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To trade, or not to trade, that is the question: New roles for incomplete contracts in dynamic settings*

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Abstract

We reexamine the role of incomplete contracts in a dynamic model of renegotiation that endogenizes the timing of investments and trade. The interaction between bargaining and investment significantly alters the lessons learned from static models. When the opportunity to trade is expected to be long lasting, contracts that exacerbate the parties’ absolute vulnerability to hold-up – especially following under-investment – are desirable. For example, joint ownership of complementary assets can be optimal, an exclusivity agreement can protect the investments of its recipient, and trade contracts can facilitate purely cooperative investment.

JEL Numbers: C73, C78, L14.

Key words: hold-up, relationship-specific investments, investment dynamics.

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1 Introduction

When an investor’s trading partner can appropriate part of the returns, a socially desirable investment may not be made. This is the well-known hold-up problem. There are two conditions that make overcoming it particularly difficult: if investments are not verifiable – for example, investments in human capital/effort – and therefore they cannot be directly contracted upon, and if they are relationship specific, in the sense that they create more surplus within the trading relationship than outwith it. The literature on incomplete contracts has explored how the hold-up problem can be mitigated under these two conditions\(^1\) and it has concluded that writing a contract about the verifiable aspects of the trading relationship (property rights, exclusivity, etc.) can incentivize investment by affecting the way surplus is shared. A main tenet of this literature is that the power of these incentives depends on the comparison of net returns to different levels of investment \textit{taking efficient agreement on trade as given}. Consequently, incomplete contracts are generally evaluated according to how well they serve the purpose of improving these intensive-margin – often referred to as “marginal” – incentives.

In this paper we put forward a dynamic model of investment and bargaining, in which we identify alternative channels through which incomplete contracts can influence investment behavior. Our primary observation is that there are realistic scenarios where the main obstacle to achieving efficient investment in equilibrium is a deviation to \textit{no agreement on trade} – rather than the fear of marginal underinvestment, taking trade for granted.\(^2\) In other words, it is the absolute level of exposure to hold-up – the extensive margin, if you will – that needs contractual attention.

The models in the literature are static, in the sense that the investment phase is finished by the time (re)negotiation occurs. In many applications, however, the processes of investment and negotiation take place in a fluid and loosely structured way over time, with (incremental) investments being feasible as long as negotiation is not concluded – with or without agreement/trade. For instance, the Department of Defense may start negoti-

\(^1\)A range of organizational and contract forms have been rationalized as safeguards against hold-up: Examples include vertical integration (Klein, Crawford and Alchian, 1978; Williamson, 1979), property rights allocation (Grossman and Hart, 1986; Hart and Moore, 1990), contracting on renegotiation rights (Chung, 1991; Aghion, Dewatripont and Rey, 1994), option contracts (Nöldeke and Schmidt, 1995), and trade contracts (Edlin and Reichelstein, 1996; Che and Hausch, 1999).

\(^2\)As we show below, in the standard models relationship specificity implies that the parties are willing to trade whenever the trade-contingent incentive constraints are satisfied.
ations to order a weapons system from a contractor, but it may decide to wait until the latter develops a better technology. A similar dynamic interaction between investment and bargaining arises in construction, or the development of advertising and software.\(^3\)

Che and Sákovics (2004a) present a dynamic model of investment and bargaining that captures this phenomenon. They show that if investment activity can continue after the negotiation has started, investment incentives may be enhanced, and the hold-up problem may be alleviated. The intuition is based on expectations. If the investor’s partner expects her to make up for today’s under-investment tomorrow, then he offers worse terms of trade today – factoring in the savings in investment cost the investor makes by agreeing today – thereby increasing her incentives to invest today. In turn, the increased incentives make the expectation of future investment rational. These dynamic incentives are the more effective the more patient the parties are. In fact, the most efficient equilibrium is constrained no longer by an incentive constraint, rather by the parties’ willingness to participate in the trade – provided they are sufficiently patient. This result shows that investment dynamics can serve as a substitute for standard incentive contracts, while it opens up new ways for contracts to foster efficiency.

