction is the development of mathematical proficiency. Kilpatrick et al. (2001) define five strands to mathematical proficiency: (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition. However, research shows that many students fail to master these competencies within the domain of GCF and LCM (Foster, 2012; Hung and Yeh, 2005). Hung and Yeh (2005) reported that many sixth-grade students in Taiwan struggled with the concepts of GCF and LCM, although they were able to apply routine procedures and algorithms successfully when performing calculations. Dias (2005, p. 1) also argued that students struggled to recognise “multiplicative relations between whole numbers, particularly when they are expressed in prime-factored form.”

It has also been reported that due to a poor understanding of the concepts of LCM and GCF, students tend to confuse GCF with LCM (Mohyuddin and Khalil, 2016). This confusion contributes to the difficulty that students have in solving real-life problems involving GCF and LCM (Triyani, Putri and Yo, 2012). This lack of problem-solving skills is concerning, since these skills are essential in helping students to think logically and in making them aware of the relevance of mathematical concepts throughout their learning (Li and Stylianides, 2018). Brown, Thomas, and Tolias (2002) contended that teaching GCF and LCM should “get students to reflect on these [procedural] actions sufficiently and in productive enough ways that they are able to move beyond successful actions to processes and objects” (p. 78).
Consequently, the need to enhance students’ understanding of GCF and LCM both procedurally and conceptually has encouraged us to conduct this research study.

Although there has been an increased emphasis on teaching mathematics in context (Liljedahl, 2015), our review shows that the number of available studies in teaching HCF and LCM in context for primary students is small and disproportionate to the number of studies that documented problems of practice for which solutions are needed. Therefore, not only do the results of this study make valuable contribution to the literature in this particular area, but it also has important implications in practice and curricula at the primary education level.

**Students’ Mathematical Development of Procedural and Conceptual Knowledge**

One of the oldest concerns in mathematics education is related to students’ mathematical development of procedural and conceptual knowledge. For instance, as Suydam and Reys (1978) stated, the development of computational skills is considered to be one of the important, primary goals of a school mathematics program. However, it has also been argued that computational fluency can largely be achieved by “a collection of isolated bonds of facts rather than an integrated set of patterns and principles” (Resnick and Ford, 1981, p. 35). Hatano (1988) called this “routine expertise,” and argued that such knowledge can only be used effectively when executing familiar tasks, but not when attempting those that contain novel elements or that require innovation.

Procedural knowledge (skills) involves “knowing how to do something”, while conceptual knowledge (understanding or concepts) involves “knowing why one should do something.” On the surface, although conceptual and procedural knowledge appear distinct, linking these two types of knowledge together during the learning process can significantly increase the speed with which they can be acquired and the ease with which they can be applied
(Tsai and Li, 2017). For instance, conceptual understanding facilitates knowing when it is appropriate to apply a skill and is critical when deploying a skill to new tasks or unfamiliar problems (Baroody and Dowker, 2003).

**Problem-Based Learning (PBL) in Mathematics Education**

As Pape and Tchoshanov (2001) suggested, in order for students to gain meaningful mathematical understanding, “teachers and students must develop classroom norms that facilitate explanation and justification and the use of representations in the service of supporting arguments” (p. 126). In recent years, constructivist perspectives on learning and teaching have become popular among mathematics educators, psychologists, and researchers (Francisco, 2013; Jančič and Hus, 2019; Levenson, 2013). One of the more popular constructivist pedagogies, one that challenges traditional teaching methods, is PBL (Li and Stylianides, 2018).

PBL has its origins in medical education and is largely focused on problem-solving. A typical PBL lesson starts with an ill-structured problem that reflects a real-life situation. It encourages students to acquire new knowledge by solving problems using existing knowledge and skills (Goodnough and Hung, 2008; Deep et al., 2019). Dahlgren and Dahlgren (2002) defined PBL as “learning in context, elaboration of knowledge through social interaction and an emphasis on meta-cognitive reasoning and self-directed learning” (p. 111). Studies have highlighted the benefits of PBL for improving problem-solving abilities, for developing mathematical knowledge in more flexible and novel ways, and for promoting a better understanding of mathematical ideas (Wijnen, Loyens, and Schaap, 2015; Wirkala and Kuhn, 2011).

