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Citation for published version:

Digital Object Identifier (DOI):
10.1016/j.omega.2020.102327

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
Omega

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Optimal Design of Retailer-Prosumer Electricity Tariffs Using Bilevel Optimization

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Abstract. We compare various flexible tariffs that have been proposed to cost-effectively govern a prosumer’s electricity management—in particular time-of-use (TOU), critical-peak-pricing (CPP), and a real-time-pricing tariff (RTP). As the outside option, we consider a fixed-price tariff (FP) that restricts the specific characteristics of TOU, CPP, and RTP, so that the flexible tariffs are at least as profitable for the prosumer as the FP tariff. We propose bilevel models to determine the optimal interplay between the retailer’s tariff design and the prosumer’s decisions on using the storage, on consumption, and on electricity purchases from as well as electricity sales to the grid. The single-level reformulations of the considered bilevel models are computationally highly challenging optimization problems since they combine bilinearities and mixed-integer aspects for modeling certain tariff structures. Based on a computational study using real-world data, we find that RTP increases retailer profits, however, leads to the largest price volatility for the prosumer. TOU and CPP only yield mild additional retailer profits and, due to the multiplicity of optimal plans on the part of the prosumer, imply uncertain revenues for the retailer.

1. Introduction

In recent years, renewable and decentralized generation has increased worldwide. Households nowadays often consume and produce energy and, at the same time, due to innovations like the internet of things, there are ample opportunities for dynamic control of demand and supply within and beyond a single household. In particular, smart meters in combination with production, storage, and smart grid technologies will considerably broaden the possibilities of so-called “prosumers” to optimize their electricity consumption, production, and storage. In the context of buildings, also heat production from electricity (e.g., through heat pumps), as well as heat storage will gain more importance since energy efficiency is a main focus of the EU’s climate action programs [15]. Since also in the mid-term prosumers will cover parts of their electricity consumption via the grid, electricity retailers have new opportunities to raise additional revenue by offering tariffs that allow prosumers to shift their electricity consumption to periods with low wholesale electricity prices. Accordingly, various contributions highlight the scope of demand response in today’s electricity markets; see, e.g., [9, 17, 23, 28, 32, 43, 50].

In this contribution, we shed light on the scope of demand response in the interaction between a retailer and prosumers. In particular, we assess a retailer’s ability to raise additional profits through flexible tariffs. Our main focus is on the design of different types of tariffs, taking into account the specific load shifting potential of a typical prosumer. To that aim, we study the interaction of two agents. We consider an electricity retailer who buys electrical power on the wholesale market at the hourly fluctuating spot-market prices and then resells it to his customers. We consider a

\textsuperscript{1}Date: August 6, 2020.
\textsuperscript{2}2010 Mathematics Subject Classification. 90-XX, 90Bxx, 90C11, 90C90.
\textsuperscript{3}Key words and phrases. Electricity tariffs, Pricing, Bilevel optimization, Mixed-integer optimization, Tariff design.
customer who possesses some small-scaled energy generation and storage facilities; in particular a photovoltaic (PV) system and a battery as well as a heat pump and a heat storage unit. We call this customer a prosumer. The prosumer operates his domestic system optimally given the electricity prices that he faces if he buys electricity to cover residual load from the retailer. By modeling both electricity generation and storage as well as heat generation and storage, we provide a model of the prosumer on which future research can build on to capture more specific approaches in the context of sector coupling. In particular, this allows an analysis of how the possibility to store both heat and electricity at different efficiencies affects tariff design.

In order to adequately account for the specific incentives of different agents, we rely on bilevel modeling techniques. This allows to explicitly consider the decisions of the agents in the model but also implies challenges in solving the models.

In our analysis, we compare various tariffs with flexible pricing in order to assess their impact on the prosumer’s decisions on system operation and electricity consumption via the grid, as well as the attractiveness of the tariffs from the perspective of the retailer.

As a benchmark, we consider a fixed-price (FP) tariff in which the electricity price is constant over time. Alternative tariffs introduce various kinds of flexible pricing aimed at shifting electricity consumption to low price intervals. As alternative tariffs we focus on time-of-use pricing (TOU), real-time-pricing (RTP), and critical-peak-pricing (CPP) tariffs. Those are most prominently discussed both by policy makers (see, e.g., [54] and [53] for the U.S. and [16] for Europe) and in the scientific community (see, e.g., [29, 41, 57, 59]). The RTP tariff implies a simple pass-through of wholesale prices. In TOU pricing, peak and off-peak periods as well as the corresponding prices are explicitly agreed on in advance among the retailer and the prosumer. This is not the case for CPP. Although CPP defines possible peak periods in advance, it gives the retailer considerable leeway in setting peak prices. The prices can therefore vary between the individual peak periods and are not contractable up front (except for possibly a price cap). In order to provide a clean comparison of the different tariffs, we assume that the prosumer must be indifferent between all tariffs, which can be achieved by a (lump-sum) compensation between the retailer and the prosumers.

Our study provides a number of new and interesting insights on flexible pricing. First, we show that the simple pass-through of wholesale prices in the RTP tariff yields the highest additional profits for the retailer. However, a detailed modeling of the prosumer’s production and storage facilities reveals that the potential for additional efficiency gains through load shifting is limited. As compared to RTP, the CPP and TOU tariffs allow only moderate additional gains for the retailer. This is mainly due to the fact that the limited possibilities for load shifting in combination with the fixed time windows for peak prices imply that only little efficiency potential can be raised. On top of the low efficiency potential in the optimistic case, in the case of CPP and TOU there is considerable uncertainty for the retailer as to whether the possible additional profits can be fully realized due to multiplicity of optimal operation plans of the prosumer. In essence, the retailer has to bear the risk that the prosumer acts against him, which makes flexible tariffs less attractive.

Our results shed light on some aspects concerning flexible prosumer tariffs that imply challenges for tariff design and are partly in contrast to some other contributions in the literature. In particular, attractiveness of flexible tariffs is only mild against the background that (i) load shifting potential might be only moderate if detailed technical configurations are accounted for and that (ii) flexible tariffs imply multiple solutions of the prosumer’s optimization problem, which raises revenue uncertainty for the retailer.

The remainder of the paper is organized as follows. We start by discussing the related literature in detail in Sect. 2 and then state the general setup in Sect. 3.
In particular, we introduce the different tariff options in Sect. 3.2 and provide a detailed description of the prosumer’s problem to operate existing facilities optimally in Sect. 3.3. We then explicitly state the optimization problems for each tariff in Sect. 4 and conduct a computational study based on real-world data both for the demand profiles and the technical data of the installed technologies. The computational study and results are presented in Sect. 5 before we conclude the paper in Sect. 6.

2. Related Literature

A large body of literature empirically analyzes data from field experiments, providing vast evidence that flexible retail tariffs indeed induce reactions of consumers. Therefore, given adequate incentive structures, such tariffs may also influence consumption and production patterns of prosumers; see, e.g., [26, 27, 34, 57] for California, [20] for the case of Ireland, [3] for Sweden, [39] for Norway, and finally [12, 37, 44] for the case of Germany. Those studies strongly motivate our general research questions, whereas the methodological focus of our contribution lies in determining the specific incentives of different tariff structures for market participants.

Another strand of literature takes a system perspective and thoroughly analyzes the overall benefits that can potentially be obtained through flexible reaction of prosumers considering different types of demand side management; see, e.g., [9, 17, 23, 43, 47, 63]. These contributions focus on system optimal solutions that maximize the cumulative welfare of all agents involved regardless of the outcome for every single agent.

Other contributions consider interactions between different agents and analyze the potential of specific contracting and pricing choices for consumers offering demand response to improve operation and investment of the distribution network; see, e.g., [24, 25, 36, 40, 55, 62]. These contributions develop interesting network-tariff structures to provide price signals for flexible demand to properly reflect congestion arising in the distribution network. However, in contrast to our work, analysis of different retail tariffs is out of scope of these contributions.

Another branch of articles concentrates on risk management by retailers through analyzing the optimal portfolio for retailers regarding long- and short-term contracts with different types of providers of demand response; see, e.g., [6, 11] as well as [21, 22, 42] for more recent work. Whereas those contributions give detailed insights on the stochastic problems arising in the context of optimal portfolio choices and risk management, they have to abstract from modeling in detail the optimal reaction of customers to changed retail tariffs. On the other hand, our work incorporates modeling of the optimal operation of all prosumer’s facilities in response to the different retail tariffs proposed by the retailer.

Our models take into account different goals of every agent and we thus directly contribute to the literature that explicitly disentangles incentives of flexible consumers and their contract partners. This is typically implemented by relying on bilevel formulations, which allows to analyze the provision and contracting of demand response and flexible smart grid technologies from different perspectives. There are some papers that model retailer and prosumer separately as different agents in a bilevel setting to properly analyze the response of the prosumer to the incentives provided by the retailer. For example, [38] or the last model in [21] analyze how optimally chosen energy prices can provide proper incentives to providers of demand response. Some literature extends the agents’ incentives to a multi-leader-multi-follower game; see, e.g., [1, 2, 56]. Finally, [5, 41, 59, 60, 64] explicitly consider optimal design of different retail tariffs. They compare fixed-price tariffs with more flexible tariffs such as TOU or time-and-level-of-use pricing for consumers who also offer demand response.

Our approach extends those studies in mainly two ways. First, when analyzing optimal tariff structures, prosumers are modeled as followers on lower levels of the multi-level models like in [5, 21, 38, 41, 59]. To the best of our knowledge, however, all
those studies of tariff design in a bilevel setting are limited to simplified formulations of the model for demand response of end customers such as load curtailment and load shifting. In contrast, we explicitly model physical smart grid technologies such as electricity generation and storage in combination with heat production and storage on the lower level. The explicit modeling of the interaction of electricity and heating technologies at the lower level of a bilevel problem makes our approach more complex and also computationally harder to solve compared to simplified formulations on the lower level or the modeling of these technologies on the upper level. The integrated analysis of power and heat in the household is particularly interesting against the background that, in recent years, energy efficiency and the integration of batteries, heat pumps, and heat storage in integrated energy systems has gained increasing attention in EU-wide and national policy initiatives; see, e.g., [14] or [15].

Note that consumers who provide demand response based on available smart grid technologies like, e.g., battery, heat storage unit, heat pumps, own electrical energy generators, etc., are of increasing importance when thinking about smart grids and new retail tariffs.

Second, we additionally analyze critical-peak-pricing. For CPP, we explicitly have to disentangle those components of the retail tariff that are already fixed at the time of signing the retail contract (mid-term perspective) and those which can be decided by the retailer after signing the contract (short-term perspective). This is particularly important in the context of flexible tariffs. Our analysis properly combines those aspects and thus, to the best of our knowledge, is the first contribution in the context of retail tariff design that brings together these two different timescales. We are thus convinced that our analysis provides an important contribution to this ongoing and highly relevant literature.

