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Oil production optimisation using piecewise linear approximations (MILP): Computational performance comparison vs. MINLP formulation

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Abstract

Typical daily operations of oil and gas production systems are characterised by numerous decisions that must be carefully made if field profitability is to be sustained. These systems are usually nonlinear, nonconvex and involve binary decision variables; hence, the application of mathematical optimisation often results in an MINLP formulation. Piecewise linearisation techniques based on Special Ordered Sets of type 2 (SOS2) constraints have been used to approximate the nonlinear functions of the optimisation problem for complexity reduction. However, a computational analysis of these MILP-based formulations in comparison to their MINLP equivalents in oil production systems is scarce in literature. In this study, the benefits of MILP reformulation are applied to a synthetic but realistic case. In comparing both formulations, we evaluate solution sensitivity to the number of breakpoints, solution time, accuracy, and ease of automation.

Keywords: Mixed-integer optimisation; piecewise linear approximation; oil production.

1. Introduction

The application of mathematical optimisation for the recovery improvement of hydrocarbon reserves is vital for a field's sustainability (Gunnerud and Foss, 2010; Epelle and Gerogiorgis, 2019a). Novel algorithmic advancements have enabled engineers model, simulate and optimise complex nonlinear phenomena characterising the production activities in the petroleum industry (Cudas et al., 2012). This results in Mixed Integer Nonlinear Programs (MINLP) which can be difficult to solve. A simplification approach involves reformulating the MINLP to a Mixed Integer Linear Program (MILP) via piecewise linear approximations (Silva and Camponogara, 2014; Kronqvist et al, 2018). Nonlinearities in the formulation are mainly attributable to the multiphase flow rate relationships in the wells, pipelines and valves (Epelle and Gerogiorgis, 2019b). These complex relationships are usually not explicitly known and are dependent on several operational parameters estimated via high fidelity simulators. Piecewise linear models have the advantage of establishing linear relationships directly from the simulator sample points; a property that reduces problem complexity. This study explores this benefit and compares the computational performance of the MILP with the corresponding MINLP. The novel analysis presented herein also enables quality assessment of the optimisation formulation on the overall oil production. Furthermore, another novel element of this study is the combination of operationally distinct well behaviours with complex flow physics and bi-level flow routings within the optimisation formulation.

2. Methodology

The surface network model is first constructed in a steady state multiphase flow simulator (PIPESIM®). As shown in Fig. 1, the model consists of the wells, chokes, flowlines, manifolds, pipelines and separators, which are all connected. Robust multiphase flow correlations are adopted to capture complex flow physics in the respective network components. Some of these phenomena include: water coning behaviour in well (W1), non-vertical/deviated well trajectories (W2 and W4), and downhole pressure assistance to maintain production by means of Progressive Cavity Pumps (PCPs) and Electrical Submersible Pumps (ESPs), as seen in Fig. 1 (denoted as W3 and W4, respectively).

Figure 1: Surface production network and routing superstructure for optimisation.

Although water coning (a change in well inflow that occurs when the water-oil interface changes to a bell shape in the reservoir) is a transient process, the steady state simulator is capable of modelling this process using data tables (implemented herein) that describe oil production rate as a function of the water cut. Several design considerations are also made during the selection of the PCP and ESP for optimal oil delivery from the wells. The surface network design procedure was followed by the generation of large data tables. This involved performing several simulations at different well and pipeline conditions, which correspond to different wellhead pressures (300 – 380 psia) and liquid production rates. Using these data, algebraic (polynomial) proxy models are developed for each network component. These proxy models are then utilised together with an objective function (Eq. 1) to optimise the oil production rate. This methodology takes advantage of the decomposable nature of the production network, in that separate equations can be written for each component, which constitute the optimisation constraints. The complexity of the optimisation problem herein also stems from the presence of discrete routing variables at different levels: the well to manifold level and the pipeline to separator level. The nonlinear pressure-rate responses of the wells and pipelines coupled with these routing decisions inevitably result in a MINLP model, described in Table 1.

Table 1: Mathematical formulations (MINLP and MILP) for oil production optimisation.