Over the past three decades, the practices used to implement PBL, practices compatible with social constructivist views, have varied from subject to subject. Various models have been
developed and implemented to meet specific instructional goals or the demands of particular subjects. In mathematics education, Li and Stylianides (2018) synthesised the results of PBL-related studies and proposed a PBL model to conduct PBL lessons in a fifth-grade mathematics classroom. As shown in Figure 1, their PBL model includes (i) the three key features of PBL: the problem-driven content and structure, the inquiry-based collaborative learning process, and the student-centred approach; (ii) the three key features of a good PBL problem: ill-structured, contextualised, and relevant to real-life situations; and (iii) the three steps of PBL: launching the problem, exploring the problem through small-group discussion, and formulating the problem by conducting a whole-class discussion.

Figure 1. The PBL model used in this study (Li & Stylianides, 2018, p. 109)
A natural characteristic of the human learning process is that people are highly motivated to learn when facing real-life problems, and that this type of learning is inevitable during the course of solving such problems (Booth et al., 2015; Kain, 2003). The intrinsic motivation that is present in a real-life setting helps to stimulate an interest in learning, and this motivation provides theoretical support for using PBL in mathematics teaching and learning. Furthermore, teachers using PBL are expected to adopt a student-centred approach, one in which students can initiate, manage, and discuss their own ideas with one another (Koh, Khoo, Wong, and Koh, 2008). Researchers have produced evidence to suggest that PBL has a positive impact on students’ achievements and on their attitudes towards the subject being taught, and, consequently, have called for further research on the implementation of PBL for K-12 (ages 5–18) populations (Goodnough and Cashion, 2006; Wirkala and Kuhn, 2011).

**Aim and Research Questions**

There is little doubt that the nature of mathematics instruction and students’ attitudes toward mathematics play an influential role in the development of students’ mathematical understanding (Györi and Czakó, 2020; McAndrew, Morris, and Fennell, 2017; Pieronkiewicz, 2017; Wang, Haertel, and Walberg, 1993). Given the growing use of PBL (Li and Tsai, 2018; Wirkala and Kuhn, 2011) and the potential for its much more widespread use in K-12 populations, the question of whether PBL is more effective than traditional methods becomes one of great significance. As argued by Wirkala and Kuhn (2011, p. 1157), “there exists little rigorous experimental evidence of its effectiveness, especially in K–12 populations.” Hence, in this study, we take a step towards addressing this question by using a quasi-experimental research design to examine the effects of PBL on students’ understanding of GCF and LCM and on their attitudes towards mathematics.
Drawing on the PBL model (see Figure 1) and on the research literature relating to GCF and LCM, the effects of PBL were studied by recruiting two classes of sixth-grade students from Taiwan, one class serving as the treatment group and the other as the control. Data were collected from pre- and post-tests that assessed the students’ performance on GCF and LCM, and from pre- and post-questionnaires that assessed their attitudes towards mathematics. The pre-test results were similar between the two groups. Accordingly, this study sought to compare the differences in scores between the control and treatment groups before and after the PBL intervention. Specifically, it aimed to address the following fundamental research questions:

1. How does students’ performance compare between the pre-test and post-test, and how can the results of this comparison be explained in terms of the intervention?
2. What is the impact of this PBL intervention on the student’s attitudes towards mathematics, and what aspects of the intervention contribute towards these changes?

Method

A quasi control-and-treatment experimental approach was adopted in this study for two main reasons. Firstly, although it is recognised that the ideal approach for this type of experiment is one of random assignment, “the children are randomised as individuals to the intervention” (Torgerson and Torgerson, 2001, p. 321), it is the case that for many educational interventions randomisation is not practicable. The quasi-experimental design was adopted because, by recruiting two or more existing groups of students, the design could be implemented far more easily than one involving randomisation at the level of the individual (Trochim and Donnelly, 2006).
Secondly, schools in Taiwan generally adopt a normalised approach to setting, which means that each class contains students with a variety of academic ability, and so each class has, collectively, a similar academic profile to that of other classes belonging to the same grade. Therefore, it was possible to select the treatment and control groups from the same school. The pre-test was used to help determine if the groups were similar in terms of the students’ academic performance before the intervention (Mertens, 2014). In addition, by selecting a control group and a treatment group from the same school, the impact of external factors on performance was controlled since both groups were exposed to the same school-based events during the course of the intervention.

Participants

In Taiwan, students are formally introduced to the concepts of GCF and LCM from the fifth grade. In grade six, the focus when teaching GCF and LCM is on the development of computational competence and problem-solving skills. Hence, this study focused on sixth-grade students since the implementation of a PBL intervention fitted in with the goals of the Taiwanese curriculum as they relate to the teaching of GCF and LCM for sixth graders.