3. General Framework

We consider a retailer who buys electricity at the spot market and resells it to his local prosumers. To this end, a contract between the retailer and the prosumer needs to be signed that specifies the type of energy supply tariff and the details of the chosen tariff.

In this section, we first introduce some preliminary notation in Sect. 3.1 and then describe the main principles of the four different considered tariff types in Sect. 3.2. Afterward, the prosumer with all associated facilities is described in Sect. 3.3. Based on these descriptions, the resulting mathematical optimization models are developed in Sect. 4.

3.1. Preliminaries. We consider a fixed time horizon \([0, t_e] \subset \mathbb{R}\) modeling the time span of the considered energy supply contracts. This time horizon is discretized as \(T = \{t_0, t_1, \ldots, t_k\}\) with \(t_0 = 0\) and \(t_k = t_e\) representing start and end of the relevant time span, e.g., one year. Thus, we have \(k + 1\) time points and \(k\) time periods with period lengths \(\tau_j := t_j - t_{j-1}, j = 1, \ldots, k\). Here, we consider an equidistant discretization with \(\tau = \tau_j\) for all \(j = 1, \ldots, k\). The discretized time horizon without the first period is denoted by \(T^0 := T \setminus \{t_0\}\). We associate each interval \([t_{j-1}, t_j]\) with its end point \(t_j\) and use \(t_j\) to denote the corresponding time period.

We assume that the time horizon exhibits some kind of periodicity and subdivide \(T\) accordingly into time intervals \(T = \bigcup_{i=1}^I T_i\) with \(T_i\) representing, e.g., days of the year. Thus, we assume that these intervals are equally long. This subdivision is also applied to the discretized horizon so that we obtain \(T_i = \{t_{i1}, \ldots, t_{in}\}\), \(i = 1, \ldots, I\), where each discretized subinterval has its start point \(t_{i1}\) and end point \(t_{in}\). Thus, it holds \(t_{i1} = t_0\) and \(t_{in} = t_e\). Each of the intervals \(T_i\), \(i = 1, \ldots, I\), comprises \(N\) time periods.

We remark that, if not stated explicitly otherwise, all quantities indexed with \(t\) have to be interpreted as quantities averaged over the corresponding time period.
3.2. Retailer and Tariff Descriptions. The retailer buys electricity at the spot market and faces the corresponding spot-market prices $\gamma_{\text{spot}}$. Afterward, this electricity is resold to the prosumer. The details of this reselling are defined in corresponding energy supply contracts (which we call tariffs in the following) between the retailer and the prosumer. In this paper we consider four different types of tariffs: (i) a conventional FP tariff (FP), (ii) a time-of-use tariff (TOU), (iii) a real-time-pricing tariff (RTP), and (iv) a critical-peak-pricing tariff (CPP).

Note that currently the most prevalently used electricity retail tariffs are FP tariffs and, to some extent, traditional TOU tariffs that are best known as day-night pricing. In the context of modern metering and information technologies, more dynamic time-based retail tariff structures are emerging. Here, RTP and CPP are the most prominent ones. These are the tariffs that are most frequently discussed in the scientific community; see, e.g., [29, 41, 57, 59]. Moreover, they are currently proposed as the most promising candidates for a large-scale implementation of dynamic electricity pricing both by governments in the US (see, e.g., [54] and [53]) as well as in the EU (see [16]). In fact, we already observe retail companies offering these tariffs in real-life electricity markets both in the EU [49] as well as in the US [48].

To compare these tariffs, we will compare each of the flexible tariffs with the conventional tariff. The conventional tariff offers the same fixed electricity price throughout the entire time horizon. In the following we will use the terms conventional tariff and FP tariff as synonyms. For our comparisons, we always assume that the prosumer has the choice between the conventional tariff and the respective flexible tariff. Since the prosumer has the conventional tariff as an outside option, the retailer has to make the prosumer indifferent between the conventional and the flexible tariff. Note that with a TOU tariff this implies that the peak and the off-peak prices have to be set such that, given expected peak and off-peak consumption, the prosumer bears the same cost of electricity consumption as under the conventional tariff. Through better adjustment of prosumers’ energy management pattern to electricity market prices, an efficiency gain will be introduced by the TOU tariff, which is earned by the retailer. The same is the case for real-time-pricing, where this efficiency gain is captured by the retailer in form of a lump-sum payment that the prosumer has to pay up-front if he chooses the RTP tariff. In the case of the CPP tariff, the retailer retains considerable flexibility with regard to pricing in peak periods. This exposes the prosumer to the risk that the sum of the volume-related payments will by far exceed the payments in the case of the conventional tariff. The retailer could only make the prosumer indifferent between CPP and the conventional tariff by taking away the pricing leeway in peak periods by means of a very restrictive cap on peak prices. This is, however, contrary to the spirit of the CPP tariff. In our analysis, we thus assume lump-sum compensations up-front to possibly induce switching to the CPP tariff.

3.2.1. Fixed-Price Tariff. In the FP tariff, the prosumer pays a fixed baseline price $\gamma_{\text{base}}$ for each MWh of electrical energy throughout the entire time horizon $T$. In this case the baseline price $\gamma_{\text{base}}$ is the only contractual element that the retailer and the prosumer need to agree on. Throughout this article we will consider an exogenously fixed level of the baseline price $\gamma_{\text{base}}$ obtained from our data; see Sect. 5.1.

The overall profit of the retailer is given by

$$ f_{\text{Conv}} := \sum_{t \in T} \left( \gamma_{\text{base}} - \gamma_{\text{spot}} \right) P_{\text{im}}^t, $$

where $P_{\text{im}}^t$ is the amount of electrical energy purchased by the prosumer from the retailer at time $t$. Note that in case of the FP tariff, the retailer does not take any decisions. This tariff serves as an important benchmark, however, in the following way. We construct all subsequent tariffs exactly such that they induce the very same
final profit for the prosumer, leaving the prosumer indifferent between choosing the FP tariff or any of the subsequently proposed tariffs.

3.2.2. Real-Time-Pricing Tariff. In contrast to the FP tariff, in a real-time-pricing tariff the prosumer agrees to directly pay the hourly day-ahead spot-market electricity prices for his consumption. These prices are exogenously specified parameters in the RTP tariff and are not controlled by the retailer. However, the retailer obtains his profits from the RTP tariff by specifying a lump-sum payment $\rho_{\text{RTP}}$. Within our framework the retailer can optimally choose $\rho_{\text{RTP}}$, but he is restricted by the fact that the FP benchmark tariff is an outside option for the prosumer. As the retailer simply passes on the spot-market prices to the prosumer without any extra charges, the entire profit of the retailer consists of the lump-sum payment made by the prosumer. We define that for $\rho_{\text{RTP}} > 0$ the payment is received by the retailer, whereas for $\rho_{\text{RTP}} < 0$ it is received by the prosumer. Thus, the retailer’s objective function reads

$$f_{\text{RTP}} := \rho_{\text{RTP}}.$$ (2)

For a discussion and comparison of the specific timing relevant for the different tariffs we refer to Sect. 3.2.5.

3.2.3. Time-of-Use Tariff. High prices and peak electricity demand are usually periodic and thus associated with particular daytimes. As noted before, we consider a time horizon $T$ (corresponding to, e.g., one year), which is divided into time-repeating intervals $T_i$ (representing, e.g., days). In time-of-use tariffs, the retailer determines a further splitting of these sub-intervals $T_i$ into a peak time and an off-peak time, i.e. $T_{i,\text{peak}} \cup T_{i,\text{off-peak}} = T_i$ with peak part $T_{i,\text{peak}}$ and off-peak part $T_{i,\text{off-peak}}$ of day $i$. To stick as close as possible to the currently used TOU tariffs in reality, we focus on the case in which we only have two switches between off-peak and peak times throughout one day. That is, we assume that at least one of these intervals, i.e., either $T_{i,\text{peak}}$ or $T_{i,\text{off-peak}}$, has to be connected. These peak and off-peak periods are assumed to be the same for every day of the year; see, e.g., [54] or [16].

Moreover, the retailer sets a TOU-specific baseline price $\gamma_{\text{ret}} \in \mathbb{R} \geq 0$, $\gamma_{\text{ret}} \leq \gamma_{\text{base}}$, for the entire time horizon and adds a markup $\gamma_{\text{tou}} \in \mathbb{R} \geq 0$ for each time block $T_{i,\text{peak}}$. The baseline price $\gamma_{\text{ret}}$ is paid by the prosumer in off-peak time periods while the price $\gamma_{\text{ret}} + \gamma_{\text{tou}}$ applies in peak time periods. Note that $\gamma_{\text{tou}}$ is independent of $i$ and $t$, i.e., the markup is the same for all peak time periods of the entire time horizon. To allow for a direct comparability of the different tariffs we furthermore impose that the prosumer has to be at least indifferent between choosing the TOU tariff as compared to the benchmark FP tariff. All details of the TOU tariff are specified and contracted already at the time of signing the TOU contract; see Sect. 3.2.5 for a detailed illustration of the specific timing of all tariffs.

In sum, the retailer, at the time of signing the contract, optimally chooses the partition of the day into peak and off-peak time periods as well as the baseline price $\gamma_{\text{ret}}$ and the markup $\gamma_{\text{tou}}$ such as to leave the prosumer indifferent as compared to the FP tariff. After contract signature, no further decisions are taken by the retailer.

The profits of the retailer are given by total revenues from electrical power sold to the prosumer minus total costs of this power bought at the spot market:

$$f_{\text{TOU}} := \gamma_{\text{ret}} \sum_{t \in T} P_{\text{im}}^t + \gamma_{\text{tou}} \sum_{i=1}^I \sum_{t \in T_{i,\text{peak}}} P_{\text{im}}^t - \sum_{t \in T} \gamma_{\text{spot}}^t P_{\text{im}}^t.$$ (3)

Note that the TOU tariff with $\gamma_{\text{ret}} = \gamma_{\text{base}}$ and $\gamma_{\text{tou}} = 0$ is equivalent to the FP tariff.

3.2.4. Critical-Peak-Pricing Tariff. Let us now describe the last considered tariff: the CPP tariff. In this tariff, the retailer again tries to exploit the periodicity of electricity demand and prices, but maintains some flexibility also after signing of the contract.
As before, all sub-intervals $T_i$ of the time horizon $T$ are split into peak $T_{i,\text{peak}} \subset T_i$ and off-peak $T_{i,\text{off-peak}} \subset T_i$ time periods, i.e., $T_i = T_{i,\text{peak}} \cup T_{i,\text{off-peak}}$ for all $i = 1, \ldots, I$.

This partition of the day into peak and off-peak time is identical for each day.

