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On risk attitude and optimal yacht racing tactics

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When the future wind direction is uncertain, the tactical decisions of a yacht skipper involve a stochastic routing problem. The objective of this problem is to maximise the probability of reaching the next mark ahead of all the other competitors. This paper describes some numerical experiments that explore the effect of the skipper’s risk attitude on their policy when match racing another boat. The tidal current at any location is assumed to be negligible, while the wind direction is modelled by a Markov chain. Boat performance in different wind conditions is defined by the output of a velocity prediction program, and we assume a known speed loss for tacking and gybing. We compare strategies that minimise the average time to sail the leg with those that seek to maximise the probability of winning, and show that by adopting different attitudes to risk when leading or trailing the competitor, a skipper can improve their chances of winning.

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1. Introduction

In this paper we model and analyse the problem faced by a skipper who wants to sail an upwind leg of a yacht race, rounding the mark before his opponent. This problem falls into the category of stochastic shortest-path problems, where the cost function to be minimised is the time needed to reach the mark, and it depends on stochastic quantities such as wind direction. Many problems fall into this category and involve routing for emergency response, both civil (Yamada, 1996) and military (Resch et al., 2003), and applications in logistics (Fleischmann et al., 2004) and transport (Shuxia, 2012). The aim is to find a path between two vertices of a graph such that the sum of its constituent edges, often representing a cost, is minimised. When cost depends on random quantities this becomes a stochastic problem, and the standard objective is to minimise expected costs (where costs include time) (Bertsekas and Tsitsiklis, 1991). For yacht races, models which minimise the expected time to finish, or to reach the next mark, have been studied in a number of papers (Philpott and Mason, 2001; Philpott, 2005). This might be appropriate in fleet races where corrected time over a number of races forms a basis for scoring points. Even so, such scoring systems assign rankings in each race and it is well known that rank-based scoring leads to different incentives than those from performance on average (Anderson, 2012).

As observed in Philpott (2005) rank-based scoring takes its most extreme form in match racing, where the objective is to maximise the probability of arriving before the competing yacht. Indeed the time difference between the two boats is not of interest, as opposed to its sign. In this context, the attitude towards risk of the skipper assumes a greater importance. The aim of this work is to show that by changing the skipper’s attitude to risk, it is possible to define a strategy that performs better in match races than strategies aimed at minimising the expected time to finish.

Of course, in most forms of match racing, the interaction between the boats is important. A leading yacht will attempt to cover a trailing yacht, not only for tactical reasons, but also to spill turbulent air on the trailing yacht’s sails to reduce their drive. Forcing another boat to tack to avoid a collision is also a tactical ploy to increase a yacht’s advantage. In this paper we choose to ignore these effects, as well as assuming identical yachts and crew expertise. This is done for modelling convenience as well as simplicity. By focusing solely on risk attitude we can see to what extent this is important, other effects being equal.

The paper is laid out as follows. In the next section we describe the model of the yacht and basic sailing strategy for the upwind leg of a match race. We then review dynamic programming as an approach to finding the strategy that minimises the expected time to reach the next mark. The following section shows how this is implemented in a routing model that accounts for different risk attitudes of the skipper. We then present the results of some simulations of the strategies that emerge from the routing model.
1.1. Sailing strategy

The speed of a sailing yacht depends on the wind speed and on the angle between boat heading and wind direction. It is usually expressed as a polar diagram like the one shown in Fig. 1. The numbers around the semicircle represent different true wind angles, while the radial ones represent the boat speed. The red line corresponds to the plot of boat speed for a particular true wind speed. While no direct course is possible straight into the wind, it is possible to sail upwind with an angle between wind direction and sailed course which is usually between 30° and 50°. Sailing closer to the wind direction (lower angle) makes the course shorter, but when sailing at higher angles a boat is faster. Velocity made good (VMG) is the component of yacht velocity in the wind direction. With a constant wind direction from the top mark, an optimal policy maximises VMG. This is typically attained at a true wind angle of around 40–45° (as in this example). In a polar diagram like the one in Fig. 1, it is possible to find the maximum VMG for a given wind speed by finding the intersection between the polar corresponding to the wind speed and the line perpendicular to the upwind direction. For this reason the common route towards an upwind mark, or in general towards the direction from which the wind blows, is a zigzag route. Such a route requires changes of direction which are called tacks. When manoeuvring for a tack, a boat points for a few seconds directly into the wind, therefore causing a temporary decrease in boat speed. If the wind is constant during the race and all over the racing area, trying to do the minimum number of tacks is the best choice. Fig. 2(a) shows two possible routes. In a constant wind, the route on the left is faster because it involves just one tack. Fig. 2(b) shows a situation in which the wind shifts towards the left over the duration of the leg. The best policy in this case is to go to the left of the course (referred to as being on starboard tack), and then tack and point towards the mark, while a myopic policy that begins the race going to the right (referred to as being on port tack) turns out to be suboptimal.