We contacted some primary school teacher acquaintances in Taiwan and invited them to participate in the study. After much searching, two sixth-grade primary school teachers (Mrs. Chan and Mrs. Tang, pseudonyms) and their classes (55 students: 29 boys and 26 girls) from a public primary school (government-owned and financed) in a suburban area of southern Taiwan agreed to participate in the study. Both Mrs. Chan and Mrs. Tang graduated from the same National Teacher College in Taiwan; they also had Master’s degrees in education from the same institution. At the time of the study, both teachers had more than 10 years of primary
school teaching experience. Informed consent was granted by the school principal, the class teacher, and the students’ parents prior to the implementation of the intervention.

The treatment group consisted of 28 students (15 boys and 13 girls) whose regular classroom teacher (Mrs. Tang) agreed to shift her customary teaching practice from a lecture-based method to a PBL style while teaching the topics of GCF and LCM. Given that Mrs. Tang was not familiar with PBL and that this was to be her first time using the PBL approach to teaching, she was invited to attend two half-day workshops run by the authors before the intervention started. The workshops covered (1) the key features of the PBL intervention, (2) the PBL process that the teacher was expected to follow in the classroom, and (3) the objectives that underpin the PBL approach to problem-solving.

During the course of this study, the control group, consisting of 27 students (14 boys and 13 girls), was taught by their regular classroom teacher (Mrs. Chan), who continued to use her customary teaching style. In conversation with Mrs. Chan, she mentioned: “I always follow the textbooks to teach. I talk most of lesson time” and “I think I would prefer to continue to use this sort of teaching pedagogies during the course of this study,” which was consistent with our classroom observation that the control group was taught using a lecture-based method, with the chalkboard and the textbook serving as the main instructional aids in the classroom.

**Intervention Design**

Based on knowledge of the Taiwanese curriculum relating to GCF and LCM, the content of this intervention was specifically organised in four major topics in order to encompass the Taiwanese curriculum in this area for sixth graders. The four topics were: (1) composite and prime numbers, (2) prime factorisation, (3) the Ladder Method for GCF and LCM, and (4) relatively prime/coprime. It is also worth mentioning that in this study it was important to make
sure that both the treatment and control groups received the same number of teaching hours in the topics relating to GCF and LCM and that they did so over the same period of time. According to the past experience of Mrs. Chan and Mrs. Tang, the anticipated lesson time for covering the four topics relating to GCF and LCM was about 280 minutes, which was allocated to seven lessons over the course of two consecutive weeks. Consequently, it was decided that both the treatment and control groups would receive around 280 minutes of lesson time in total, and would cover the topics of GCF and LCM over two consecutive weeks. Both teachers also agreed to ensure that the only difference during this period was in the teaching approach: the students in the treatment group were taught using the PBL approach, while the students in the control group continued to be taught by the teacher-centred and textbook-based method. Figure 2 illustrates the differences in teaching approaches between the control and treatment groups.

![Diagram](image.png)

Figure 2. The differences in teaching approaches between the control and treatment groups
Through a synthesis of the relevant literature on GCF and LCM, six PBL problems were designed by the authors and implemented by Mrs. Tang within the treatment group. A typical procedure for tackling a PBL problem, as illustrated in Figure 1, followed three phases: (i) launching the problem (the problem was assigned to the students by the teacher who verbally introduced the problem to the students); (ii) exploring the problem (the students were asked to work on the problem collaboratively in small-group discussions, with the teacher circulating between the groups at all times); and (iii) closing the problem (the teacher facilitated a whole-class discussion to arrive at a consensus as to the nature of the problem). A brief description of each of the six PBL problems appears in Table 1.

Table 1
The PBL problems used in this study

<table>
<thead>
<tr>
<th>The GCF and LCM topics in the Taiwanese curriculum</th>
<th>PBL problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite and prime numbers</td>
<td>Problem 1: Factors and Multiples Game – Do you have any winning strategies?</td>
</tr>
<tr>
<td></td>
<td>Problem 2: Is 1 a composite number or a prime number?</td>
</tr>
<tr>
<td></td>
<td>Problem 3: What would be the best strategy to find all the composite and prime numbers on a 100 square grid?</td>
</tr>
<tr>
<td>Prime factorisation</td>
<td>Problem 4: How can we use Scratch to do prime factorisation?</td>
</tr>
<tr>
<td>Ladder Method for GCF and LCM</td>
<td>Problem 5a: Supermarket challenge – what best strategy do you have?</td>
</tr>
<tr>
<td></td>
<td>Problem 5b: Science Experiment – how to make an efficient experiment check schedule?</td>
</tr>
<tr>
<td>Relatively Prime/coprime</td>
<td>Problem 6a: Why is it possible that there is a connection between prime numbers and the life circles of cicadas?</td>
</tr>
<tr>
<td></td>
<td>Problem 6b: How does RSA public-key cryptography relate to relatively prime/coprime?</td>
</tr>
</tbody>
</table>

Note. Problem 6b was not intended to cover the full complexity of the public-key cryptography. Rather, small prime numbers were used to help students make sense of it.
Mrs. Tang managed and controlled the pace at which each PBL problem was explored, taking into account the time limitation and the nature of the students’ responses. Typically, the time taken to explore a PBL problem was in the range 25–55 minutes. The total number of teaching hours spent on the PBL problems was around 280 minutes, similar to the amount of time spent on these topics by the control group.