Similar to the TOU tariff, at the time of signing the CPP contract the retailer and the prosumer agree on a baseline price, which applies during off-peak times. Unlike the TOU tariff, however, at the time of signing the CPP contract the retailer and the prosumer do not fix the specific level of the extra markup charged during peak times. In the contract, they agree that later on flexible markups $\gamma_{\text{cpp}}^t$ with an upper bound $\bar{\gamma}_{\text{cpp}} \geq \gamma_{\text{cpp}}^t$ can be announced by the retailer in peak-time periods $t \in T_{i,\text{peak}}$, $i = 1, \ldots, I$. More formally, we obtain
\[
\gamma_{\text{cpp}}^t = 0 \quad \text{for all } t \in T_{i,\text{off-peak}}, \quad i = 1, \ldots, I, \quad \text{and} \quad \gamma_{\text{cpp}}^t \leq \bar{\gamma}_{\text{cpp}} \quad \text{for all } t \in T.
\]

The precise markups can be announced flexibly by the retailer long after signing the retail contract. In the TOU tariff, on the contrary, the retailer does not have this degree of freedom but has to fix a constant markup for all peak time periods already at the time of signing the contract. The CPP tariff is thus considered to allow the retailer to have considerably more flexibility by allowing him to directly react to changing conditions in electricity markets; see, e.g., [58] or [30].

The optimal design of all components of the CPP tariff in principle results in a both theoretically and computationally rather involved trilevel problem: First, all contract details are determined optimally such as to guarantee indifference of the prosumer. Second, after contract signature, peak prices are determined by the retailer. Finally, the prosumer takes all operational decisions. To keep our computations tractable, we have to exogenously fix those components of the CPP contract, which have a direct impact on the decisions on the two later stages. This allows us to solve the overall problem as a bilevel problem; see also Section 4.5. In our setup, we thus have to assume that the partition in peak and off-peak times as well as the baseline price and the upper bound for peak prices $\bar{\gamma}_{\text{cpp}}$ are given exogenously. (In our computational analysis in Section 5 we will use the partition that is optimal for the TOU tariff). This clearly implies that the specific CPP tariff determined is not the best possible CPP tariff and a CPP tariff for which all components are chosen optimally might perform slightly better.

As for the RTP and the TOU tariff, we also construct the CPP tariff such that the prosumer is indifferent as compared to choosing the conventional FP tariff. To maintain tractability, as discussed above, in case of the CPP tariff the retailer has to specify a lump-sum compensation $\rho_{\text{CPP}}$ up-front in the CPP energy supply contract. We define that for $\rho_{\text{CPP}} > 0$ the payment is received by the retailer, whereas for $\rho_{\text{CPP}} < 0$ it is received by the prosumer.

The profits of the retailer are given by total revenues from electricity sold to the prosumer minus total spot-market costs, i.e.,
\[
f_{\text{CPP}} := \sum_{t \in T} (\gamma_{\text{base}} + \gamma_{\text{cpp}}^t) P_{\text{im}}^t - \sum_{t \in T} \gamma_{\text{spot}}^t P_{\text{im}}^t,
\]
plus the lump-sum payment, i.e., $f_{\text{CPP-all}} := f_{\text{CPP}} + \rho_{\text{CPP}}$.

For a detailed illustration of the specific timing of the CPP tariff as compared to the other tariffs see Sect. 3.2.5. The explicit mathematical model is stated in Sect. 4.5. Obviously, the CPP tariff with $\gamma_{\text{cpp}}^t = 0$ for all $t \in T$ is equivalent to the FP tariff.
Figure 1. Timeline of decision making. Retailer and prosumer choices are printed in regular letters, parameters are shaded gray.
3.2.5. Discussion of the Tariffs. Before we introduce the mathematical optimization models of the four tariffs, let us first provide a quick survey of the timing of the decision-making processes for the different tariffs shown in Figure 1. Decisions of the retailer regarding the different tariff parameters are all taken before the start of the supply-time horizon, i.e., at the time of signing the supply contracts. In case of the flexible CPP tariff, however, the retailer additionally specifies peak prices $\gamma_{cpp}^t$, which are announced to the prosumer after contract signature on a day-ahead basis. Once all tariff details are known by the prosumer, he can proceed with optimizing his day-ahead operation plans. The analysis of our setup thus results in a sequence of linked optimization problems. The retailer decides on different tariff structures by anticipating the prosumer’s purchase and operational decisions, which take place later on. Thus, we will consider bilevel models in which the prosumer’s decisions are always considered at the lower level; see also Section 4.

Note that in our setup, the conventional FP tariff serves as a benchmark regarding prosumer profits. That is, we explicitly construct the other tariffs such that they induce the same final profits for the prosumer. Thus, the prosumer will be indifferent between signing either of the proposed supply contracts. In case of the TOU tariff, we directly obtain indifference by optimally adjusting the baseline price $\gamma_{ret}$ and the TOU markup $\gamma_{tou}$. In case of the RTP and CPP tariff, this is obtained by appropriately adjusting the level of the lump-sum payments $\rho_{RTP}$ and $\rho_{CPP}$. Since the prosumer is kept indifferent, the final profit differences resulting for the retailer for the different tariffs should be interpreted as the potentially obtained overall benefit, which, in practice, will be shared in one way or another according to the prevailing competitive situation at the market or, e.g., according to varying risks induced by the different tariffs in the context of risk sharing. For a more detailed discussion of possible implications in case of risk and uncertainty see Section 4.6.

3.3. Prosumer. The prosumer signs a contract for a specific electricity supply tariff and then controls his facilities to cover his overall demand for electricity and heat in a way that maximizes his overall profit obtained from selling PV-generated electrical energy. The model describes electricity production, storage, and consumption as well as heat production, storage, and consumption together with the optimal coupling of these two energy systems. It is a particular feature of our approach that we explicitly consider the interaction of electricity and heat production as well as storage within the household. This allows us to directly assess the current political tendencies to promote energy efficiency by integrating households’ heat and electricity systems; see, e.g., [14], or [15]. Our setup thus allows to explicitly analyze how the operation of the different components interact with the different tariff structures.

3.3.1. Electricity Production (Photovoltaic System). Electricity production of the prosumer is assumed to take place by a rooftop photovoltaic (PV) power station. This PV station generates electricity $P_{pv}^t \geq 0$ during time period $t \in T$ that the prosumer can either consume directly, store in the battery, or feed into the main grid. The nominal capacity $P_{pv}$ of the considered PV module provides an upper bound for the power output, i.e., $P_{pv}^t \leq \tau P_{pv}$ for all $t \in T$. A list of all controllable and given quantities of the PV, as well as of all other facilities, is given in Table 1 at the end of this section.

Since PV production mainly depends on solar radiation, it cannot be controlled by the prosumer. Thus, $P_{pv}^t$ is a given parameter. On the other hand, the amount of PV generated power $P_{pv-ex}^t$ that is fed into the main grid during each time period $t \in T$ can be controlled and is subject to the condition

$$P_{pv-ex}^t \leq P_{pv}^t \quad \text{for all} \quad t \in T. \quad (6)$$

3.3.2. Electricity Storage (Battery). Electricity can be stored in a battery to better manage the prosumer’s overall energy system such as, e.g., to store solar generated
energy or electricity purchased from the retailer when it is available at a cheaper price. The battery can either be charged ($P_{\text{bat-c}}^t > 0$) or discharged ($P_{\text{bat-d}}^t > 0$). For both cases, specific upper bounds limit the amount of charged and discharged power, i.e.,

$$P_{\text{bat-c}}^t \leq \tau P_{\text{bat-c}}^t, \quad P_{\text{bat-d}}^t \leq \tau P_{\text{bat-d}}^t \quad \text{for all } t \in T. \quad (7)$$

Moreover, the power balance in the battery over time is given by

$$E^t = (1 - \alpha_{\text{bat}})E^{t-1} = \frac{P_{\text{bat-d}}^{t-1}}{\eta_{\text{bat-d}}} + \eta_{\text{bat-c}}P_{\text{bat-c}}^{t-1} \quad \text{for all } t \in T^0, \quad (8)$$

where $E^t$ is the amount of electrical energy stored in the battery at the beginning of time period $t$. $\alpha_{\text{bat}} \in [0, 1]$ is the self-discharge rate of the battery, and $\eta_{\text{bat-c}}, \eta_{\text{bat-d}} \in (0, 1]$ are the efficiencies of the charging and discharging process.

Finally, electricity stored in the battery is limited by its capacity $\bar{E} > 0$, i.e.,

$$0 \leq E^t \leq \bar{E} \quad \text{for all } t \in T. \quad (9)$$

3.3.3. Heat Production (Heat Pump). For heat production, we assume that the prosumer owns a heat pump that uses electrical power to provide heat at some desired temperature level. We denote the amount of electrical power that the heat pump consumes by $P_{\text{h-pump}}^t \geq 0$. The amount of the resulting heat power that the heat pump delivers to the heat storage depends on its coefficient of performance $\eta_{\text{h-pump}}$, which is considered in Equation (12) in Sect. 3.3.4. We denote the maximum power output of the heat pump by $\bar{P}_{\text{h-pump}}$, i.e.,

$$P_{\text{h-pump}}^t \leq \tau \bar{P}_{\text{h-pump}} \quad \text{for all } t \in T. \quad (10)$$

3.3.4. Heat Storage (Heat Storage Tank). For heat storage, we assume that the prosumer can use a thermal water tank as a heat storage unit (HSU) to store heat. Depending on the capacity of the tank as well as on the minimum and maximum water temperature, the stored heat $H^t$ is bounded from below and above:

$$0 \leq H \leq H^t \leq \bar{H} \quad \text{for all } t \in T. \quad (11)$$

Charging ($P_{\text{hsu-c}}^t > 0$) of the HSU in time period $t$ is modeled by

$$P_{\text{hsu-c}}^t = \eta_{\text{h-pump}}P_{\text{h-pump}}^t \quad \text{for all } t \in T, \quad (12)$$

where $\eta_{\text{h-pump}}$ is the coefficient of performance and $P_{\text{h-pump}}^t$ is the electricity consumed by the heat pump. Finally, the heat balance in the HSU over time is modeled similarly to the energy balance in the battery, i.e.,

$$H^t = (1 - \alpha_{\text{hsu}})H^{t-1} = \frac{P_{\text{hsu-d}}^{t-1}}{\eta_{\text{hsu-d}}} + \eta_{\text{hsu-c}}P_{\text{hsu-c}}^{t-1} \quad \text{for all } t \in T^0. \quad (13)$$

Here, $H^t$ is the heat stored in the HSU at the beginning of time period $t$, $\alpha_{\text{hsu}} \in [0, 1]$ is the self-discharge rate of the HSU, and $\eta_{\text{hsu-c}}, \eta_{\text{hsu-d}} \in (0, 1]$ are the respective charging and discharging efficiencies.