In real races the evolution of the wind can be much more complicated than these examples, with temporary shifts or gusts that a sailor seeks to take advantage of. Moreover wind has a random component. While racing, it is difficult to know how the wind is behaving at another location, or to foresee how it will behave once that point is reached. In the presence of randomness the optimal course in Fig. 2(b) might turn out to be worse than a myopic policy that tacks on every wind shift. For this reason sailors tend to try and stay in the centre of the course to enable shifts in wind direction to be exploited by tacking, while avoiding the risk of overlaying the mark.

In the presence of a competitor, a policy that avoids the course boundaries while staying close to the competitor reduces the risk of being beaten, at least when the competitor is the trailing boat. On the other hand, when the competitor is leading, it can make sense for a skipper to take a risk and explore the corners of the course hoping for a favourable wind shift. This is the phenomenon that we seek to model in this paper.

1.2. Dynamic programming

Finding an optimal set of tacks when the wind varies randomly requires a stochastic dynamic optimisation model. In contrast to the deterministic case, a solution does not consist of a single optimal path for a specific wind realisation, but a policy that is optimal over a range of wind realisations. Policies can be computed a priori and respect the principle of optimality: an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (Bellman, 1957). A policy that respects this principle can be found with dynamic programming (Bertsekas, 1995). Dynamic programming has been successfully applied in sailing in both ocean races and short course racing (see Philpott and Mason, 2001; Philpott, 2005). In this work we adapt the short-course model described in Philpott and Mason (2001) and Philpott (2005) with the aim of incorporating the skipper’s attitude towards risk in their actions.

The risk that a skipper is willing to take is usually influenced by his position with respect to the opponent. A common behavioural pattern is to be conservative, or risk averse, when in a leading position, while being risk seeking when losing. Here we interpret risk aversion as being pessimistic about wind shifts, believing that any shifts we observe will not be to our advantage. In contrast, a risk-seeking skipper will be optimistic about wind shifts and act as if these are more likely to be to his advantage. Such attitudes can be modelled by altering the transition probabilities of the process that defines wind shifts.

To understand the effect of risk-averse or risk-seeking skippers, we develop a race modelling program (RMP) for simulating races between two boats. The first RMP was developed in 1987 for the America’s Cup syndicate Stars and Stripes and is described in Letcher et al. (1987). Since then, RMPs have been used mainly in America’s Cup applications to compare different designs (see e.g. Philpott et al., 2004). In our case, since we are interested in comparing tactical choices, we model two identical boats (i.e. they have the same polar diagram).
2. Method

2.1. Dynamic programming

We consider an upwind leg of 6000 m (corresponding to 3.24 nautical miles, which approximates the length of the 2013 America’s Cup course), and 4000 m wide. In the coordinate system used the starting line is located on the x-axis, and centred around the origin, while the upwind mark is located on the y-axis. The racing area is discretised into a rectangular grid with $N=20$ increments $\Delta x$ across the course and $M=400$ increments $\Delta y$ in the direction of the course, as shown in Fig. 3. The $N-1$ lines defining the grid that are perpendicular to the y-axis will be referred to in the following as “cross sections”. The dynamic program is at stage $i$ when the yacht crosses the $i$th cross section.

The state variables are the yacht’s position $x_i$, the wind direction $w_i$ observed at stage $i$, and the current tack $z$ (where $z=\text{0}$ denotes starboard tack and $z=1$ denotes port tack). The wind direction $w_i$ is random and satisfies the Markov property, namely that the probability distribution for the variable $w_i$, conditioned on all the previous values, is equal to the distribution for the variable $w_i$ conditioned just on the last event:

$$P(w_i = w|w_{i-1} = w_{i-1},w_{i-2} = w_{i-2},\ldots,w_0 = w_0) = P(w_i = w|w_{i-1} = w_{i-1})$$

(1)

for every $i > 0$ and for every $w_i$ in the state space.

The actions at each stage are whether to tack the boat (i.e. change $z$ to $1 - z$) or continue on the same tack. As mentioned in the Introduction, a tacking manœuvre implies a time loss that will change with all the previous values, is equal to the distribution for the variable $w_i$ that are perpendicular to the

$$F$$

$$\frac{\Delta y}{C_0}$$

the probability distribution for the variable $w_i$ that is random and satisfies the Markov property.