Objective function	
Max (NPV) = ROP - CWP	(1)
$ROP = r_o \times \sum_{w=1}^{N_{prod}} q_o$	(2)
$CWP = r_{wt} \times \sum_{w=1}^{N_{prod}} q_w$	(3)
Constraints of the MINLP formulation	
$P_{w,min}^{wh} \leq P_w^{wh} \leq P_{w,max}^{wh} \quad \forall w$	(4)
$q_{i,w,NF} = f(P_w^{wh}) \quad \forall i, \forall w$	(5)
$q_{i,w,ESP} = f(P_w^{wh}, f_{i,w,ESP}) \quad \forall i, \forall w$	(6)
$q_{i,w,PCP} = f(P_w^{wh}, \Omega_{i,w,PCP}) \quad \forall i, \forall w$	(7)
$\Delta P_p = f(q_{p,o}, q_{p,wt}) \quad \forall p$	(8)
$x_{w,p} P^m \leq P_w^{wh} \quad \forall w, \forall p$	(9)
$z_{p,s} P^s \leq P^m \quad \forall p, \forall s$	(10)
$Q_{i,p} = \sum_w (x_{w,p} \times q_{i,w}) \quad \forall i, \forall p$	(11)
$LC^s = \sum_p (z_{p,s} \times Q_{i,p}) \quad \forall i, \forall s$	(12)
$\sum_p x_{w,p} = 1$	(13)
$\sum_s z_{p,s} = 1$	(14)
$P^s = P^m - \Delta P$	(15)
$\sum_p q_p \leq LC^s$	(16)
Piecewise linearization in 2 dimensions	
$P_w^{wh} = \sum_{j \in J} \sum_{k \in K} \lambda_{j,k} P_{(w)j}^{wh}$	(17)
$f_{w,ESP} = \sum_{j \in J} \sum_{k \in K} \lambda_{j,k} f_{(w,ESP)k}$	(18)
Proxy model structure	
$q_{o,ESP} = \alpha_0 + \alpha_1 P_{wh} + \alpha_2 f_{ESP} + \alpha_3 P_{wh}^2 + \alpha_4 f_{ESP}^2 + \alpha_5 P_{wh} f_{ESP}$	(37)
$q_{w,ESP} = \sum_{j \in J} \sum_{k \in K} \lambda_{j,k} q_{(w,ESP)j,k}$	(19)
$\sum_{j \in J} \sum_{k \in K} \lambda_{j,k} = 1$	(20)
$\delta_j = \sum_{k \in K} \lambda_{j,k} \quad \forall j$	(21)
$\delta_k = \sum_{j \in J} \lambda_{j,k} \quad \forall k$	(22)
$\lambda_{j,k}, \delta_j, \delta_k \geq 0$	(23)
δ_j and δ_k are SOS2	(24)
Piecewise linearization in 1 dimension	
$P_w^{wh} = \sum_{j \in J} \lambda_j P_{(w)j}^{wh}$	(25)
$q_w = \sum_{j \in J} \lambda_j q_{(w)j}$	(26)
$\sum_{j \in J} \lambda_j = 1$	(27)
λ_j is SOS2	(28)
Linearizing bilinear terms of type (C1-C2 and B-C)	
$C_1 \cdot C_2 = \xi_1^2 - \xi_2^2$	(29)
$L_1 \leq C_1 \leq U_1; \quad L_2 \leq C_2 \leq U_2$	(30)
$\xi_1 = 0.5(C_1 + C_2); \quad 0.5(L_1 + L_2) \leq \xi_1 \leq 0.5(U_1 + U_2)$	(31)
$\xi_1 = 0.5(C_1 - C_2); \quad 0.5(L_1 - U_2) \leq \xi_2 \leq 0.5(U_1 - L_2)$	(32)
$\tau = B \cdot C; \quad 0 \leq C \leq U$	(33)
$\tau \leq U \cdot B$	(34)
$\tau \geq C - U(1 - B)$	(35)
$\tau \geq 0; \quad \tau \leq C$	(36)