3.3.5. Balance Equations. We finally impose general energy and heat balance equations of the considered system. Electrical energy balance reads

$$0 \leq P_{\text{im}}^t = P_{\text{le}}^t + P_{\text{h-pump}}^t + P_{\text{bat-c}}^t - P_{\text{bat-d}}^t + P_{\text{pv-ex}}^t - P_{\text{pv}}^t \quad \text{for all } t \in T, \quad (14)$$

where $P_{\text{im}}^t \geq 0$ is the power purchased by the prosumer from the retailer in time period $t$ and $P_{\text{le}}^t$ is the total electricity demand in time period $t$. We assume the prosumer’s heat demand to be covered entirely by the HSU at each corresponding time period. Thus, heat balance is modeled by

$$P_{\text{fh}}^t = P_{\text{hsu-d}}^t \quad \text{for all } t \in T, \quad (15)$$

where $P_{\text{fh}}^t$ is the heat load of the prosumer in time period $t$. 

Table 1. PV, battery, heat storage unit, and heat pump variables (top) and parameters (bottom)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{in}}$</td>
<td>Power imported from the main grid</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$P_{\text{pv-ex}}$</td>
<td>PV power fed into main grid</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$E$</td>
<td>Energy stored in the battery</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$P_{\text{bat-c}}$</td>
<td>Power charged to the battery</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$P_{\text{bat-d}}$</td>
<td>Power discharged from the battery</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$H$</td>
<td>Heat power stored in the HSU</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$P_{\text{hsu-c}}$</td>
<td>Heat power charged to the HSU</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$P_{\text{hsu-d}}$</td>
<td>Heat power discharged from the HSU</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$P_{\text{h-pump}}$</td>
<td>Power required to run the heat pump</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$\overline{P}_{\text{pv}}$</td>
<td>Nominal capacity of the PV module</td>
<td>MW</td>
<td>0.01008</td>
</tr>
<tr>
<td>$P_{\text{pv}}$</td>
<td>Power generated by the PV module</td>
<td>MWh</td>
<td>—</td>
</tr>
<tr>
<td>$\gamma_{\text{pv}}$</td>
<td>Feed-in tariff for PV-generated electricity</td>
<td>EUR/MWh</td>
<td>125</td>
</tr>
<tr>
<td>$\eta_{\text{bat-c}}$</td>
<td>Charging efficiency of the battery</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>$\eta_{\text{bat-d}}$</td>
<td>Discharging efficiency of the battery</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>$\overline{P}_{\text{bat-c}}$</td>
<td>Maximum charge capacity of the battery</td>
<td>MW</td>
<td>0.0046</td>
</tr>
<tr>
<td>$\overline{P}_{\text{bat-d}}$</td>
<td>Maximum discharge capacity of the battery</td>
<td>MW</td>
<td>0.0046</td>
</tr>
<tr>
<td>$\overline{E}$</td>
<td>Maximum capacity of the battery</td>
<td>MWh</td>
<td>0.0135</td>
</tr>
<tr>
<td>$\alpha_{\text{bat}}$</td>
<td>Self-discharge rate of the battery</td>
<td>1</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\eta_{\text{hsu-c}}$</td>
<td>Charging efficiency of the HSU</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{\text{hsu-d}}$</td>
<td>Discharging efficiency of the HSU</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{\text{h-pump}}$</td>
<td>Performance coefficient of the heat pump</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha_{\text{hsu}}$</td>
<td>Self-discharge rate of the HSU</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>$\hat{H}$</td>
<td>Initial &amp; minimal final heat in the HSU</td>
<td>MWh</td>
<td>$\hat{H}$</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>Initial &amp; minimal final energy in the battery</td>
<td>MWh</td>
<td>0</td>
</tr>
<tr>
<td>$\overline{H}$</td>
<td>Minimum energy level of the HSU</td>
<td>MWh</td>
<td>0.017919</td>
</tr>
<tr>
<td>$\overline{\dot{H}}$</td>
<td>Maximum energy level of the HSU</td>
<td>MWh</td>
<td>0.033276</td>
</tr>
<tr>
<td>$\overline{P}_{\text{h-pump}}$</td>
<td>Maximum power output of the heat pump</td>
<td>MW</td>
<td>0.006</td>
</tr>
</tbody>
</table>

3.3.6. Objective Function. The goal of the prosumer is to maximize net profit, i.e., total revenues from electricity produced by the PV unit that is fed into the main grid minus total costs for electricity bought from the retailer. With the feed-in price $\gamma_{\text{pv}}$ of the PV generated energy, the prosumer’s revenues from selling solar generated electricity are given by

$$f_{\text{rev}} := \gamma_{\text{pv}} \sum_{t \in T} P_{\text{pv-ex}}^t.$$ 

For each MWh purchased from the retailer the prosumer has to pay a fixed amount of taxes and fees $\gamma_{\text{tax}}$ to the authorities, i.e.,

$$f_{\text{tax}} := \gamma_{\text{tax}} \sum_{t \in T} P_{\text{in}}^t.$$ 

Finally, the prosumer buys electrical power from the retailer and pays for it according to the particular tariff. We denote this part of the prosumer’s objective function by $f_{\text{im}}$ and specify it further in the Sections 4.2–4.5 since it depends on the specifically chosen tariff.

Thus, the overall optimization goal of the prosumer is to maximize

$$f_{\text{low}} := f_{\text{rev}} - f_{\text{tax}} - f_{\text{im}}.$$  (16)
This objective function will later be used as (part of) the objective function of the lower level in bilevel models. This is why we already use the index “low”.

Note that all controllable and exogenously given quantities of the prosumer’s facilities are listed in Table 1. If specific values are given, their origin is discussed in Sect. 5.1.

4. Optimization Models

In this section we collect the descriptions of the tariffs and the prosumer’s facilities of the last section and state a specific optimization problem for every tariff. A simplified summary of the models used is given in Figure 2. We start by describing the optimization problem that the prosumer faces after a specific tariff has been chosen. This is done in Sect. 4.1. Afterward, we combine this prosumer model with tailored models for the specific tariffs to obtain models for the entire situation combining the decisions of the retailer and the prosumer. Depending on the specific tariff, the problem might be a single-level optimization or a multilevel problem. In addition, it also depends on the specific tariff whether we need to incorporate mixed-integer aspects in the model or not.

Note that the modeling of the design problem for the different tariffs as described in the previous section only results in a single- or bilevel problem, if we impose the assumption of perfect foresight for all agents. That is, throughout our formal analysis we assume that all parameters of our setup are known by the agents at the time of their decision making. Without this assumption of perfect foresight, the analysis of the above described tariffs results in highly complicated stochastic multilevel optimization problems, which are far from being tractable in practice.

Under perfect foresight all model parameters including the spot-market prices, electricity and heat demand, as well as PV power output of the prosumer are known beforehand. This allows us to solve the entire sequence of optimization problems of the retailer and the prosumer after the signing of the contract simultaneously. Consequently, the lump-sum payments \( \rho_{\text{RTP}} \) and \( \rho_{\text{CPP}} \) that are due at the contract signing stage can be determined with the full knowledge of the prosumer’s optimal operating plan for the entire time horizon. The same is valid for all variables of the retailer in each of the four tariffs—in particular also for \( \gamma_{\text{cpp}} \).

As mentioned above, we obtain either single- or bilevel optimization problems depending on the tariff that we analyze. The resulting problems are challenging, but we will later show that we can still solve them for instances of relevant size.

4.1. The Prosumer’s Linear Optimization Problem. In this section we formulate a linear optimization model (LP) for the prosumer, which is based on the descriptions given in Sect. 3.3. In addition to the equations given there we furthermore need the following constraints that allow us to state the complete model for the prosumer problem.

Both for the battery and the heat storage unit we require initial and terminal conditions to avoid undesired finite horizon effects:

\[
H^{t_0} = \bar{H}, \quad \dot{H} \leq (1 - \alpha_{\text{hsu}})H^{t_e} + \eta_{\text{hsu-c}}P_{\text{hsu-c}}^{t_e} - \frac{P_{\text{lh}}^{t_e}}{\eta_{\text{hsu-d}}} \leq \ddot{H}, \quad (17)
\]

and

\[
E^{t_0} = 0, \quad \dot{E} \leq (1 - \alpha_{\text{bat}})E^{t_e} - \frac{P_{\text{bat-d}}^{t_e}}{\eta_{\text{bat-d}}} + \eta_{\text{bat-c}}P_{\text{bat-c}}^{t_e} \leq \ddot{E}. \quad (18)
\]

Recall that \( H^t \) and \( E^t \) denote the amount of power stored at the beginning of the corresponding time period \( t \); cf. Sect. 3.3.2 and 3.3.4.
### Figure 2

Overview over the optimization models used to model the tariffs. The dashed lines denote the boundary of the different levels of the optimization models.

<table>
<thead>
<tr>
<th>Prosumer</th>
<th>Conventional</th>
<th>RTP</th>
<th>TOU</th>
<th>CPP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max $f_{low}$</td>
<td>max $f_{RTP}$</td>
<td>max $f_{TOU}$</td>
<td>max $f_{CPP}$</td>
</tr>
<tr>
<td></td>
<td>s.t. PV model</td>
<td>s.t. Competition constraint</td>
<td>s.t. Competition constraint</td>
<td>s.t. CPP markup bounds</td>
</tr>
<tr>
<td></td>
<td>Battery model</td>
<td>TOU model</td>
<td>TOU model</td>
<td>Battery model</td>
</tr>
<tr>
<td></td>
<td>Heat pump and HSU model</td>
<td></td>
<td>Heat pump and HSU model</td>
<td>Heat pump and HSU model</td>
</tr>
<tr>
<td></td>
<td>Balance equations</td>
<td></td>
<td>Balance equations</td>
<td>Balance equations</td>
</tr>
</tbody>
</table>
The above constraints (17) and (18) impose conditions on the state of charge at the beginning and the end of the time horizon. Here, we claim that the final state of charge of the HSU and the battery should be at least the amount stored at the beginning of the time horizon and not larger than the maximal capacity.

Throughout this paper we assume that in any given time period \( t \) the battery can either be charged or discharged, but not both at the same time:

\[
P_{\text{bat-c}}^t = 0 \quad \text{or} \quad P_{\text{bat-d}}^t = 0 \quad \text{for all} \quad t \in T.
\]

(19)

It can be shown that, if \( \eta_{\text{bat-c}} < 1 \) or \( \eta_{\text{bat-d}} < 1 \), every optimal solution of the prosumer’s problem satisfies this assumption. As our data satisfies the above stated condition, i.e., the battery does incur power losses in the charge/discharge process, no additional mixed-integer techniques are required to model the disjunction in (19).

We are now ready to state the complete linear model of the prosumer’s optimization problem:

\[
\begin{align*}
\max & \quad f_{\text{low}} \\
\text{s.t.} & \quad \text{PV model: (6)}, \\
& \quad \text{battery model: (7)–(9), (18)}, \\
& \quad \text{heat pump and HSU model: (10)–(13), (17)}, \\
& \quad \text{balance equations: (14), (15)}. \\
\end{align*}
\]

(20a–e)

Note that all constraints are linear. The linearity of the objective function depends on the specific tariff model, which we discuss next.