Although more refined wind models are being developed (see for instance the recent reviews by Costa et al., 2008 and Bitner-Gregersen et al., 2014), Markov models are computationally very efficient and can still capture most of the statistical properties that are relevant in certain applications (Shamsbad et al., 2005; Sahin and Sen, 2001).

For tactical purposes we are interested in changes in wind direction that significantly affect the racing time. We therefore define a finite number of wind direction states: namely $-45^\circ$, $-40^\circ$, $-35^\circ$, $0^\circ$, $5^\circ$, $\ldots$, $45^\circ$, where $0^\circ$ represents the wind direction at which the upwind mark is set, and the other states represent shifts of $\pm 5^\circ$ from that direction.

For a system with a finite number of states the stochastic process is uniquely defined with an initial distribution for $w_0$ and a transition matrix $P$. The matrix elements $P_{jk}$ represent the probability that the system at time step $i$ is in state $k$ conditioned on the fact that it was in state $j$ at the previous time step $i-1$:

$$P_{jk} = P(w_i = k|w_{i-1} = j)$$

In order to obtain a realistic transition matrix we considered a time series of wind measurements from a weather station installed on the Newcastle University research vessel, and then built the matrix $P$ using a maximum likelihood estimator. As we use for the model a grid with 15 m resolution in the upwind direction and the decisions are taken every time the yacht reaches a cross section, the wind is modelled using a time step of 3 s, which is the time spent on average to move between two consecutive cross sections. The recorded wind direction signal was sampled every three seconds, and the corresponding wind directions were placed in $K$ bins of amplitude $5^\circ$. The number of jumps from bin $j$ to bin $k$ divided by the total number of jumps out of bin $j$ defines the value $P_{jk}, j = 1, 2, \ldots, K$, in the transition matrix.

Given a transition matrix $P$, Eq. (2) becomes

$$F(i, x, x', w, z) = F(i, x, x', w_j, z) + \sum_{k = 1}^{K} P_{jk} T_{i-1}(x', w_k, z).$$

(4)

2.2. Wind modelling

We assume the wind speed to be constant during the race, focusing on the changes in wind direction. As discussed in the previous section the dynamic programming algorithm we use assumes that the wind direction satisfies the Markov property. Although more refined wind models are being developed (see for instance the recent reviews by Costa et al., 2008 and Bitner-Gregersen et al., 2014), Markov models are computationally very efficient and can still capture most of the statistical properties that are relevant in certain applications (Shamsbad et al., 2005; Sahin and Sen, 2001).

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(4)

2.3. Risk modelling

We now turn our attention to the risk attitude of the yacht skipper. There is an enormous literature on modelling risk (for a recent introduction see Anderson, 2013). To model risk aversion, we adopt an approach based on the theory of coherent risk measures (Artzner et al., 1999). As shown in Artzner et al. (1999) coherent risk measures can be expressed as the worst-case expectation over a convex set of probability distributions to give a risk-adjusted expectation. Given the current wind direction state,
the probability distribution that we work with is the corresponding row of the transition matrix. To model risk aversion we choose the worst possible transition probabilities from a convex set \( \mathcal{D} \) of transition matrices. In other words, \( (4) \) becomes

\[
\hat{F}(i, x, x', w_j, z) = t(i, x, x', w_j, z) + \max_{\mathcal{P} \in \mathcal{D}} \sum_{k=1}^{K} P_{jk} T_{i+1}(x', w_k, z).
\]

An interpretation of \( (5) \) is illuminating. A boat skipper who is winning will be risk averse. She will try to behave safely, trying to stay ahead and to minimise her losses in bad wind outcomes. Using \( (5) \) in a recursion will be optimistic about the possible outcomes (i.e. heading shifts). Being pessimistic about random outcomes reduces risk, at some loss in expected performance.

Risk seeking behaviour has been less well studied, although it is often given as an explanation for participation in lotteries and negative expectation gambles, where optimistic participants place greater weight on winning probabilities than their real values. In our context we model risk seeking by choosing the best possible transition probabilities from a convex set \( \mathcal{D} \) of transition matrices. In other words, \( (4) \) becomes

\[
\hat{F}(i, x, x', w_j, z) = t(i, x, x', w_j, z) + \min_{\mathcal{P} \in \mathcal{D}} \sum_{k=1}^{K} P_{jk} T_{i+1}(x', w_k, z).
\]

This has the following interpretation. A boat skipper who is losing will seek risk. If she adopts a minimum expected finish time strategy against another skipper who minimises his expected time to finish, then she will tend to make the same decisions (unless the boats see very different winds) and lose the race almost certainly. She will instead seek different wind conditions from the competitor. Using \( (6) \) in a recursion will be optimistic about the possible advantageous wind shifts and assign a higher probability to these outcomes (i.e. lifting shifts). Being optimistic about random outcomes increases risk, as well as incurring some loss in expected performance.