**Pre-test and Post-test**

There is no doubt that assessing the effectiveness of learning in PBL environments can be challenging (Jones, 2002). PBL and traditional teacher-centred approaches pursue different goals: the former focuses on developing competence, while the latter aims to demonstrate it. Different assessment methods may produce “varying” results regarding the effectiveness of the PBL approach (Hung, 2011). The more efficiently an assessment measures students’ competence in knowledge application, the more effective PBL will be shown to be compared to traditional teacher-centred methods (Dochy, Segers, Van den Bossche, and Gijbels, 2003).

For most primary schools in Taiwan, there are two internal school examinations per semester, one that takes place in the middle of the semester and one at the end. These examinations assess the students’ academic performance in subjects such as Mathematics, Mandarin, Science, Social Studies, and English. All the students who participated in the treatment and control groups were tested for their knowledge of, and skills in, GCF and LCM in examinations that were administered both before and after the intervention. Hence, the students’ performance on these two school examinations in the areas of GCF and LCM, the pre-test and the post-test, provided us with the means to examine the effects of the intervention on students’ performance in standardised school examinations.
Pre- and post-questionnaires

A short version of the Attitudes Towards Mathematics Inventory (short ATMI), proposed by Lim and Chapman (2013), was adopted as the pre- and post-questionnaire to investigate the students’ attitudes towards mathematics. The short ATMI comprises 19 items that rate a student on four subscales: Enjoyment of Mathematics, Motivation to do Mathematics, Self-confidence in Mathematics, and Perceived Value of Mathematics. The questionnaire is based on a 5-point Likert scale that ranges from Strongly Disagree (1) to Strongly Agree (5). Lim and Chapman (2013) found that the participants in their study were able to finish the short ATMI in less than 10 minutes, a time short enough to make it possible to administer the questionnaire during the course of a busy school day.

We first translated the 19 items in the short ATMI from English to Mandarin and then invited Mrs. Chan and Mrs. Tang to review the Mandarin version with the aim of ensuring that the wording was appropriate for sixth graders. A pilot study of the short ATMI was also conducted in an primary school within the same geographic area. This pilot enrolled 38 boys and 36 girls whose backgrounds and ages were comparable to those of the students participating in the formal study. Cronbach’s alpha coefficient for the short ATMI was .712, indicating that the internal consistency of the test was rated as acceptable (Pallant, 2020).

The pre- and post-questionnaires were identical and they were administered anonymously to all the students in the treatment and control groups with the aim of encouraging honest reporting. The pre- and post-questionnaires were given to the students before the first lesson was commenced and, again, after the last lesson was completed. Each teacher followed a similar procedure when administering the questionnaires to their students. Students were first invited to read the instructions before they responded to the questionnaire and no time limit
was set for its completion. Most students completed the questionnaires within 10 minutes and then handed them to their teacher immediately on completion.

Cronbach’s alpha coefficients for the pre- and post-questionnaires were .804 and .774, respectively, indicating that the internal consistency of the questionnaires used in the study was rated as good and acceptable, respectively.

**Data Analysis**

IBM SPSS Statistics was used to perform the statistical analyses. The result of the normality test suggested that the data was not normally distributed. Therefore, it was decided to use non-parametric tests, the Mann–Whitney U test and the Wilcoxon signed-rank test in this study. The former was used to determine whether there were statistically significant differences between the treatment and control groups, while the latter was used to determine whether there were statistically significant differences between variables within a group. The significance level for the analyses was set at .05. Descriptive statistics were calculated to provide information on the means and standard deviations of students’ performances on the pre-test, post-test, pre-questionnaire, and post-questionnaire.

**Results and Discussion**

Despite the careful control of the variables and, in particular, the systematic manipulation of the treatment variable, it is important to note that this study falls within the quasi-experimental paradigm and that the validity of the results has only been established at a certain probability level and only for the conditions that existed during the experiment. Another limitation is the short-term nature of the PBL intervention: the present study looked at the impact of a 2-week
PBL intervention implemented in an primary school context. Future studies may evaluate the long-term outcomes of this type of intervention.