4.2. The Entire Optimization Problem for the Fixed-Price Tariff. As already discussed in Sect. 3.2.1, the conventional tariff’s baseline price \( \gamma_{\text{base}} \) is given exogenously as the average price of such tariffs in Germany. Given this baseline price, the costs of the prosumer for electricity—as part of the objective function in (16)—are given by

\[
f_{\text{im}} = f_{\text{im}}^{\text{conv}} = \gamma_{\text{base}} \sum_{t \in T} P_{\text{im}}^t,
\]

i.e., the prosumer’s objective function reads

\[
f_{\text{low}} = f_{\text{conv}}^{\text{low}} = f_{\text{rev}} - f_{\text{tax}} - f_{\text{im}}^{\text{conv}}.
\]

(21)

As the retailer cannot make any decisions in this case, the solution in case of the FP tariff is obtained by simply solving the resulting single-level problem of the prosumer, determining the optimal operation of his facilities. This optimization problem is given by

\[
\begin{align*}
\max & \quad f_{\text{conv}}^{\text{low}} \\
\text{s.t.} & \quad (20b)–(20e).
\end{align*}
\]

(22)

4.3. The Entire Optimization Problem for the Real-Time-Pricing Tariff. In the real-time-pricing tariff the spot-market prices \( \gamma_{\text{spot}}^t \) are simply forwarded to the prosumer. Thus, for the prosumer’s objective function (16) we specify the cost term \( f_{\text{im}} \) as

\[
f_{\text{im}} = f_{\text{RTP}}^{\text{im}} = \sum_{t \in T} \gamma_{\text{spot}}^t P_{\text{im}}^t + \rho_{\text{RTP}},
\]

where \( \rho_{\text{RTP}} \) is the lump-sum payment discussed in Sect. 3.2.2. Hence, the prosumer’s objective function reads

\[
f_{\text{low}} = f_{\text{RTP}}^{\text{low}} = f_{\text{rev}} - f_{\text{tax}} - f_{\text{RTP}}^{\text{im}}.
\]

(23)

As already mentioned in Sect. 3.2.2, the lump-sum payment \( \rho_{\text{RTP}} \) is the only profit the retailer makes out of the RTP tariff contract. Consequently, the retailer’s objective function just maximizes \( \rho_{\text{RTP}} \); see (2) in Sect. 3.2.2. However, the RTP tariff is constructed such as to leave the prosumer indifferent as compared to the FP tariff. Thus, the RTP tariff must satisfy the competition constraint

\[
f_{\text{conv}}^{\text{low}} (y_{\text{conv}}^*) \leq f_{\text{low}}^{\text{RTP}},
\]

(24)
Table 2. Variables (top) and parameters (bottom) of the TOU tariff upper level model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_{\text{ret}})</td>
<td>Electricity baseline price of the retailer</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>(\gamma_{\text{tou}})</td>
<td>Markup during a non-baseline (peak) time period</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>(z_j^{\text{peak}})</td>
<td>Non-baseline pricing in (t_j)</td>
<td>—</td>
</tr>
<tr>
<td>(z_j^{\text{s-peak}})</td>
<td>Non-baseline pricing switched on in (t_j)</td>
<td>—</td>
</tr>
<tr>
<td>(z_j^{\text{s-off-peak}})</td>
<td>Baseline pricing switched on in (t_j)</td>
<td>—</td>
</tr>
<tr>
<td>(\gamma_{\text{spot}})</td>
<td>Spot-market price in (t \in T)</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>(\gamma_{\text{tax}})</td>
<td>Electricity taxes and fees payed by prosumer</td>
<td>EUR/MWh</td>
</tr>
</tbody>
</table>

where \(y_{\text{conv}}^*\) is an optimal solution of the FP tariff model stated in Sect. 4.2. This competition constraint provides an upper bound for the lump-sum payment. Thus, the overall optimization problem with the retailer acting in the upper level and the prosumer in the lower level reads

\[
\begin{align*}
\max_{\rho_{\text{RTP}}} & \quad f_{\text{RTP}} = \rho_{\text{RTP}} \\
\text{s.t.} & \quad \text{competition constraint: (24)}, \\
\max_{f_{\text{RTP}}} & \quad f_{\text{RTP}} : (23) \\
\text{s.t.} & \quad \text{prosumer constraints: (20b)} - (20e),
\end{align*}
\]

(25)

Note that the only decision variable of the retailer is the lump-sum payment \(\rho_{\text{RTP}}\) that constitutes the profit of the retailer and evens out the profits of the prosumer in the RTP tariff compared to the FP tariff. However, the lump-sum payment appears in the lower level just as a constant term added to the prosumer’s objective function. Thus, due to the simple way of price forwarding and the lack of other tariff mechanisms, the retailer cannot influence the actions of the prosumer. This means that the solution of the lower level is independent of the upper level’s decision (i.e., the lump sum). Therefore, the problem can be solved sequentially as follows. First, we optimize the lower-level problem and get an optimal solution \(y_{\text{RTP}}^*\). Based on \(y_{\text{RTP}}^*\) and an optimal solution \(y_{\text{conv}}^*\) of the FP tariff model we compute the optimal lump-sum payment \(\rho_{\text{RTP}}^*\) that satisfies the competition constraint (24). As the retailer maximizes \(\rho_{\text{RTP}}\), we get

\[
\rho_{\text{RTP}}^* = f_{\text{rev}}(y_{\text{RTP}}^*) - f_{\text{tax}}(y_{\text{conv}}^*) - \sum_{t \in T} z_t^{\text{spot}}(P_{\text{im}}^*) - f_{\text{low}}(y_{\text{conv}}^*),
\]

(26)

where the variables \((P_{\text{im}}^*)^t\) are part of the solution \(y_{\text{RTP}}^*\).

4.4. The Entire Optimization Problem for the Time-of-Use Tariff. We now describe the model of the time-of-use tariff. Since we interpret the intervals \(T_i\) as days, we assume that they have the same number of time periods, i.e., \(|T_i| = |T_j| = N\) for all \(i, j \in \{1, \ldots, I\}\). We also impose that the splitting into peak and off-peak times is the same for every day, i.e., for every time interval \(T_i\). Thus, if \(t_j \in T_{i,\text{peak}}\), then \(t_{(j+N)} \in T_{(i+1),\text{peak}}\) for all \(j = 1, \ldots, k - N\) and \(i = 1, \ldots, I - 1\). Given this implication, it is sufficient to model the peak/off-peak split for the first time interval and then expand it to the other time intervals. To this end, we introduce three binary
variables per time period \( t_j \in T_1 = \{0, \ldots, N-1\} \):

\[
z_{\text{peak}}^j = \begin{cases} 
1, & \text{if } t_j \in T_{\text{peak}}, \\
0, & \text{if } t_j \notin T_{\text{peak}}, \text{i.e., } t_j \in T_{\text{off-peak}},
\end{cases}
\] (27a)

\[
z_{\text{s-peak}}^j = \begin{cases} 
1, & \text{if peak pricing is switched on in } t_j, \\
0, & \text{otherwise},
\end{cases}
\] (27b)

\[
z_{\text{s-off-peak}}^j = \begin{cases} 
1, & \text{if off-peak pricing is switched on in } t_j, \\
0, & \text{otherwise}.
\end{cases}
\] (27c)

Next, we model that the peak and off-peak pricing mode needs to be switched on exactly once during the day using the SOS-1-like constraints

\[
\sum_{j=0}^{N-1} z_{\text{s-peak}}^j = \sum_{j=0}^{N-1} z_{\text{s-off-peak}}^j = 1.
\] (28)

Given peak or off-peak pricing in the time period \( t_{j-1} \), it continues in the following time period, unless the pricing mode is switched in \( t_j \):

\[
z_{\text{peak}}^j - z_{\text{peak}}^{(j-1) \text{ mod } N} = z_{\text{s-peak}}^j - z_{\text{s-off-peak}}^j \quad \text{for all } j = 0, \ldots, N-1,
\] (29a)

\[
z_{\text{s-peak}}^j + z_{\text{s-off-peak}}^j \leq 1 \quad \text{for all } j = 0, \ldots, N-1.
\] (29b)

With this TOU model we reformulate the prosumer’s objective function from Sect. 3.3.6, i.e.,

\[
f_{\text{TOU}}^\text{low} = f_{\text{rev}} - f_{\text{tax}} - f_{\text{TOU}}^\text{im}.
\] (30)

with

\[
f_{\text{im}}^\text{TOU} = \sum_{t \in T} \left( \gamma_{\text{ret}} + z_{\text{peak}}^t \gamma_{\text{tou}} \right) P_{\text{im}}^t, \quad \tilde{t} := t \text{ mod } N.
\] (31)

Recall the retailer’s competition constraint from Sect. 3.2.3, which ensures that the prosumer’s profit in the RTP tariff is not less than the profits under the FP tariff. We adapt this constraint now for the TOU tariff and obtain

\[
f_{\text{conv}}^\text{low} (y^*_{\text{conv}}) \leq f_{\text{TOU}}^\text{low},
\] (32)

where \( y^*_{\text{conv}} \) denotes an optimal solution for the single-level prosumer model (20). Finally, we can rephrase the retailer’s objective function in (3) as

\[
f_{\text{TOU}} = \sum_{t \in T} \left( \gamma_{\text{ret}} + z_{\text{peak}}^t \gamma_{\text{tou}} \right) P_{\text{im}}^t - \sum_{t \in T} \gamma_{\text{spot}}^t P_{\text{im}}^t, \quad \tilde{t} := t \text{ mod } N.
\] (33)

Together with the competition constraint (32) we are able to state the TOU tariff bilevel model as

\[
\begin{align*}
\text{max} \quad & f_{\text{TOU}} \\
\text{s.t.} \quad & \text{TOU model: (27)–(29)}, \\
& \text{competition constraint: (32)}, \\
\text{max} \quad & f_{\text{TOU}} \\
\text{s.t.} \quad & \text{prosumer constraints: (20b)–(20e)}.
\end{align*}
\] (34)

