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Conformal vs confining scenario in $SU(2)$ with adjoint fermions

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The masses of the lowest-lying states in the meson and in the gluonic sector of an $SU(2)$ gauge theory with two Dirac flavors in the adjoint representation are measured on the lattice at a fixed value of the lattice coupling $\beta = 4/g_0^2 = 2.25$ for values of the bare fermion mass m_0 that span a range between the quenched regime and the massless limit, and for various lattice volumes. Even for light constituent fermions the lightest glueballs are found to be lighter than the lightest mesons. Moreover, the string tension between two static fundamental sources strongly depends on the mass of the dynamical fermions and becomes of the order of the inverse squared lattice linear size before the chiral limit is reached. The implications of these findings for the phase of the theory in the massless limit are discussed and a strategy for discriminating between the (near-)conformal and the confining scenario is outlined.

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The existence of a strongly-interacting sector beyond the Standard Model that is responsible for the breaking of electroweak symmetry was proposed many years ago [1, 2]. This scenario is referred to as Technicolor. The constituent fields of this sector describe techniquarks and technigluons. The dynamically-generated mass scale – i.e. the equivalent of the typical hadronic mass scale in QCD – separates the low-energy regime of the theory, of which the Standard Model is an effective theory, from the regime in which the technicolor sector becomes manifest; this scale is of the order of the TeV. However, a simple rescaling of QCD to the TeV scale and successive extensions to account for non-zero masses of Standard Model fermions prove to be inadequate to describe the phenomenology of the Standard Model itself (see *e.g.* [3, 4] for a review). Walking Technicolor provides a suggestive framework to cure some of these problems [5, 6, 7]. In this scenario, the technicolor theory behaves like QCD both in the ultraviolet and in the infrared regimes; asymptotic freedom characterizes the behavior at high-energies, while in the infrared domain the theory is confining, and chiral symmetry is broken. However, there is an intermediate range of energies in which the dependence of the coupling on the energy scale is supposed to be very mild. This is the walking regime, which should characterize theories that are close to the conformal window. Although this framework solves phenomenological problems of the original Technicolor model, its most straightforward realization, as an $SU(N)$ gauge theory with fundamental fermions, requires a large number of flavors to be compatible with electroweak precision data [8]: the phenomenologically favored theories with a low number of flavors N_f and a low number of colors N have necessarily fermions in higher representations. Another interesting scenario, known as Conformal Technicolor [9], assumes the existence of an underlying theory with a strongly-interacting IR fixed point. Recent analytical investigations using a wide range of techniques have tried to characterize such (near-)conformal theories [10, 11, 12, 13, 14].

An interesting possibility for reconciling the Walking Technicolor scenario with the experimental data is to consider gauge theories coupled with fermions transforming in higher representations of the gauge group [15], in which the conformal phase (and as a consequence the near-conformal phase) can be reached at values of N_f and N that are not excluded by phenomenology [16]. In particular, the theory with $N = N_f = 2$, and Dirac fermions in the adjoint representation is a likely candidate for the realization of the (near-)conformal scenario. Evidence for such (near-)conformal behavior is based on analytical calculations performed relying on uncontrolled approximations, or educated guesses. Hence, it is mandatory to investigate the phase structure of those theories from first principles. This has prompted several lattice calculations of bound state masses [17, 18, 19, 20, 21], of the Dirac operator spectrum [22, 23, 24] and of the renormalization group flow [25, 26, 27] of candidate walking theories. These calculations have to be interpreted with great care, since lattice systematic errors can obscure the physical behavior. Nonetheless the picture emerging from these studies is that the physics of gauge theories with fermions in the adjoint or in the symmetric representation has a different signature than QCD. Whether this is an indication of possible (near-)conformal behavior or a manifestation of the limitations of the current calculations is a question that can be answered only by gaining better control on the chiral, the infinite volume, and the continuum limit extrapolations. Interesting discussions of possible lattice signatures have been presented in Refs. [28, 29].