**Comparison of Students’ Performance on the Pre- and Post-tests**

As mentioned earlier, this study examined the effects of a PBL intervention on students’ performance using standardised school examinations in the areas of GCF and LCM. Given the inherent limitations of using school examinations as the basis for the pre- and post-tests, this study is limited to a more cognitive approach to the learning of students, and the students’ performance on the tests may be interwoven with other variables, which were not identified in this study.

Table 2 shows some general descriptive statistics and the results of Mann–Whitney U tests that compared the students’ mean scores between the treatment and control groups for both the pre- and post-tests. Students’ mean scores in the treatment group were higher than their peers in the control group in both tests (mean=34.93, SD=2.340; mean=34.07, SD=2.956, respectively). The results of the Mann–Whitney U tests show that there was no statistically significant difference between the two groups in either test.

**Table 2**

*Mann-Whitney U test of the pre- and post-test*

<table>
<thead>
<tr>
<th>Test</th>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
<th>Z score</th>
<th>Asymp. Sig. (2-tailed)</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>Treatment Group</td>
<td>28</td>
<td>34.93</td>
<td>2.340</td>
<td>-.408</td>
<td>.683</td>
<td>-.055</td>
</tr>
<tr>
<td></td>
<td>Control Group</td>
<td>27</td>
<td>34.81</td>
<td>2.896</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-test</td>
<td>Treatment Group</td>
<td>28</td>
<td>34.07</td>
<td>2.956</td>
<td>-.943</td>
<td>.346</td>
<td>-.127</td>
</tr>
<tr>
<td></td>
<td>Control Group</td>
<td>27</td>
<td>32.74</td>
<td>4.654</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. The full score of the pre-test and post-test was 36. Statistical significance was set at the *p < .05 level*
It is noticeable that the treatment and control groups had similar mean scores on the pre-test but slightly different mean scores on the post-test, implying that the changes in the scores may be due to the treatment, and not to time-related factors or to environmental factors, as both groups were drawn from the same school. This finding is consistent with previous studies in mathematics education which found that groups receiving PBL-aligned instruction performed better than those receiving teacher-centred instruction (Chianson, Kurumeh and Obida; 2011; Wirkala and Kuhn, 2011).

PBL is associated with the Vygotskian school of thought, which claims that, as Sfard, Forman, and Kieran (2001) stated, “human thinking [is] essentially social in its origins and […] inextricably dependent on historical, cultural, and situational factors” (p. 5). This PBL intervention followed Vygotsky’s social constructivism by using Li and Stylianides’s (2018) PBL model to examine its effects on the students’ performance on the pre- and post-tests. Evidence from this study suggests that learning can occur through interacting with other students, by actively processing what they have to say, by challenging their perspectives, and by offering interpretations and explanations by way of discussion.

Nevertheless, it is argued that PBL makes it difficult for teachers to be sure that all students develop necessary, or useful, skills. Hence, it is considered by some researchers to be ineffective when compared to the traditional method of direct instruction (Kirschner et al., 2006). Results from previous research in mathematics education have been varied. Some researchers have found that students who engaged in PBL-aligned practices performed better in mathematical reasoning, in communication, and in their ability to apply new material than did groups who received teacher-centred instruction (Wirkala and Kuhn, 2011). However, others (Ewing, 2011) have concluded that there is no convincing evidence that PBL improves students’ mathematical performance.
It is also worth mentioning that in mathematics education, as argued by McGee, Wang and Polly (2013), teachers often believe that student-centred approaches contradict the culture of standardised school tests, a sentiment that makes it difficult to introduce reform. As shown in Table 2, the results from the comparison of students’ mean scores on the pre- and post-tests between the treatment and control groups contradict those studies that found teachers’ direct instruction to be the most effective approach to achieving a high level of competence in mathematics (e.g., Ewing, 2011).

In line with Wirkala and Kuhn (2011), this study, therefore, calls for more theoretical and, in particular, more empirical work on PBL to be conducted at the primary school level, since much of the PBL-related research has, to date, been conducted at the graduate and postgraduate levels.