This MINLP formulation is linearised in 3 ways to generate MILPs; the computational performance of these 4 formulations (including the MINLP) are compared. The first MILP formulation (MILP-3) applies standard algebraic transformation and Special Ordered Sets of type 2 (SOS2) to linearise nonlinear terms (quadratic and bilinear terms – products of 2 continuous variables and products of a continuous and binary variable) in the MINLP formulation using 3 breakpoints. The second MILP formulation (MILP-5) uses 5 breakpoints instead; the third (MILP-LKT) directly utilises the look-up data tables for linear interpolation in 1 and 2 dimensions. Table 1 presents the detailed formulations for the MINLP and MILP, respectively. The aim is to maximise the objection function (in terms of the Net Present Value – NPV, Eq. 1); where the Revenue from Oil Production (ROP) and Cost of Water Production (CWP) are given by Eqs. 2 and 3 respectively; r_o is the oil price (USD/STB), r_{wt} denotes the water production unit cost (USD/STB) and N_{prod}

is the number of wells. Eq. 4 ensures that the wellhead pressure (P^{wh}) is tightly bounded. The proxy models for Naturally Flowing (NF) well, ESP well, PCP well and pipelines are given by Eqs. 4–7 respectively; q represents the flowrate, f_{ESP} the ESP frequency, Ω , the PCP impeller rotation speed and ΔP_p , the pipeline pressure drop. Indices o , wt , i , w , p , wh , m , s denote oil phase, water phase, all phases, wells, pipelines, wellheads, manifolds, and separators, respectively. P_m and P_s are manifold and separator pressure, respectively.

Binary variables $x_{w,p}$ assigned to each well, ensure that the produced fluids from a well are routed by the choke (Eq. 9) to one of the pipelines. Similarly, $z_{p,s}$ in Eq. 10 ensures that the pipelines are routed to the separator. The mass balance constraint between wells and pipelines is represented by Eq. 11; Eq. 12 ensures material balance between the pipelines and the separators (which operate at a fixed pressure). The selection of only 1 binary variable is enforced using Eqs. 13–14. The constraint defined by Eq. 15 ensures the target separator pressure is met, while the liquid capacity constraints of the separators are represented by Eqs. 16. The procedure for linearising functions in 2D and 1D are shown in Eqs. 17–28 respectively; where j and k represent the breakpoints associated with the different variables. Bilinear terms which occur in the MINLP formulation as shown in the typical proxy model structure (Eq. 37) are linearised using Eqs. 29–36. In these equations, C represents, a continuous variable, and B a binary variable; L and U denote the lower and upper bounds of a continuous variable. ξ and τ are additional variables introduced in the linearisation procedure. BONMIN (v.1.8.6) and CPLEX (v.12.8.0.0) have been employed in order to solve the MINLP and MILP formulations, respectively.

3. Optimisation results

The proposed formulations were programmed in MATLAB® R2016a (using OptiToolbox v2.28) and solved with BONMIN and CPLEX on an Intel Core i7-6700 processor at 3.40 GHz running on a 64 bit Windows workstation with 16GB of RAM. The MILP-LKT formulation consists of 25 polytopes (squares) for the well performance function (5 breakpoints for the ESP frequency/rotational speed and 5 breakpoints for the wellhead pressure). For the pipelines, 144 polytopes (squares) were adopted (12 breakpoints for the oil and water phases respectively). This resulted in a total of 8,740 variables; this is significantly larger than the number of variables required in the other formulations (as shown in Table 2). Despite this considerably large number of variables, the MILP model is solved in a shorter time, compared to the MINLP formulation which has 36 variables.

This increase in problem size (number of constraints and variables) that ensues with an increasing number of data points is a major drawback of the SOS formulation; hence, it is only suitable for low-dimensional problems. With the MINLP, the increase in number of data points would hardly affect the approximations of the simulator output. In this regard, the MINLP formulation can be regarded as more robust and scalable compared to the MILP reformulation. The convergence of proposed formulations to different optimal solutions (Table 1) is an indication of the non-convexity of the optimisation problem. Nevertheless, high-quality, reliable solutions have been obtained from all formulations as demonstrated in the relative gap obtained (Table 2). The MINLP formulation has provided the best solution in terms of the NPV. Our computational analysis has also shown that the improvement in resolution quality affects the solution quality of both formulations. With 5 breakpoints (MILP-5), the NPV obtained is closer to that of MINLP compared to the lower resolution formulation, consisting of 3 breakpoints (MILP-3).