We implement \( (5) \) and \( (6) \) in the recursion by adding a transformation in the solver that post-multiplies the transition matrix by another matrix which redistributes the probabilities. The resulting matrix has to be normalised in order to represent again a probability distribution.

3. Results

Fig. 4 shows a graphical representation of the transition matrix for the Markov model obtained with the maximum likelihood estimator as described in the previous section. With a notation that will be used throughout this paper, we use a grey scale to represent values in the interval \([0, 1]\) where white represents 0 and black represents 1. It can be noticed that the diagonal is dominant, meaning that, in general, if the wind is in state \( i \), the most probable state for the next step is to remain in state \( i \).

Moreover, when the wind has deviated from the mean, the event of a shift back towards the mean value is more likely than one in the same direction.

The wind for the simulations was generated as described in the previous section. The Markov chain defines a discrete wind direction. This can be made continuous by superimposing a mean-reversion noise process (see Philpott et al., 2004). However, we did not do this as we found that the behaviour of the simulated wind signal, achieved with no additional noise component, was similar to the empirical one, as can be seen in Fig. 5, with close values of mean and variance on different sub-intervals. A wind history of 400 values was generated for each of the 4000 simulated races.

Fig. 6 shows a histogram of the time needed by a yacht following the policy generated to minimise the expected time of arrival, according to the wind distribution previously modelled. The distribution is asymmetric, and this is due to the fact that even with a very favourable evolution of the wind there is a minimum time needed to complete the course. On the other hand, even with a policy which is effective in the majority of the cases, it is possible to be very unlucky and need a much higher time.

This policy was generated using a risk-neutral transition matrix for wind direction as pictured in Fig. 4. When the skipper is risk seeking or risk averse we replace this with a modified transition matrix. A sailor who is losing will seek risk. This corresponds to increasing her confidence of a lifting wind shift while discounting the likelihood of a heading wind shift. The transition matrices we use to represent a risk-seeking skipper are shown in Fig. 7(a) and (b). As shown in the figures, advantageous shifts (cells below the diagonal when the skipper is to the left of the opposition, and cells above when on the right) happen with higher probability than in the risk-neutral case. The remaining probabilities in each row are reduced to add to one.

The transition matrices for a risk-averse skipper are constructed similarly. Here bad wind shifts (above the diagonal when the skipper is to the left of the opposition, and below the diagonal

![Fig. 4. Representation of the transition matrix obtained for the wind model.](image)

![Fig. 5. Sixty-minute example of artificially generated wind and sixty-minute example of recorded wind.](image)

![Fig. 6. Distribution of arrival time of boat following the optimum policy.](image)
than \(d_{\text{max}}\) in independent sample if their distance is between 

\[d_{\text{min}}\] and \(d_{\text{max}}\). A t

\[d_{\text{min}}\].

The results presented in this paper underline that when trying to optimise a policy in order to win a competition, looking at average values is rarely the best approach, and accounting for differing risk attitudes might give policies that perform significantly better. Further work is being carried out in order to validate the model with data registered during America’s Cup races, and we are developing methodologies for learning risk parameters that yield maximum win probabilities.

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\section*{Appendix}

\begin{proposition}
Minimising the expected arrival time over all strategies will give a policy that is slower than a perfect skipper by the least amount on average.
\end{proposition}
Proof. Suppose a perfect skipper sails races in wind that she predicts perfectly. Each race is a random sample of wind and so her time to finish is an independent identically distributed random variable $T$.

Suppose she now sails a strategy $s$ that is not clairvoyant in each of these same wind conditions. The time to finish under this strategy is an independent identically distributed random variable $S(s)$.

Now the delay in finishing under strategy $s$ versus the perfect strategy is also an independent identically distributed random variable $D(s) = S(s) - T$. The expected delay from sailing $s$ is then $E[D(s)] = E[S(s)] - E[T]$.

To minimise this we should minimise $E[S(s)]$ as $E[T]$ is a constant. So the strategy that minimises expected delay after a clairvoyant skipper is the one that minimises expected arrival time. □

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