**Comparison of Students’ Attitudes towards Mathematics between Different Groups**

Students’ responses to the pre- and post-questionnaires were analysed and divided into four subscales: (1) Enjoyment, (2) Motivation, (3) Self-Confidence, and (4) Value. Analyses of the Full Scale category were also made. Table 3 shows the results of Mann–Whitney U tests of the students’ responses to the pre-questionnaire, suggesting that there were no statistically significant differences between the treatment and control groups in any of the categories before the implementation of the intervention. The two groups had similar scores on the pre-questionnaire, although the treatment group had a slightly higher mean score than the control group in the Motivation, Self-Confidence, and Full Scale categories, though not in the Enjoyment and Value categories.
Table 3

**Mann-Whitney U test of the pre-questionnaire between the treatment and control groups**

<table>
<thead>
<tr>
<th>Pre-Questionnaire</th>
<th>Group</th>
<th>N</th>
<th>Mean (SD)</th>
<th>U score</th>
<th>Z score</th>
<th>Asymp. Sig. (2-tailed)</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enjoyment (5 items)</td>
<td>Treatment</td>
<td>28</td>
<td>14.86 (1.820)</td>
<td>319.000</td>
<td>-1.109</td>
<td>.268</td>
<td>-.150</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>14.89 (1.934)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motivation (4 items)</td>
<td>Treatment</td>
<td>28</td>
<td>8.54 (1.972)</td>
<td>377.000</td>
<td>-.037</td>
<td>.970</td>
<td>-.005</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>8.37 (1.363)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Confidence (5 items)</td>
<td>Treatment</td>
<td>28</td>
<td>17.68 (2.262)</td>
<td>321.500</td>
<td>-.984</td>
<td>.325</td>
<td>-.133</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>16.70 (2.757)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value (5 items)</td>
<td>Treatment</td>
<td>28</td>
<td>18.68 (1.492)</td>
<td>371.000</td>
<td>-.124</td>
<td>.901</td>
<td>-.017</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>18.78 (1.121)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Scale (19 items)</td>
<td>Treatment</td>
<td>28</td>
<td>59.75 (5.104)</td>
<td>356.000</td>
<td>-.373</td>
<td>.709</td>
<td>-.050</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>58.74 (2.917)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Statistical significance was set at the *p < .05 level.

Table 4 shows the results of the students’ responses to the post-questionnaire. They demonstrate that following the intervention the treatment group had a higher mean score in all categories compared to the control group. In particular, the results of the Mann–Whitney U tests indicate that there are statistically significant differences between the treatment and control groups for the Value and Full Scale categories (U(28, 27)=198.500, Z=-3.188, p=.001, Effect Size=.430; U(28, 27)=239.500, Z=-2.355, p=.019, Effect Size=.318, respectively). However, there are no statistically significant differences between the two groups in the categories of Enjoyment, Motivation, and Self-Confidence. The effect size of both the Value and Full Scale categories can be classified as moderate according to Cohen (1988) (.1=small effect, .3=moderate effect, .5=large effect).
In order to further understand the difference between the treatment and control groups in the Value subscale, each of its five items was analysed using a Mann–Whitney U test, as shown in Table 5. Following the intervention, there were statistically significant differences between the treatment and control groups in the responses given to questions 16 and 18 (U(28, 27)=135.000, Z=-4.793, p=.000, Effect Size=.646; U(28, 27)=287.000, Z=-2.068, p=.039, Effect Size=.279, respectively), with the former having the higher score. The magnitudes of the differences in the means for the question 16 and 18 scores were large and moderate, respectively, according to the effect size.
Table 5

*Mann-Whitney U test of the Value category in the post-questionnaire*

<table>
<thead>
<tr>
<th>Items in the Value category</th>
<th>Group</th>
<th>N</th>
<th>Mean (SD)</th>
<th>U score</th>
<th>Z score</th>
<th>Asymp. Sig. (2-tailed)</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q15: Mathematics is a very worthwhile and necessary subject</td>
<td>Treatment</td>
<td>28</td>
<td>4.04 (.189)</td>
<td>351.000</td>
<td>-1.402</td>
<td>.161</td>
<td>.189</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>3.96 (.192)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q16: Mathematics is important in everyday life</td>
<td>Treatment</td>
<td>28</td>
<td>3.96 (.331)</td>
<td>135.000</td>
<td>-4.793</td>
<td>.000*</td>
<td>.646</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>3.30 (.465)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q17: Mathematics is one of the most important subjects for people to study</td>
<td>Treatment</td>
<td>28</td>
<td>4.04 (.189)</td>
<td>337.500</td>
<td>-1.733</td>
<td>.083</td>
<td>.234</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>3.93 (.192)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q18: College mathematics lessons would be very helpful no matter what I decide to study in future</td>
<td>Treatment</td>
<td>28</td>
<td>4.00 (.385)</td>
<td>287.000</td>
<td>-2.068</td>
<td>.039*</td>
<td>.279</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>3.74 (.526)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q19: A strong mathematics background could help me in my professional</td>
<td>Treatment</td>
<td>28</td>
<td>3.68 (.612)</td>
<td>356.500</td>
<td>-.416</td>
<td>.677</td>
<td>.056</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>27</td>
<td>3.59 (.501)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Statistical significance was set at the *p < .05 level