Most of the spectrum-based studies have looked for signatures of conformal or walking behavior in various observables; mesons have been studied in Refs. [17, 18, 19, 20, 21], baryons in Ref. [20] and Creutz ratios in Ref. [19]. In a gauge theory with massless fermions and chiral symmetry breaking, the vector meson has a mass of the order of the dynamical scale of the theory Λ and the pseudoscalar meson is massless, since it is the Goldstone boson associated to the spontaneous breaking of chiral

symmetry. If the fermions have a small mass $m \ll \Lambda$, the masses of the pseudoscalar and vector states are given by

$$m_{\text{PS}} = a_{\text{PS}}\sqrt{m}, \quad m_{\text{V}} = a_{\text{V}}m + b_{\text{V}}, \quad (1)$$

where b_{V} is the mass of the vector meson in the chiral limit. The other states in the spectrum are expected to behave like the vector state. In the case of a conformal theory, at $m = 0$ bound states cannot form. When a small mass is added, the theory is driven away from conformality. Masses of bound states then scale as

$$m_{\text{PS}} = \alpha_{\text{PS}}m^{\rho}, \quad m_{\text{V}} = \alpha_{\text{V}}m^{\rho}, \quad (2)$$

where ρ is related to the anomalous dimension of the mass γ by

$$\rho = 1/(1 + \gamma). \quad (3)$$

Since in ordinary Monte Carlo calculations it is impossible to simulate directly the massless case, indications of (near-)conformality can be sought using Eqs. (1-2). In particular, one would measure the ratio $m_{\text{V}}/m_{\text{PS}}$ and check whether it behaves like in QCD (i.e. whether it goes to infinity as $m^{-1/2}$, or it stays constant at least for a wide interval of masses before diverging). The problem with this approach is that a constant ratio $m_{\text{V}}/m_{\text{PS}} \simeq 1$ is also a characteristic of the large fermion mass limit of $\text{SU}(N)$ gauge theories, and for a theory that is not QCD (i.e. for which we can not rely on guidance coming from experiments) it is not clear a priori which bare mass would be small enough for the onset of the chiral behavior to be visible. Hence, there is a risk to confuse a confining, chiral symmetry breaking theory with heavy fermions, and a (near-)conformal theory in the infrared. The central point of this work is to show that this ambiguity can be successfully resolved by comparing the spectrum of the lightest mesons with gluonic observables.

In this note, we report on a numerical investigation of $\text{SU}(2)$ gauge theory with two flavors of Dirac fermions in the adjoint representation for a fixed value of the lattice coupling β . With this calculation, we aim to make a relevant step towards the understanding of the chiral limit while keeping under control finite size effects, but we will not be addressing the issue of the extrapolation towards the continuum, our calculation being at fixed lattice spacing. This point is crucial to understand the scope of the conclusions that can be drawn from our calculations, since all our results can potentially be affected by lattice artifacts, which could distort the numerical results. On the other hand, it is also worth stressing that the results presented here explore a range of masses much lighter than the ones in previous studies of this theory. At the same time care is taken at taming finite volume effects at such small masses.

In order to keep our presentation contained and accessible to a more general audience, we defer the discussion of the lattice observables and the presentation of our results in full to future publications. Here, we limit our exposition to the essential aspects, referring for the moment

the interested reader to the quoted literature for general discussions of the techniques. Simulations are performed on a spacetime lattice with geometry $(2L) \times L^3$, where $L = 8, 12, 16$. The long direction plays the role of Euclidean time, while the others are spatial directions. Fermions have antiperiodic boundary conditions in the temporal direction, and periodic boundary conditions in the spatial directions. We use the Wilson action for the gauge field and the Wilson discretization for the Dirac operator (the details of the implementation are given in Ref. [18]). The bare parameters are the bare coupling g_0 and the bare mass in lattice units am_0 . The coupling g_0 controls the size of the lattice spacing a . Our calculation is performed at $\beta = 4/g_0^2 = 2.25$, which has been found to be in the region of the phase diagram connected with the continuum limit [19, 20].

We measure masses of non-singlet mesons, the quark mass from the axial Ward identity (PCAC mass), the masses of the 0^{++} and of the 2^{++} glueballs and the string tension from the large distance exponential decay of correlators of operators with the appropriate quantum numbers. More details on the meson observables discussed in this work can be found in Ref. [18], which also contains a description of the simulation techniques we use to generate the configurations, while for measurements of quantities in the gluonic sector we follow Ref. [30], from which we also borrow results for gluonic observables in the $\text{SU}(2)$ Yang-Mills theory. The data shown are those obtained on the largest available lattice after checking that results on the smaller lattices were compatible. Whenever this request was not fulfilled we have discarded the data for the corresponding observable. As a consequence, gluonic observables that are more sensitive to finite-size effects could not be computed at the lighter masses.