Table 2: Computational performance of optimisation formulations

Optimisation formulation	MINLP	MILP-3 (SOS2)	MILP-5 (SOS2)	MILP-LKT (SOS2)
Solver used	BONMIN	CPLEX	CPLEX	CPLEX
Number of constraints	34	184	184	340
Number of variables	36	134	170	8702
Relative gap (%)	0.00	0.00	0.00	0.00
Solution time (s)	0.536	0.111	0.152	0.287
Number of nodes	0	229	253	292
NPV (USD)	989,228	979,934	986,832	979,261
Total oil production rate (STB/day)	15,219	15,076	15,182	15,066
Total water production rate (STB/day)	3,803	3,767	3,794	3,766

The MILP-LKT formulation has provided the lowest NPV (1% lower than the MINLP). This may be attributed to the fact the solutions are always approximated by linear segments (in the SOS formulation – with inherently limited extrapolation capabilities when flowing conditions change), which are generated between nonlinear data points. Water coning behaviour is a broadly known source of nonlinearity in the wellbore model. Another important observation from results is that the high-pressure separator with lower capacity (S1) is the least preferred option for routing fluids from manifolds (Fig. 2).

Figure 2: Optimal discrete routing structure for all optimisation formulations considered.

The pipeline diameter and length, and the high pressure drop that ensues, makes it difficult for fluids to be delivered to S1 which operates at 45 psia, compared to S2 and S3 at 35 psia and 25 psia respectively. The algorithm has shown good utilisation of separator capacities for routing the fluids. As shown in Table 2, the MILP formulations converge

faster than the MINLP. Furthermore, the time required for proxy model development if incorporated will further make the MINLP slower compared to the MILP (which directly uses the table data points). It can also be observed that the differences between the oil and water production rates of the respective formulations are minimal. Despite this, very different optimal routing structures are obtained. This indicates that the algorithmic treatment of discrete variables is complicated, especially when they exist at different levels. However, the low number of nodes utilised for finding the optimal solution in all formulations reflects the efficiency of the CPLEX solver (which uses the Branch and Bound algorithm). On applying the CBC solver (based on the Branch and Cut algorithm) to our problem, the number of nodes reduces by an order of magnitude, although with a higher relative gap and a longer computational time. While a detailed comparison of solver performances is beyond the scope of this study, such analysis may be conducted to further evaluate the performances of these formulations especially for bigger-sized problems. Although the computational time requirement is expected to increase with larger production networks, the fast solution times obtained herein should be attributed to the nature of our optimisation formulation, rather than just problem size. On performing a network optimisation task (on Fig. 1) using PIPESIM's v2019.3[®] optimisation module, we observed the required runtime is over 30 s, resulting in a 10% lower NPV obtained: this attests to the superior performance of our proposed and implemented formulation.

4. Conclusion

This study proposes MINLP and MILP formulations for optimising production from a synthetic oil field consisting of 3 separators, 2 manifolds and 4 wells with complex downhole/multiphase flow physics. The nonlinear models were developed using regression analysis that resulted in algebraic polynomial models; whereas piecewise linear models were developed from production points sampled from a look-up table. A computational analysis performed on the formulations showed superior performance of the MILP formulation in terms of the runtime despite the significantly increased number of variables involved in comparison to the MINLP. Improved resolution of the MILP formulation from 3 to 5 breakpoints resulted in a better NPV. However, compared to other formulations, the MINLP yielded the highest NPV. Despite the similarity in the production rates, different optimal routing strategies are obtained for each formulation.

5. References

- A. Cudas, S. Campos, E. Camponogara, V. Gunnerud and S. Sunjerga, 2012. Integrated production optimization of oil fields with pressure and routing constraints: The Urucu field. *Comput. Chem. Eng.*, 46, 178–189.
- E.I. Epelle and D.I. Gerogiorgis, 2019a. Optimal Rate Allocation for Production and Injection Wells in an Oil and Gas Field for Enhanced Profitability. *AIChE J.*, 65, 1, (DOI: 10.1002/aic.16592).
- E.I. Epelle and D.I. Gerogiorgis, 2019b. Mixed-Integer Nonlinear Programming (MINLP) for production optimisation of naturally flowing and artificial lift wells with routing constraints. *Chem. Res Des.*, 152, 134–148.
- V. Gunnerud and B. Foss, 2010. Oil production optimization—A piecewise linear model, solved with two decomposition strategies. *Comput. Chem. Eng.*, 34, 11, 1803–1812.
- J. Kronqvist, D.E. Bernal, A. Lundell, and I.E. Grossmann, 2018. A review and comparison of solvers for convex MINLP. *Optimization Eng.*, 1–59.
- T.L. Silva, and E. Camponogara, 2014. A computational analysis of multidimensional piecewise-linear models with applications to oil production optimization. *Eur. J. Op. Res.*, 232, 3, 630–642.