It is noticeable that after the intervention more students in the treatment group, compared to the control group, agreed or strongly agreed with Q16: “Mathematics is important in everyday life.” Using real-life situations in the problem-solving process is essential in PBL (Cunningham and Corderio, 2000) because learning comes from “the process of working towards [an] understanding or resolution of a problem” (Barrows and Tamblyn, 1980, p. 1). Recognising the usefulness of mathematics in their everyday lives may also affect their views on Q18: “College mathematics lessons would be very helpful no matter what I decide to study in future,”
views that may explain why statistically significant results were observed for the improvement in their attitudes towards mathematics in the future.

**Comparison of Students’ Attitudes towards Mathematics within the Same Group**

A Wilcoxon signed-rank test was performed to measure whether there were significant differences between the pre- and post-questionnaires within the same group. Table 6 shows the results of the Wilcoxon signed-rank test for the control group, indicating that there is no statistically significant difference in any of the categories between the pre- and post-questionnaires. The control group had similar mean scores in each of the categories, showing a slightly lower mean score in the Full Scale category on the post-questionnaire than in the pre-questionnaire.

Table 6

**Wilcoxon Signed-Rank test of the pre- and post-questionnaires within the control group**

<table>
<thead>
<tr>
<th>Group</th>
<th>Questionnaire</th>
<th>Mean</th>
<th>SD</th>
<th>Z score</th>
<th>Asymp. Sig. (2-tailed)</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>Pre-Enjoyment</td>
<td>14.89</td>
<td>.934</td>
<td>-1.508</td>
<td>.132</td>
<td>.290</td>
</tr>
<tr>
<td></td>
<td>Post-Enjoyment</td>
<td>15.07</td>
<td>1.035</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pre-Motivation</td>
<td>8.37</td>
<td>1.363</td>
<td>.000</td>
<td>1.000</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Post-Motivation</td>
<td>8.39</td>
<td>1.367</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td>Pre-Self-Confidence</td>
<td>16.70</td>
<td>2.757</td>
<td>-1.732</td>
<td>.083</td>
<td>.333</td>
</tr>
<tr>
<td>(N=27)</td>
<td>Post-Self-Confidence</td>
<td>16.59</td>
<td>2.805</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pre-Value</td>
<td>18.78</td>
<td>1.121</td>
<td>-1.611</td>
<td>.107</td>
<td>.310</td>
</tr>
<tr>
<td></td>
<td>Post-Value</td>
<td>18.52</td>
<td>1.221</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pre-Full Scale</td>
<td>58.74</td>
<td>2.917</td>
<td>-.894</td>
<td>.371</td>
<td>.172</td>
</tr>
<tr>
<td></td>
<td>Post-Full Scale</td>
<td>58.56</td>
<td>2.778</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Statistical significance was set at the *p < .05 level
Table 7 shows the results of the Wilcoxon signed-rank test for the treatment group, indicating that there are statistically significant differences for the Value and Full Scale categories (Z=-3.616, p=.000, Effect Size=.683; Z=-2.934, p=.003, Effect Size=.554, respectively). The magnitudes of the differences in the means within the treatment group for the Value and Full Scale categories are large, according to the effect size.