A - Hierarchy in the spectrum. A global overview of our numerical results is presented in Fig. 1. The plot shows that a small, but clearly non-vanishing string tension exists at least down to PCAC masses of the order of 0.1 in units of the inverse lattice spacing, and that such string tension decreases with decreasing PCAC mass. The string tension shown in the figure is extracted using correlators of Polyakov loops, but compatible numbers are obtained from the static potential. For a lattice of fixed size L , the string tension decreases when decreasing the PCAC mass until it becomes $\mathcal{O}(1/L^2)$ at which value a plateau is reached. Tests on lattices of different sizes show that this is a finite size effect. As the lattice size is increased, the string tension keeps decreasing as the fermion mass is decreased. A non-zero string tension is expected in the massive case even in the conformal window, since a non-zero quark mass moves the theory away from the attraction basin of the IR fixed point. What is remarkable is that even at our lowest PCAC masses there is a well-defined hierarchy in the spectrum: the string tension defines the lowest mass scale in the system, and the meson spectrum is well above the lowest-lying glueballs. At this stage, it is worth noticing that states with

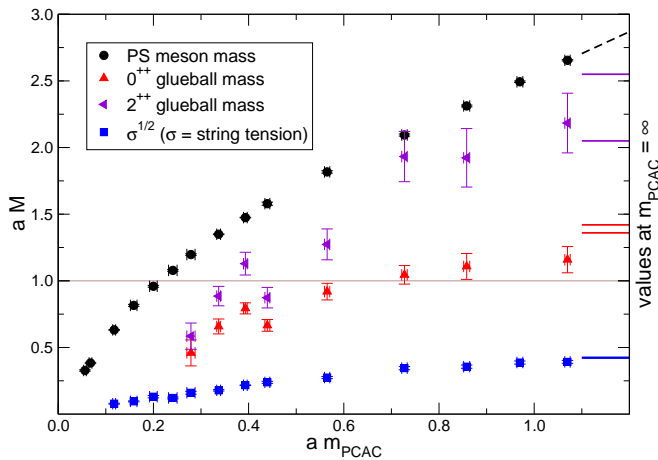


FIG. 1: Comparative plot of the various observables as a function of m_{PCAC} . The lines at high PCAC mass show the quenched behavior of the various observables. The horizontal line at $aM = 1$ visually shows the separation between values of masses that are affected by lattice artifacts ($aM > 1$) and values for which the states are below the ultraviolet cut-off ($aM < 1$). Although ideally one wants all the states of interest to be free from lattice artifacts, due to the big separation of the scales, this condition is difficult to accommodate in practice.

mass of the order or above a^{-1} are expected to be significantly affected by discretization artifacts; while we can reach small masses for the mesons, the extraction of the gluonic spectrum becomes very expensive for light fermions. As a consequence we do not have results for the glueballs and the string tension at the smaller values of the mass. Despite the fact that a significant portion of our spectrum falls in the region where discretization artifacts are not under control, the hierarchy of the spectrum seems to be a robust conclusion, as it can be extrapolated smoothly to the region where discretization errors are expected to be under control. To investigate in more detail the observed hierarchy of scales, we plot in Fig. 2 the ratio $m_{\text{PS}}/\sqrt{\sigma}$ as a function of the pseudoscalar mass in lattice units. For a standard confining and chiral symmetry breaking theory, this ratio goes to zero in the chiral limit. For our theory, even when varying the pseudoscalar mass by a factor of six to a region where it is well below the cutoff scale, this ratio is always of order 10, and does not extrapolate to zero in the chiral limit. This behavior is at odds with the one expected for QCD, and indeed it is not observed in QCD simulations for similar variations of the pseudoscalar mass. Fermion loops seem to strongly affect the gluonic sector, keeping the corresponding scale always well below the scale of mesonic physics. Let us discuss now the main features emerging from the numerical calculations.