The current paper considers contracts – signed before the game starts – that specify the disagreement/no-trade payoffs following any feasible level of investment. We adapt the above dynamic modelling framework, by replacing discounting with an exogenous probability of breakdown in negotiations, so that it can explicitly handle contracts as the determinants of the payoffs in case of a breakdown.\(^4\) When negotiation is certain to break down after the first period, this model nests the standard/static one as a special case. We show that the predictions of the existing theory are robust to small perturbations: when the future counts, but little (the breakdown probability is high but less than one). At the same time, when the future looms large (the breakdown probability is low), the investment dynamics becomes important and harnessing the power of expectations results in the role of contracts being to relax the participation constraint rather than to provide marginal incentives. This is when a new insight on contract design emerges.

Before we discuss our results, it is useful to revisit the standard effects of contracts, which

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\(^3\) The (original) editorial procedure at the B.E. Press can also be seen in this light. Here a submission was simultaneous to four vertically differentiated journals. Barring rejection, the author was offered a choice between immediate acceptance at a lower level, or, acceptance at a higher level, conditional on a substantial revision (that is, incremental investment).

\(^4\) For parsimony, we concomitantly simplify the investment strategies to be binary.
indirectly also determine the trade payoffs themselves: party 1’s trade payoff increases in her own disagreement payoff and decreases in party 2’s one. From here it is intuitive what the standard recipe is: the optimal contract maximizes the increase in an investor’s contract payoff as a result of her investment, while minimizing the concomitant increase in her partner’s contract payoff (the “leakage” of investment return).

Our flagship result\(^5\) is for contracts that provide the same investment returns in the absence of trade – that is, the above mentioned effect is held constant, rendering these contracts equivalent according to the standard recipe. We show that a binding participation constraint – arising for a sufficiently low probability of breakdown – can be relaxed, by choosing from these contracts one that provisions a “punishment” (low contract payoffs) for not trading (not for not investing!)\(^6\). Significantly, it is the reduction in the aggregate no-trade payoff that counts: it does not matter how this decrease is distributed across the parties – a useful characteristic when both parties need to invest. We call this result the No-trade Payoff Minimization Principle. It has been observed (c.f. Holmström and Roberts, 1998) that many large systems with complementary activities (satellite broadcasting – e.g. BSkyB –, software – e.g. Microsoft – biotechnology – e.g. Genentech) operate as an intricate network of contracts, resisting the pressure for integration that the standard hold-up logic would require. Holmström and Roberts note that this “lack of integration” comes about despite the fact that the break-up costs of these networks would be very large. The No-trade Payoff Minimization Principle provides a ready answer to why this should not be surprising.

Considering contracts with differing investment returns, our result becomes more nuanced, but its main thrust continues to hold. There are three effects that relax the participation constraint. The first two agree with the standard insight that follows from relaxing the incentive constraint: it is beneficial to increase (decrease) the investor’s (her partner’s) contract payoff following investment. This simply captures their effect on the sharing of the surplus in equilibrium. The last, negative, effect is that of the investor’s contract payoff following no investment. Note that – unlike in the standard case – it is irrelevant what the partner’s contract payoff is in case the investor does not invest. This becomes particularly

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\(^5\)For clarity, in the body of this paper we present a model (and results) with one-sided investment and symmetric bargaining power. We generalize to two-sided investment and asymmetric bargaining power in the Appendix.

\(^6\)Since investment returns are assumed to be the same across contracts, if one provides a lower payoff following investment it also provides a lower payoff (by the same amount) in the absence of it.
relevant when both parties are supposed to invest, as the first two effects are rival: moving them to relax one party’s participation constraint, tightens her partner’s. However, the third effect is not rival: decreasing the contract payoff of a player if she fails to invest relaxes her own participation constraint but it does not affect her partner’s. Consequently, the No-Trade Payoff Minimization Principle generalizes, but it is refined: the prescription that works without fault is to reduce the no-trade payoff of each player in case they underinvest.⁷

Next, we turn to some well-known applications: asset ownership and exclusivity of trading relationships. We explore how such explicit contract terms can be designed to harness the power of expectation as a source of incentive. We obtain some surprising new insights: joint ownership of complementary assets can be optimal and an exclusivity agreement can protect the investments of its recipient, in contrast to the results in the literature.