Table 7

<table>
<thead>
<tr>
<th>Group</th>
<th>Questionnaire</th>
<th>Mean</th>
<th>SD</th>
<th>Z-score</th>
<th>Asymp. Sig. (2-tailed)</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment Group (N=28)</td>
<td>Pre-Enjoyment</td>
<td>14.86</td>
<td>1.820</td>
<td>-0.632</td>
<td>.527</td>
<td>.119</td>
</tr>
<tr>
<td></td>
<td>Post-Enjoyment</td>
<td>14.93</td>
<td>1.609</td>
<td>.698</td>
<td>.487</td>
<td>.119</td>
</tr>
<tr>
<td></td>
<td>Pre-Motivation</td>
<td>8.54</td>
<td>1.972</td>
<td>-0.514</td>
<td>.514</td>
<td>.097</td>
</tr>
<tr>
<td></td>
<td>Post-Motivation</td>
<td>8.75</td>
<td>2.066</td>
<td>.487</td>
<td>.487</td>
<td>.097</td>
</tr>
<tr>
<td></td>
<td>Pre-Self-Confidence</td>
<td>17.68</td>
<td>2.262</td>
<td>-1.414</td>
<td>.157</td>
<td>.267</td>
</tr>
<tr>
<td></td>
<td>Post-Self-Confidence</td>
<td>17.54</td>
<td>2.333</td>
<td>.084</td>
<td>.497</td>
<td>.267</td>
</tr>
<tr>
<td></td>
<td>Pre-Value</td>
<td>18.68</td>
<td>1.492</td>
<td>-3.616</td>
<td>.000*</td>
<td>.683</td>
</tr>
<tr>
<td></td>
<td>Post-Value</td>
<td>19.71</td>
<td>1.272</td>
<td>.000</td>
<td>.000</td>
<td>.683</td>
</tr>
<tr>
<td></td>
<td>Pre-Full Scale</td>
<td>59.75</td>
<td>5.104</td>
<td>-2.934</td>
<td>.003*</td>
<td>.554</td>
</tr>
<tr>
<td></td>
<td>Post-Full Scale</td>
<td>60.93</td>
<td>4.570</td>
<td>.000</td>
<td>.000</td>
<td>.554</td>
</tr>
</tbody>
</table>

Note. Statistical significance was set at the *p < .05 level.

It is noticeable that within the treatment group there are higher mean scores in the Value and Full Scale categories on the post-questionnaire than on the pre-questionnaire, a finding that echoes the results presented in Table 4: the PBL intervention appears to have an impact on the students’ attitudes towards the perceived value of mathematics. Moreover, this finding from the treatment group contradicts the results from the control group, which show a decrease in
the mean score within each of these two categories on the post-questionnaire as compared to the pre-questionnaire.

**Implications**

This study by no means claims to be a fully comprehensive analysis of the effects of PBL on students’ performance on GCF and LCM and on their attitudes towards mathematics. It is also recognised that the findings of this study are subject to the limitations of the methodology used. However, its results suggest the positive effects that PBL may have on the development of mathematical competencies by primary school students.

Researchers have highlighted the importance of positive attitudes in enhancing students’ learning experience (Dowker, Bennett and Smith, 2012; Pieronkiewicz, 2017). This study, conducted with sixth grade students within their “real-life” classroom, has pedagogical implications for teachers to use PBL to promote students’ positive attitudes towards mathematics. The evidence presented in this study shows that the treatment group recognised and valued the role of mathematics in real-life situations more than did the control group. The features of PBL (problem-driven, inquiry-based, collaborative, and student-centred), as illustrated in Figures 1 and 2, could help to explain the changes in students’ attitudes towards mathematics that have been observed in this study.

Another implication of this study is that PBL could be a powerful tool to promote students’ mathematical performance and to prepare them for standardised school examinations. Admittedly, “pressures are increasing to provide evidence-based descriptions of effective mathematics teaching” (Hiebert and Grouws, 2007, p. 373). In this study, the application of a quantitative and quasi-experimental approach has enabled us to shed light on certain aspects of a short-term PBL intervention on students’ attitudes towards mathematics in the areas of
Enjoyment, Motivation, Self-Confidence, and Value. It also shows that the treatment group performed slightly better than the control group in the post-test, which consisted of standardised school examinations on the topics of GCF and LCM. It implies that PBL has the potential to enhance students’ academic performance.

**Conclusion**

A key feature of PBL is that its approach is completely different from those employed by traditional pedagogies. Its great strength, as explained earlier, is that it provides students with more opportunities to construct knowledge that can subsequently be utilised, and to do so in a meaningful way (Pape, Bell, and Yetkin, 2003). PBL is expected to generate true expertise by encouraging learners to seek out an understanding that can be used in a flexible manner to solve novel problems, whereas traditional pedagogies have been criticised to focus on memorising the steps involved in solving a narrow range of predefined problems (Gijbels et al., 2005). Maaß and Artigue (2013) noted that one aim of a more student-centred approach to learning mathematics is to promote “a learning culture in which students are invited to work in ways similar to how mathematicians and scientists work” (p. 781).

The findings of this study present a general picture of how the students’ performance varied between the pre- and post-tests within the treatment and control groups. We hope, as a result of this investigation, that further research will be stimulated, especially research conducted at a larger scale and over longer periods of time, and that it will lead to a broader and more representative understanding of the effects of PBL on students’ mathematical performance and on their attitudes towards mathematics. PBL can be beneficial for learners in all age groups (Wirkala and Kuhn, 2011). Ongoing research into PBL classroom practice will
no doubt contribute further to the knowledge base in this highly complex and demanding area of education.

**Reference**