B - IR effective dynamics. The vector meson is not displayed in Fig. 1, since it is degenerate with the pseudoscalar meson on the scale of the plot (the approximate

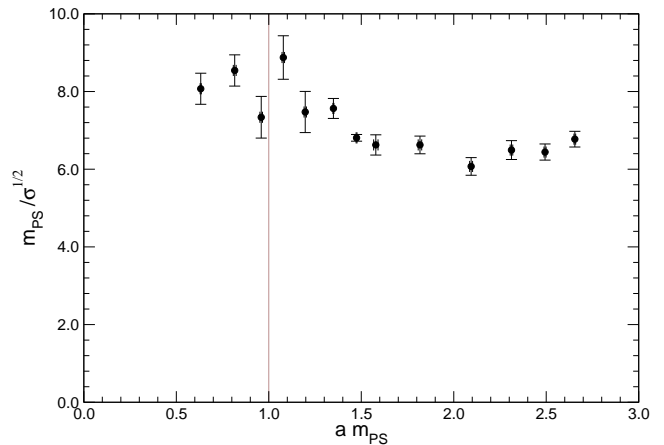


FIG. 2: Pseudoscalar mass in units of $\sqrt{\sigma}$ as a function of the pseudoscalar mass in units of a^{-1} . Points on the left of the vertical line at $aM = 1$ are expected to be reasonably free from finite lattice spacing effects.

degeneracy of the pseudoscalar and vector mesons was already observed in previous simulations, starting from Ref. [17]). Having a better control on the massless limit allows us to investigate more closely the degeneracy between the pseudoscalar and the vector meson observed in previous studies. Our data in Fig. 3 show that, as the fermion mass is reduced, the ratio m_V/m_{PS} progressively rises from 1 (which is the expected result in the heavy quark effective theory) to 1.04, where it seems to stabilize. Understanding whether this 4% variation is significant will require a more systematic study. To shed more light on this ratio, we performed a comparison with data obtained in quenched SU(2) simulations. The bare coupling and the fermion mass of the quenched theory need to be fine-tuned in such a way that the string tension and the pseudoscalar mass of the dynamical simula-

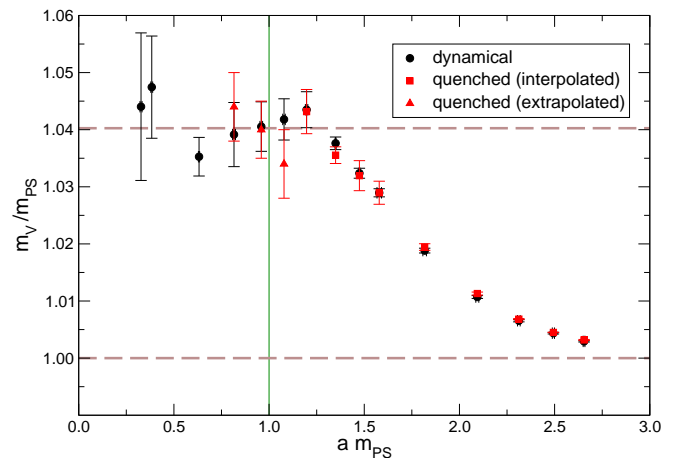


FIG. 3: The ratio of the vector mass m_V over the pseudoscalar mass m_{PS} as a function of the pseudoscalar mass. Quenched data at equivalent bare lattice parameters are also shown.

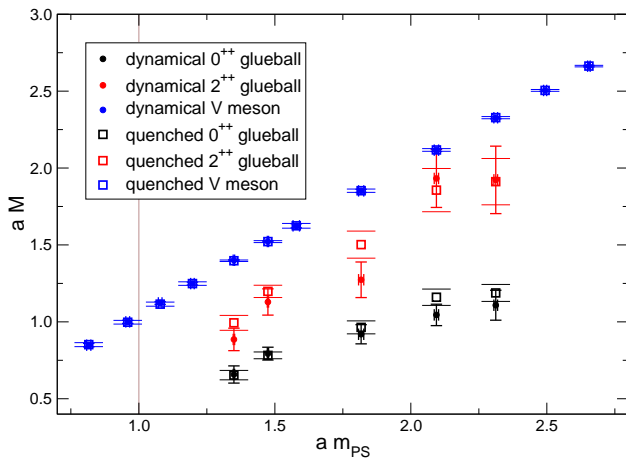


FIG. 4: Comparison of the lightest pseudoscalar and glueball masses in the quenched and in the dynamical theory as a function of the pseudoscalar mass.

tions are matched. The comparison in Fig. 3 shows that the splitting between the pseudoscalar and vector meson masses is due purely to gluonic effects. To see how much the spectrum resembles the pure Yang–Mills one, we have compared in Fig. 4 the glueball masses obtained from our simulations to the ones of pure gauge SU(2) after matching the string tensions. Data for the Yang–Mills theory are obtained by interpolating (or slightly extrapolating) the data in Ref. [30]. Once the string tension and the pseudoscalar mass are tuned, all the states that we studied are reproduced by the quenched data. Note that the value of the bare lattice coupling β needs to be adjusted as a function of the fermion mass used in the dynamical simulations. A possible explanation of this behavior is that the mesons decouple in the infrared, and the effective long–distance theory is SU(2) Yang–Mills with a hadronic scale smaller than the fermion mass of the full theory.