Finally, we extend our model to include contracts that specify the quantity or some aspects of quality of goods/services to be traded even if negotiation breaks down – and thus disagreement no longer means no trade. We use the extension to model two further applications: purely cooperative investment, where the investor does not directly benefit from her investment; and contracting in a complex environment, where the investment requires specialization before it is revealed what the optimal specialization would have been. We show, by appealing to the No-trade Payoff Minimization Principle, that incomplete contracts may be useful in these environments – by setting the worst possible disagreement payoff – overturning the accepted wisdom once more.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 looks at the necessary and sufficient conditions that lead to the efficient investment in both the standard/static and our dynamic contract model and establishes the effects of contracts. Section 4 then applies the results to some well-known contracts. Section 5 contains the extension to trade contracts. Section 6 reviews the related literature and Section 7 concludes. All the proofs are collected in the Appendix.

2 The model

Two patient, risk-neutral parties, i = 1, 2, can create a surplus from some productive activity that we simply label as “trade”. Party 1 can increase the trade surplus by investing

⁷But their partner does not, as we are deterring unilateral deviations from both of them investing.
prior to trade. Her investment decision is assumed to be binary, represented by an indicator function, $I$, which takes the value 1 if she has invested and zero if she has not. The cost of investment is $c > 0$. Investment $I$, leads to a bounded aggregate (gross) Return, denoted by $R(I) < K$. We assume that investment is socially desirable:

**Assumption 1. (Investment is efficient)**

$$R(1) - R(0) > c.$$  

A pair of $R(.)$ and $c$ define a *scenario*. In light of Assumption 1, our main focus is to investigate how different contracts compare in enabling investment for the largest set of scenarios – in a dynamic investment/renegotiation game.

The difficulty of ensuring efficient investment stems from the inability to contract on the terms of trade as a function of investment. Thus, in keeping with the prevailing view of the contract literature, we assume that investment is not verifiable. Nonetheless, the parties can contract on other aspects, such as asset ownership, or the exclusivity of trading relationships. These “incomplete” contracts determine the default outcome in case the parties do not trade: they take effect whenever the trade negotiations break down. It is worth pointing out that these contracts are not assumed to be fine tuned to a specific scenario, rather they are long-term solutions potentially applying to a sequence of different (investment and) trade opportunities.

For a general analysis, it is convenient to identify a contract by the payoffs it yields to the parties (in case of disagreement). Specifically, suppose they have signed contract $\gamma$. Then, the parties collect gross payoffs of $d_1(I; \gamma)$ and $d_2(I; \gamma)$, henceforth referred to as disagreement or no-trade payoffs, in case they choose not, or fail, to renegotiate it (and trade). We denote the aggregate disagreement payoff by $D(I; \gamma) = d_1(I; \gamma) + d_2(I; \gamma)$. A particularly important feature of these contract payoffs is how they vary with investment. To capture this, we let $\Delta d_i(\gamma) := d_i(1; \gamma) - d_i(0; \gamma)$ denote the changes in the contract payoffs of party $i = 1, 2$, that result from party 1’s investing.

In case investments accrued a net benefit even in the absence of trade, they would be relatively easy to incentivize. Therefore, we – as the literature – concentrate on the situation where investments are relationship specific. The following assumption makes this precise. Let the set of available contracts be denoted by $\Gamma$.

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$^8$Following the literature, we do not model how the parties negotiate the original contract, it is considered exogenous.
Assumption 2. (Specificity) For each \( \gamma \in \Gamma \)

(a) \( 0 \leq \Delta d_2(\gamma) \);
(b) \( 0 \leq \Delta d_1(\gamma) < c \);
(c) \( 0 \leq \varepsilon < R(0) - D(0; \gamma) < R(1) - D(1; \gamma) \).

Assumption 2-a postulates that the partner’s investment cannot lower the non-investor’s disagreement payoff. Not only is this realistic in most situations, but if this were not the case, there would be an additional reason to invest: to harm the bargaining position of the trading partner.

Assumption 2-b implies that \( 1 \) will not invest unless she expects trade between the partners. In other words, investment is worthwhile only in the expectation of reaping its benefits through the trading relationship. Assumption 2-c means that the parties’ investments are specific, in the absolute sense (on the extensive margin) that they generate higher aggregate surplus when the parties trade efficiently than when they disagree; and in the marginal sense (on the intensive margin) that this difference increases with additional investment. Furthermore, Assumption 2-c also implies that the disagreement outcome can never be efficient, so the social optimum can be characterized independently of the contract in place, in harmony with Assumption 1. Henceforth we will take it for granted that whenever we mention scenarios and contracts they satisfy Assumptions 1 and 2.

Our model is general enough to accommodate a broad set of circumstances in terms of the underlying environment and the allowed contracts/organizations. In particular, several well-known contracts are included.