All TOU-specific variables and parameters of this model are listed in Table 2. Taking a closer look at the model one observes that the only nonlinear terms in the model are the products \( \gamma_{\text{ret}} P_{\text{im}}^t \) as well as the products \( z_{\text{peak}}^t \gamma_{\text{tou}} P_{\text{im}}^t \) in the objective functions (31) and (33). The variables \( \gamma_{\text{ret}}, \gamma_{\text{tou}}, \text{ and } z_{\text{peak}}^t \text{ are upper and } P_{\text{im}}^t \text{ are lower-level variables. Thus, the upper level is a mixed-integer quadratic problem in the upper-level variables due to the product } z_{\text{peak}}^t \gamma_{\text{tou}}, \text{ which, fortunately, can be linearized using standard techniques. After such a linearization the upper level becomes a mixed-integer linear problem. For fixed upper-level variables, the lower-level problem is a parametric LP.
Table 3. Variables (top) and parameters (bottom) of the first two levels of the CPP model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Lump-sum payment from the prosumer to the retailer</td>
<td>EUR</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>Extra charge during critical peaks time period</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>$\gamma_{\text{base}}$</td>
<td>Electricity baseline price of the retailer</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>$\bar{\gamma}_{\text{cpp}}$</td>
<td>Upper bound for the CPP markup, $\gamma_{\text{base}}$</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>$\gamma_t^{\text{spot}}$</td>
<td>Spot-market price in $t \in T$</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>$\gamma_{\text{tax}}$</td>
<td>Electricity taxes and fees payed by prosumer</td>
<td>EUR/MWh</td>
</tr>
<tr>
<td>$T_{i,\text{off-peak}}$</td>
<td>Off-peak time period</td>
<td>—</td>
</tr>
<tr>
<td>$T_{i,\text{peak}}$</td>
<td>Peak time period</td>
<td>—</td>
</tr>
</tbody>
</table>

depending on the upper-level decisions. From the latter it follows that we can state optimality conditions for the lower-level LP exploiting the strong duality theorem of linear optimization; see, e.g., [7]. By adding these optimality conditions together with the original primal and dual lower-level constraints to the upper level, we obtain a single-level mixed-integer nonlinear problem (MINLP), which is an equivalent reformulation of the bilevel problem (34). In addition to the fact that such reformulations are usually difficult to solve for practically relevant, i.e., large, instances, we have the additional challenge here that the single-level reformulations contain bilinear terms of products of primal upper and lower level variables. This situation always appears in bilevel pricing problems; see, e.g., [33]. Thus, these problems are nonconvex, i.e., global, optimization problems. More information on single-level reformulations of bilevel problems can be found in, e.g., [18]. Note that solving this single-level reformulation always gives optimistic solutions; cf., e.g., [33] or [10].

4.5. The Entire Optimization Problem for the Critical-Peak-Pricing Tariff.

For the critical-peak-pricing tariff model, the objective function of the prosumer from Sect. 3.3.6 reads

$$ f^{\text{CPP}}_{\text{low}} := f^{\text{rev}} - f^{\text{tax}} - f^{\text{CPP}}_{\text{im}} $$

with

$$ f^{\text{CPP}}_{\text{im}} := \sum_{t \in T} (\gamma_{\text{base}} + \gamma_t^{\text{cpp}}) P^{\text{im}}_t + \rho^{\text{CPP}}. $$

Again, to even out the profits of the prosumer under the conventional and the CPP tariff, the retailer has to satisfy the competition constraint

$$ f^{\text{low}}_{\text{conv}} (y^{\text{conv}}_{\text{conv}}) \leq f^{\text{CPP}}_{\text{low}}, $$

where $y^{\text{conv}}_{\text{conv}}$ is an optimal solution of the FP tariff model as stated in Sect. 4.2.

Recall the timing of the CPP decision making process as stated in Figure 1. When signing the contract both the baseline price $\gamma_{\text{base}}$ and the partition of the days into peak and off-peak periods, $T_{i} = T_{i,\text{peak}} \cup T_{i,\text{off-peak}}$, are exogenously given, and the retailer only sets the lump-sum payment $\rho^{\text{CPP}}$. Overall profit maximization under the competition constraint (37) thus constitutes the first level of the CPP tariff model. In the second level, the retailer sets CPP markups in order to maximize the profit from selling electrical energy to the prosumer as stated in (5), while satisfying the upper bounds for the CPP markup in (4). The third level is the optimization problem of the prosumer. Thus, the overall resulting optimization model for the CPP tariff
reads
\[
\begin{align*}
\max & \quad f_{\text{CPP-all}} \\
\text{s.t.} & \quad \text{competition constraint: (37),} \\
& \quad \max f_{\text{CPP}} \\
\text{s.t.} & \quad \text{CPP markup bounds: (4),} \\
& \quad \max f_{\text{CPP}} \\
\text{s.t.} & \quad \text{prosumer constraints: (20b)–(20e).}
\end{align*}
\]

Recall from Sect. 3.2.4 that both the baseline price \( \gamma_{\text{base}} \) and the identical partition of the day into peak and off-peak periods, \( T_i = T_i,\text{peak} \cup T_i,\text{off-peak} \), are exogenously given for the CPP tariff. Thus, the only decision variable of the retailer in the first level is the lump-sum payment \( \rho_{\text{CPP}} \). However, on levels two and three the lump-sum payment appears only once—namely in the third level as a constant added to the prosumer’s objective function. As a consequence, the first level does not influence decision making either on the second or third level. Therefore, we can solve the second and the third level jointly as a bilevel problem—similarly as in Sect. 4.4. Based on the outcome, we then compute the lump sum of the first level of Problem (38) analogously to the lump-sum computation for the RTP tariff in (26).

The only nonlinear terms in the model are \( \gamma_{\text{cpp}}^t P_{\text{im}}^t \) for \( t \in T \). These terms appear in the objective functions (5) and (36) of both levels. As \( \gamma_{\text{cpp}}^t \) are upper and \( P_{\text{im}}^t \) lower-level variables, these terms are quadratic only with regard to variables from both levels. Thus, for fixed upper level variables \( \gamma_{\text{cpp}}^t \), the lower-level problem is a parametric LP and the upper level is also linear in the upper-level variables. We can thus solve the discussed CPP bilevel problem using the same reformulation as described in Sect. 4.4.

4.6. Discussion of the Models. Observe, finally, that our formal analysis relies on the assumption of perfect foresight of the retailer and the prosumer with regard to all model parameters. In reality the retailer and the prosumer are likely to hold only incomplete information regarding the evolution of, e.g., spot-market prices \( \gamma_{\text{spot}}^t \) or renewable energy production of the prosumer. The consideration of stochasticity in our multilevel modeling would lead to extremely challenging stochastic, multilevel, and mixed-integer optimization problems. As it turns out, see Section 5.1, the considered multilevel problems in their deterministic version are very hard to solve in practice and, thus, are already at the frontier of computational tractability. Solving a stochastic variant of these models would, e.g., lead to bilevel problems with the same characteristics but on huge scenario trees; see, e.g., [31]. Due to the curse of dimensionality, the resulting nonconvex mixed-integer nonlinear problems would be by far too hard to be solved for the settings considered in this paper.

On the other hand, the deterministic results obtained in our analysis provide important benchmark results regarding the desirability of the different tariffs. Indeed, stochasticity would induce further important aspects regarding the desirability of the different tariffs. Risk for the prosumer in our setup would come from two major sources: First, quantity risk due to PV feed-in. This, however, remains unchanged for different tariff structures and is thus likely to affect all tariffs analyzed in a similar way. Second, price risk directly induced by the tariff structure itself will affect the different tariffs in a different way, since the FP and TOU tariff do not exhibit any price risk for the prosumer by construction, whereas the RTP and the CPP tariff do. Even for a risk-neutral prosumer, price risk will result in a less efficient operation of storage devices (as compared to our perfect foresight analysis) and thus would also require more prosumer-friendly contract terms in case of the RTP and the CPP tariff. That is, since the FP and the TOU tariff by construction do not exhibit any price risk for the prosumer, in case of risky spot-market prices the RTP and the CPP
tariff should perform relatively worse as indicated by our quantitative results obtained later; see Section 5.1. Furthermore, for a risk-averse prosumer this will be even more pronounced and both the RTP and the CPP tariff become even less desirable.

5. Computational Study

5.1. Data Description. We use real-world data for the calibration of our models. Most of the parameters regarding the prosumer are calibrated according to the research project “Smart Grid Solar”, which has been funded by the Bavarian Ministry of Economic Affairs and Media, Energy and Technology and the European Regional Development Fund. The project was carried out in the period 2012–2017. In the project, various partners from industry and economy worked together with research institutions and local network operators under the coordination of the Bavarian Center for Applied Energy Research (ZAE). During the mentioned period, the participating project partners installed a smart grid infrastructure in a small village of northern Bavaria. The infrastructure was operated and controlled in order to solve economic and electrotechnical problems as well as to identify benefits and potentials of smart grid systems. During the operation of the installed smart grid infrastructure, precise data on different aspects (such as PV production, storage devices, and consumption) have been collected.1

For our analysis we focus on a typical farmstead in that village considering an exemplary prosumer with a photovoltaic system and an electrically driven heat pump. For our computations we use actual hourly measured values from the year 2015 for the output $P_{\text{pv}}^t$ generated by the PV system, for the heat demand $P_{\text{lh}}^t$, and for the electricity demand $P_{\text{le}}^t$ of the prosumer.

We furthermore consider an installed PV capacity of 0.010 08 MWp, which is on the verge of eligible size for the German feed-in tariff $\gamma_{\text{pv}}$ for small-sized PV panels; see [45] for detailed information on governmental regulation and subsidization of photovoltaic energy generation in Germany. The heat storage unit of the prosumer is a water tank with a maximum capacity of 0.033 276 MWh and a self-discharge rate of 1% per hour; cf. Sect. 3.3.4. The coefficient of performance $\eta_{\text{h-pump}}$ of the heat pump is assumed to be 3; cf. [35]. To better reflect current conditions in Germany, where own consumption of solar energy is more profitable now as it was the case when the PV system of the considered prosumer was installed, in our model we equip the prosumer with a battery; cf. Sect. 3.3.2. This is supposed to be a Tesla Powerwall with technical information taken accordingly to [52].

For the calibration of the FP tariff $\gamma_{\text{base}}$ we take the average market price of FP energy supply tariffs in Germany; see [4]. The spot-market prices $\gamma_{\text{spot}}^t$ are the publicly available EEX prices of the [13].

Electricity consumers pay a considerable amount of fees and taxes for electricity purchased from their retailers. This critically influences the operation of the facilities of the prosumer when solving the trade-off between (i) immediate own consumption and storage for later own consumption versus (ii) feed-in to and later purchase from the grid. To realistically assess the decisions of the prosumer in our setup we explicitly have to take into account the amount $\gamma_{\text{tax}}$ of taxes and fees paid for the different tariffs. For the levels of fees and taxes payed by the prosumer we adopt the values for final household consumers from the year 2017 in Germany; see [4].

The computational hardness of the considered bilevel models, which result in MINLP and nonconvex NLP reformulations for the TOU and CPP tariffs, respectively, does not allow to solve the instances on an hourly discretization of the entire time horizon of one year. Thus, from the available data of the year 2015 we pick one representative week for winter, the mid-season, and the summer period and join them consecutively in this order. The problems as described in Sect. 4 are solved on this time horizon.

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1 For further information on the project see http://www.smart-grid-solar.de.
with objective functions scaled to a year (proportionally to the actual length of the corresponding seasons). Thus, we obtain computational results for a year based on the three chosen and representative weeks of every season.