Another explanation for the features observed in the mesonic spectrum could be that the fermions are simply too massive and as a consequence the theory is quenched; however, this conclusion is unlikely to accommodate the observed hierarchy between mesonic and gluonic states in the spectrum, which seems to reflect the importance of fermion loops in the UV. In fact, the simultaneous presence of these two phenomena could be seen as a signature of a conformal point in the massless limit. Such a scenario has been proposed by Miransky [31], who has shown that in SU(N) gauge theories with fundamental fermions in the conformal phase but close to the higher end of the conformal window (where the infrared fixed point is perturbative) the low–energy effective spectrum coincides with the spectrum of the SU(N) Yang–Mills theory with a dynamically generated mass scale that is proportional to the pseudoscalar meson mass. The proportionality constant is exponentially small in the inverse of the squared gauge coupling at the IR fixed point. While the details of the calculation can not be trusted at the lower

end of the conformal window (in which our theory would be if an infrared fixed point existed), it is conceivable that features like the hierarchy in the spectrum and the suppressed infrared scale will survive also when a perturbative analysis of the physics near the infrared fixed point is not reliable.

In conclusion, our numerical data are the first systematic study of the chiral regime for the SU(2) gauge theory with two Dirac adjoint fermions. They support a scenario in which the spectrum is determined by a pure gauge dynamics, whose dynamically–generated scale *slides* with the fermion mass and is always well below the scale of the meson physics (the separation being about one order of magnitude). Moreover all the states in the mesonic spectrum become lighter as the fermion mass is decreased; the separation of the pseudoscalar Goldstone bosons from the rest of the spectrum, which would characterize the spontaneous breaking of chiral symmetry, is not observed. These peculiar features of the spectrum provide a stronger evidence in favor of the conformality of the theory in the massless limit.

Indeed if the behavior observed over the limited range of quark masses in the scaling region of our present simulations does extrapolate to the continuum and chiral limit without any significant qualitative difference, the conclusion that this theory lies in the conformal window will be natural.

At the moment, however our conclusions are limited by a number of factors: gluonic observables are very expensive to measure accurately at small quark masses so at present we only have two points with glueball masses below the cutoff scale. Moreover finite volume effects have to be under control even for the mesonic spectrum to be extrapolated with confidence to the small mass regime. This makes it very difficult to get closer to the chiral limit, and explicitly check that we are not in a heavy quark regime. Finally, we lack control on the continuum limit, since all of our numerical simulations were performed at a single lattice spacing. Scaling towards the continuum limit is beyond the scope of the present paper and it will be addressed by forthcoming simulations.

In view of these limitations a QCD–like scenario still remains possible, as well as the intermediate Walking scenario. At present however, taking into account the available information presented in this paper as well as in previous publications, the alternative which is more likely is that this theory is IR conformal. To put this statement on solid grounds, more accurate lattice studies are still necessary.

The discussion presented in this paper outlines a strategy to understand the phase of the massless theory in the continuum limit. Simulations will be extended to other values of the lattice spacing a , and smaller pseudoscalar masses, with the twofold aim to extrapolate the results to the continuum and to keep under control cutoff effects on the spectrum. This should allow us to resolve the issues related to the possibility of our constituent fermions being still too heavy, and of our spectrum being influ-

enced by an IR fixed point not related to the continuum theory. The comparison of the gluonic and the mesonic sector looks like a promising way to address the issue of conformality by lattice simulations. Finally, following the recent numerical studies of the conformal window in SU(3) gauge theory with fermions in the fundamental representation [32, 33, 34, 35, 36], we notice that applying a combined analysis of the meson and glueball spectrum to that theory at both ends of the conformal phase could help to clarify the physics of theories with IR fixed points.

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