Both the TOU and the CPP tariff have peak and off-peak day partitions. For the TOU tariff, this partitioning is determined endogenously. To obtain a clear comparison between these two tariffs, we take the peak and off-peak time periods as well as the off-peak price from the optimal solution of the TOU model as input parameters for the CPP model.

5.2. Computational Setup. All computations were performed on a 4 core Intel(R) Core(TM) i7-4600U CPU with 2.1 GHz and 4 MB cache each as well as 8 GB RAM under Ubuntu 18.04.1 LTS OS. For the data handling we used Python 3.7.0 with Anaconda 5.3.0. The optimization models have been modeled using GAMS 25.1.2 [19]. All (mixed-integer) linear models have been solved with CPLEX 12.8.0.0 [8] and all (mixed-integer) nonlinear models have been solved with BARON 18.5.8 [46, 51]. The linear models were solved up to a relative optimality gap of $10^{-6}$ without imposing any time limit. The running time for solving each linear model did not exceed 1 second. The time limit for all nonlinear instances was 15 minutes and the optimality gap was set to 1%. Often, it was not possible to solve the (MI)NLP instances with a gap of less than 1% in the given time limit. If this was the case, we always specify the achieved optimality gap individually in the text or in the corresponding tables. Let us note at this point that we also used significantly larger time limits in our preliminary numerical experiments. However, it turned out that BARON (as well as other tested global solvers) get stuck after some minutes and made no more progress. Thus, the achieved gap after a significantly longer computation time was still the same as after a few minutes. This is why we decided to use 15 minutes as the time limit for the final computations.

5.3. Optimistic Prosumer Solutions. It is important to note that the single-level reformulations of Sect. 4.4 and 4.5 used to solve the multilevel TOU and CPP models imply the so-called optimistic assumption; cf., e.g., [10]. This assumption means that if more than one optimal solution of the prosumer’s problem exists, the most advantageous for the retailer is selected. In order to compare the computational results properly, we need to consider optimistic behavior of the prosumer in the FP and the RTP tariff as well.

For the FP tariff we achieve this by artificially extending the FP tariff model by an upper level comprising just the corresponding objective function (1) of the retailer:

$$\begin{align*}
\max & \quad f_{\text{Conv}} \\
\max & \quad f_{\text{low-conv}} \\
\text{s.t.} & \quad (20b)-(20e).
\end{align*}$$

(39)

We solve this model using the same strong-duality based lower-level reformulation as applied to the multilevel TOU and CPP models. Since (39) does not have any upper-level variables, the resulting reformulation of the bilevel problem (39) remains an LP and is solved up to a relative optimality gap of $10^{-6}$. In what follows, all presented results (unless specified otherwise) in the tables and figures for the FP tariff are obtained for the model (39) that incorporates the optimistic assumption on the prosumer’s behavior. The prosumer’s optimal objective function value remains the same as in the model (22) from Sect. 4.2.

The RTP tariff exhibits the optimistic behavior of the prosumer naturally, since the goal of both the prosumer and the retailer is to maximize the prosumer’s profit. Recall that the entire profit of the retailer in the RTP case is just the lump-sum payment $\rho_{\text{RTP}}$ charged from the prosumer; see Sect. 3.2.2. This lump-sum payment $\rho_{\text{RTP}}$ allows the retailer to extract all prosumer’s profits beyond those the prosumer would
Table 4. Model sizes for single-level reformulations including optimistic assumption.

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Conv.</th>
<th>RTP</th>
<th>TOU</th>
<th>CPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilevel model?</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Overall number of variables</td>
<td>8574</td>
<td>3529</td>
<td>10 160</td>
<td>10 086</td>
</tr>
<tr>
<td>Number of discrete variables</td>
<td>0</td>
<td>0</td>
<td>72</td>
<td>0</td>
</tr>
<tr>
<td>Number of nonlinear terms</td>
<td>0</td>
<td>0</td>
<td>4032</td>
<td>2016</td>
</tr>
<tr>
<td>Number of constraints</td>
<td>5047</td>
<td>1517</td>
<td>7618</td>
<td>7063</td>
</tr>
</tbody>
</table>

Table 5. Results for three exemplary weeks scaled up to a year (with the optimistic assumption).

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Conv.</th>
<th>RTP</th>
<th>TOU</th>
<th>CPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer profit (EUR)</td>
<td>505.60</td>
<td>530.38</td>
<td>513.19</td>
<td>520.50</td>
</tr>
<tr>
<td>Gain (compared to conv. tariff) (%)</td>
<td>—</td>
<td>4.67</td>
<td>1.48</td>
<td>2.86</td>
</tr>
<tr>
<td>Relative optimality gap (%)</td>
<td>0.00</td>
<td>0.00</td>
<td>3.24</td>
<td>2.14</td>
</tr>
<tr>
<td>Lump-sum payment (EUR)</td>
<td>0</td>
<td>530.38</td>
<td>0</td>
<td>−46.50</td>
</tr>
<tr>
<td>Average price (EUR/MWh)</td>
<td>64.20</td>
<td>31.10</td>
<td>63.89</td>
<td>66.61</td>
</tr>
<tr>
<td>Lowest price (EUR/MWh)</td>
<td>64.20</td>
<td>20.07</td>
<td>57.76</td>
<td>57.76</td>
</tr>
<tr>
<td>Highest price (EUR/MWh)</td>
<td>64.20</td>
<td>75.87</td>
<td>66.70</td>
<td>71.17</td>
</tr>
</tbody>
</table>

have obtained under the FP tariff, cf. (24) and also (26) for the explicit formula for calculating the lump-sum payment. Thus, in case of the RTP tariff, it is not necessary to reformulate an extended problem version analogous to (39): The solution of the problem (25) directly yields optimistic behavior of the prosumer.

In the optimal solution of the RTP and CPP tariff problems the corresponding competition constraints (24) and (37) are satisfied with equality due to the profit maximization objective of the retailer. Hence, the prosumer’s profit is the same for the FP, the RTP, and the CPP tariff. In the TOU tariff the prosumer cannot earn less than in the FP tariff case due to the TOU competition constraint (32). Consequently, the TOU tariff is at least as attractive for the prosumer as the other three tariffs. In all presented computational results the TOU competition constraint is also satisfied with equality, making the prosumer’s profit equal for all four considered tariffs.

Before we move on to the actual computational results and their discussion, we provide some details on model sizes for all four tariffs in Table 4. Note that these details are always given for the corresponding single-level reformulation incorporating the optimistic assumption.

5.4. Results for the Optimistic Prosumer Solution. Before we discuss the results in detail let us first note that all results presented in the tables in this section have been scaled up to a time horizon of one year while the figures comprise the three exemplary weeks without scaling.

Table 5 presents the most important properties of the optimal solutions under the optimistic assumption for all four considered tariffs. The gain compared to the conventional tariff (in percent) is the difference between the considered and the conventional retailer profits divided by the retailer profit of the alternative tariff considered. The relative optimality gap is the one obtained after the time limit was exceeded. Thus, the retailer profit (and the respective gains) might be larger in those columns with a positive optimality gap, i.e., for the TOU and the CPP tariff. As expected, the retailer’s profits with flexible tariffs are higher than with the FP tariff.
Table 6. Aggregated prosumer parameters (above) and results (below; all scaled up to a year in MWh).

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Conv.</th>
<th>RTP</th>
<th>TOU</th>
<th>CPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical load</td>
<td>5.748</td>
<td>5.748</td>
<td>5.748</td>
<td>5.748</td>
</tr>
<tr>
<td>Heat load</td>
<td>15.607</td>
<td>15.607</td>
<td>15.607</td>
<td>15.607</td>
</tr>
<tr>
<td>Total power bought</td>
<td>16.476</td>
<td>16.533</td>
<td>16.493</td>
<td>16.505</td>
</tr>
<tr>
<td>Total power sold</td>
<td>3.488</td>
<td>3.488</td>
<td>3.488</td>
<td>3.488</td>
</tr>
<tr>
<td>Battery and HSU losses</td>
<td>0.267</td>
<td>0.324</td>
<td>0.284</td>
<td>0.296</td>
</tr>
<tr>
<td>Total profit</td>
<td>−4483.885</td>
<td>−4483.885</td>
<td>−4483.885</td>
<td>−4483.885</td>
</tr>
</tbody>
</table>

The RTP as the “flexibility benchmark” shows the highest profits compared to the conventional tariff. It is followed by the CPP tariff for which retailer profit is at least 2.86% higher than for the fixed-price tariff. The TOU tariff offers a gain of at least 1.48%, which is smaller than in the CPP case. However, the larger optimality gap of 3.24% in the TOU solution still leaves some room for a larger gain that, in the best case (given the optimality gap), would be comparable to the CPP result. Prices faced by the prosumer differ substantially across tariffs. The lowest average price is obtained for the RTP, the highest average price for the CPP tariff. In both cases, the compensation payment $\rho$ adjusts the profit of the prosumer to the level of the fixed-price tariff so that the retailer profits do not differ substantially in spite of the huge price differences as compared to the benchmark. TOU results in approximately the same price level as the fixed-price tariff. Whereas prices do not vary significantly between time periods in TOU, there is more difference in the case of CPP. Not surprisingly, the highest price fluctuation is shown by RTP.

One purpose of designing flexible energy supply tariffs is to provide incentives for prosumers to apply load shifting. Thus, we now consider the load management of the prosumer in more detail. To this end, we first present relevant aggregated input data such as the total electrical and heat load as well as the total PV power production scaled up to a year in Table 6. This input data is the same for all considered tariffs. Moreover, we also specify aggregated results like the total yearly amount of electrical power bought from or fed into the main grid as well as the aggregated losses of the battery and the HSU caused by load shifting. As expected, these results may differ between the different tariffs. The imported electrical power differs, with the smallest amount for the FP tariff and then (in ascending order) for the TOU, the CPP, and the RTP tariff. This order of amounts of electrical power bought from the main grid exactly corresponds to the order of increasing losses in the battery and the HSU across the tariffs (provided in the penultimate row of the table). Indeed, the more power shifting occurs due to higher import price fluctuation of a certain tariff, the higher are the losses in the battery and the HSU of the prosumer. The RTP tariff is the most flexible tariff (w.r.t. the price of electricity) and thus corresponds to the highest losses. Consequently, the RTP tariff drives the prosumer to buy the largest amount of power from the main grid. The TOU and the CPP cause smaller losses and result in less power bought from the main grid as the RTP—but still more than the FP tariff. Note that the amount of PV power fed into the main grid is the same for all four tariffs. The reason is that PV energy is used primarily for self-consumption, which is due to

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2 The average price is calculated by first multiplying the amount of purchased electrical energy at each time period by the corresponding retailer price and weighting the products according to the duration of the corresponding season. Then we sum these products over all time periods and finally divide this sum by the total amount of electrical energy purchased throughout the entire time horizon, which is also weighted according to different seasons.
the very favorable treatment of self-consumption under the German regulation. Only when the prosumer’s demand is satisfied, the remaining amount is sold. Thus, the amount of sold PV energy does not depend on the electricity prices (and thus not on the specific tariff). The prosumer’s profit is obviously the same for all four tariffs, cf. Sect. 5.3, which is due to the constraint that all tariffs have to yield at least the profits from the fixed-price tariff for the consumer.

To sum up, we observed that the higher the flexibility of the tariff, the more load shifting and, consequently, power losses occur. To study the respective seasonal dependencies more clearly, we now present the respective optimal load managements of the prosumer for the three exemplary weeks. We start by discussing the FP tariff under the optimistic assumption. The corresponding load management of the prosumer is illustrated in Figure 3. The energy sources such as power bought from the main grid, PV generated power, and power discharged from the battery are shown going down below zero in the upper part of the figure. The power used by the prosumer to cover electrical load, for the heat pump, for feed-in to the main grid, and for charging the battery are plotted above zero. The second figure below shows the price for electrical energy bought by the prosumer during each time period (red line; in EUR/MWh). Of course, this is a flat line for the FP tariff. The other black line indicates the hourly costs (negative values) or profits (positive values) of the prosumer during each time period in EUR.

Here, larger amounts of electricity bought from the main grid correspond to higher heat demand (and thus higher electrical demand used for the heat pump) in winter and in the mid-season whereas smaller amounts of purchased electricity coincide with larger amounts of self-generated PV power—particularly in the summer. Since the price of electricity is constant over time in this tariff, the cost/profit curve of the
The optimal prosumer’s load management in the TOU tariff is shown in Figure 5. The peak time starts at 6 a.m. and lasts until midnight. The off-peak time accordingly starts at midnight and lasts until 6 a.m. We can see that the price difference between the peak- and off-peak times is much lower as compared to the RTP tariff in which the spot-market prices are directly forwarded to the prosumer. It is also visible that the TOU tariff induces a certain regular pattern of the behavior of the prosumer. Directly before the end of the off-peak time period, the prosumer buys electricity and directly uses it for the heat pump in order to store heat in the HSU that is then used in the peak time period. Thus, for TOU as well as RTP, the main storage in colder periods is the heat storage unit, as is the case for all four considered tariffs. Let us also note that the specific moment of purchase is reasonable since earlier purchases would lead to losses in the HSU. The battery does not play an important role during the colder weeks. This changes in warmer periods in which the battery is mainly used to store PV power in the mid-season and the summer.

The results under the CPP tariff are illustrated in Figure 6. The curves are less jagged than with the RTP tariff, but show more peaks than the TOU tariff, and—as it is also the case for the TOU tariff—have a more periodic pattern due to the periodicity...
of peak and off-peak times. Recall that peak and off-peak times of the CPP are the same as those in the TOU, i.e., peak time from 6 a.m. to midnight, and off-peak time from midnight to 6 a.m. The increased amount of peaks for the energy bought from the main grid compared to the TOU tariff is caused by variations in the CPP markup values during peak time periods. Note that the CPP markup values are highly ambiguous if the prosumer does not buy any electrical power during the corresponding period, e.g., during the summer. Here, the optimal markup level may adopt a high value, but this does not affect the prosumer, whose energy demand is covered by the PV panel in combination with the battery.

Moreover, note that the profitability of the CPP tariff is highly dependent on the fixed upper bound for the CPP markups. Unless specified otherwise, all computational results presented in this paper are given for $\bar{\gamma}_{cpp} = 1.5\gamma_{tou}^*$, where $\gamma_{tou}^*$ is the TOU markup from the optimal TOU solution. By choosing a higher upper bound value, e.g., $\gamma_{cpp} = \gamma_{base}$, the retailer profit would be lower (517.95 EUR) and the lump-sum payment would increase to -611.08 EUR. With $\gamma_{cpp} = 2\gamma_{base}$, the retailer’s overall profit would amount to just 444.782 EUR, being 13% less than the profit under the FP tariff.

5.5. Results under a Worst-Case Assumption. All results of the last section are obtained under the optimistic assumption for the behavior of the prosumer. However, this optimistic behavior is not a necessarily realistic assumption in practice. Unfortunately, computing pessimistic solutions of bilevel problems is even harder than computing optimistic solutions; see, e.g., [61], where a method is presented for computing pessimistic solutions that builds upon methods for computing optimistic solutions. Since the bilevel instances that we solve already are at the frontier of computational tractability in the optimistic case, we have no hope on tackling the
pessimistic case as well. Thus, we are also interested in some kind of worst case. To this end, we proceed as follows. We fix the values of retailer's variables from the optimal solutions of the multilevel models for the TOU as well as the CPP tariff and then solve the corresponding bilevel models minimizing the retailer's objective function:

$$
\begin{align*}
\min \; & f_{\text{TOU}}: (33) \text{ with fixed upper-level variables} \\
\max \; & f_{\text{low-TOU}}: (30) \\
\text{s.t.} \; & (20b)-(20e).
\end{align*}
$$

As the lump-sum payment in the CPP tariff only depends on the optimal objective function value of the lowest level, i.e., of the profits of the prosumer, it is also fixed once the CPP model (38) is solved. Thus, it is sufficient for the CPP tariff to consider only the second and third level of the CPP model, i.e.,

$$
\begin{align*}
\min \; & f_{\text{CPP}}: (5) \text{ with fixed second-level variables} \\
\max \; & f_{\text{low-CPP}}: (35) \\
\text{s.t.} \; & (20b)-(20e).
\end{align*}
$$

For the conventional tariff we modify the artificial bilevel formulation (39) accordingly and obtain

$$
\begin{align*}
\min \; & f_{\text{Conv}}: (1) \\
\max \; & f_{\text{low-conv}}: (21) \\
\text{s.t.} \; & (20b)-(20e).
\end{align*}
$$
Table 7. Total retailer profit for three exemplary weeks scaled to a year (all values in EUR): optimistic assumption vs. worst case.

<table>
<thead>
<tr>
<th></th>
<th>Conv.</th>
<th>RTP</th>
<th>TOU</th>
<th>CPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>505.60</td>
<td>530.38</td>
<td>513.19</td>
<td>520.50</td>
</tr>
<tr>
<td>Worst case</td>
<td>505.60</td>
<td>530.38</td>
<td>508.01</td>
<td>510.44</td>
</tr>
</tbody>
</table>

We note that this does not give the pessimistic solution (in the classical sense of bilevel optimization) but leads to a special notion of the worst case for the retailer. Thus, it gives an insight on the dependence of the load management of the prosumer if the latter faces some ambiguities regarding the control of his utilities.

Before we discuss the differences between these worst-case solutions and the solutions obtained under the optimistic assumption, let us point out that in our computational results the solution for the RTP and the FP tariff are the same for the optimistic and the worst-case solution. The latter depends on the parameters of the model since there could also exist instantiations in which these results may differ. For both the TOU and the CPP tariff, however, corresponding gains in the worst case are below the solutions with the optimistic assumption; cf. Table 7.

The mathematical reason for the differences between the optimistic and worst-case setting is that in some cases the lower level of the considered multilevel problems does not possess a unique solution for every upper-level decision. It is well-known that this situation may lead to issues in bilevel optimization that are hard to analyze; cf., e.g., Chapter 7 of [10]. Regarding the application of bilevel optimization discussed in this paper, a word of caution is thus appropriate. First, using usual optimization models, i.e., single-level problems, for analyzing the impact of different tariff types may not be appropriate since the different goals of the retailer and the prosumer cannot be modeled properly in such a framework. Moreover, just applying bilevel optimization using standard single-level reformulation techniques may also lead to contestable results because the “most profitable” tariff may depend on how a prosumer controls his utilities. If such a control is not unique, a profitable and flexible tariff may even get worse than the standard FP tariff—depending on how “cooperative” the prosumer behaves.

6. Conclusion

In this paper we analyze the potential to incentivize prosumers through flexible tariffs in a smart energy system and thereby assess the possibilities of a retailer to raise additional profits by using them. We focus on tariff design by the retailer who takes into account the specific load shifting potential of prosumers who possess small-scaled energy generation and storage facilities. It is a particular feature of the presented approach that our prosumer model explicitly considers the interaction of electricity as well as heat production and storage within a household.

The prosumer operates his domestic system optimally given the electricity prices that he faces if he buys electricity from the retailer to cover residual load. In order to appropriately model the specific decision environments and the interaction of retailer and prosumer, we rely on bilevel modeling techniques. In our analysis we consider four tariffs: a conventional fixed-price tariff (FP) in which the electricity price is constant over time, real-time-pricing (RTP), time-of-use pricing (TOU), and critical-peak-pricing (CPP). In order to allow for a proper comparison, we ensure that the prosumer will maintain his level of profit if he switches from the conventional to either of the three more flexible tariffs.

Our study provides a number of new and interesting insights on flexible pricing. We show that the simple pass-through of wholesale prices in the RTP tariff yields...
the highest additional profits for the retailer within our framework. This is intuitive since the price signal from the market induces the optimal use of the prosumer’s flexibility. CPP and TOU only yield moderate additional gains in our analysis. This is mainly due to the fact that load shifting potential turns out to be moderate if detailed technical configurations are accounted for in combination with fixed time windows for peak prices. In CPP, the prosumer will be aware of the risk of high peak prices and therefore has to be compensated up-front by a lump-sum payment. On top of the low efficiency potential in the optimistic case, in the case of CPP and TOU, flexible tariffs imply multiple solutions of the prosumer’s optimization problem, which raises revenue uncertainty for the retailer. In essence, the retailer has to bear the risk that the prosumer acts against him, which makes flexible tariffs such as TOU and CPP less attractive.

Our framework creates the basis for looking at prosumers with alternative technology parks as well. If, for example, the prosumer has devices where energy production is controllable, the efficiency potential of a flexible tariff may be higher.

Our results clearly illustrate that the design of flexible tariffs is extremely complex. The performance of tariffs depends on many different details of the tariff structure and also on the behavioral patterns of prosumers. In order to achieve significant efficiency gains (which ultimately benefit the retailer or the prosumer), it is necessary to optimize further aspects of the tariff beyond the multiple aspects that we already addressed in our analysis. Possible extensions of our approach would be consideration of further technologies and possibilities of the prosumer allowing for an explicit demand reduction (additionally to demand shifting), optimal choice of duration and number of peak intervals specifically for CPP or the consideration of direct load control. These questions will be addressed in future work.

ACKNOWLEDGMENTS

This research has been performed as part of the Energie Campus Nürnberg and is supported by funding of the Bavarian State Government. Moreover, the research of the first, fourth, and fifth author has been supported by the Emerging Field Initiative (EFI) of the Friedrich-Alexander-Universität Erlangen-Nürnberg through the project “Sustainable Business Models in Energy Markets”. Finally, all but the second author thank the DFG for their support within the subprojects A05, B07, B08, and B09 in CRC TRR 154.

REFERENCES


30 REFERENCES


REFERENCES


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