Example 1. (Asset Ownership) The Grossman-Hart-Moore (GHM) model\(^{12} \) of asset ownership deals with how different ways of allocating productive assets to the parties affects their incentives for relationship-specific investments. They postulate that asset ownership directly affects the disagreement payoffs of the parties when they negotiate, à la Nash Bargaining, the terms of the trade between them. This model is clearly subsumed in our current setup when the disagreement payoffs depend on the allocation of asset ownership.

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\(^9\)\( \varepsilon \) is just a parameter that will be useful in our characterization results.

\(^{10}\)Our interpretation is that the parties already have a going concern that generates a surplus and the investment being considered is an additional one and thus it should be viewed as marginal.

\(^{11}\)In principle, a more sophisticated contract, for example, one requiring exchanges of messages, can be incorporated into our model, with \( d_i \) interpreted as the equilibrium payoff of party \( i \) in that contract (sub)game. Of course, there is the issue of how these latter payoffs are determined and what contract payoffs are feasible. These are difficult questions to address even in the static model.

Example 2. **(Exclusive Dealing)** An agreement prohibiting a trade partner from dealing with a third party is often justified by the protection that it may provide for relationship-specific investment. Segal and Whinston (2000) investigate this hypothesis using an incomplete contract model where trade partners negotiate the terms of internal trade, and their disagreement payoffs depend on the presence of an exclusivity agreement. Our setup accommodates such a model, with the contract payoff varying with the extent to which a trade partner is allowed to trade with an external third party.

As Example 2 illustrates, the absence of a contract is a special case of our model. In Che and Sákovics (2004a), where no contracts are allowed/feasible, the status quo payoffs are assumed to be zero. But this is just a normalization, and the reader should not interpret the current paper as assuming that the status quo payoffs will indeed rise as the parties sign some *ex ante* contract. As in the exclusivity example, a contract may increase or decrease the parties’ status quo payoffs.

Most previous authors studying these problems have employed the framework in which the parties invest first and then bargain over the terms of trade according to the Nash Bargaining Solution (NBS), with the contract payoffs \((d_1(I; \gamma), d_2(I; \gamma))\) serving as the disagreement point.\(^{13}\) For ease of comparison, it is useful to establish the resulting outcome as a benchmark. Suppose party 1 first chooses her investment level, and subsequently the parties bargain over the terms of trade according to the NBS. Then, party \(i = 1, 2\), will collect the gross payoff consisting of their disagreement payoff plus half of the bargaining surplus:

\[
U_i^0(I; \gamma) := d_i(I; \gamma) + \frac{R(I) - D(I; \gamma)}{2}.
\]

We consider a natural dynamic extension of this ‘investment followed by NBS’ framework. We first fix a contract \(\gamma \in \Gamma\), satisfying Assumption 2. The parties subsequently play the following investment and bargaining game:

In the first period, party 1 decides whether to make a sunk investment, \(I \in \{0, 1\}\). One of the parties is then randomly chosen to make a proposal on the terms of trade, dividing the available revenue \(R(I)\). If the proposed terms are accepted by the responder, the game ends. If not, then with probability \(1 - \delta \in (0, 1]\) the bargaining breaks down for some

\(^{13}\)Examples include Grossman and Hart (1986), Hart and Moore (1990), Edlin and Reichelstein (1996), Che and Hausch (1999), Hart and Moore (1999), Segal (1999), and Segal and Whinston (2000, 2002). An alternative approach treats contracts as affecting the outside option payoffs of non-cooperative bargaining (MacLeod and Malcomson, 1993; Chiu, 1998; De Meza and Lockwood, 1998).
exogenous reason,\textsuperscript{14} and the parties collect their contract payoffs, $d_1(I; \gamma), d_2(I; \gamma)$. With probability $\delta$, the game moves on to the next period, and the same process is repeated as in the first period, i.e., party 1 may invest (if she hasn’t done so before) followed by random-proposer bargaining, and so on and so forth, until either there is an agreement/trade or the bargaining breaks down. Our solution concept is that of Subgame Perfect Nash Equilibrium (SPNE).

Note that, when $\delta = 0$, the game ends after the first period, resulting in the standard two-stage investment-trade model. In that case, our random-proposer bargaining procedure has a unique SPNE that replicates the NBS, yielding (expected) gross payoffs $U_i^0(I; \gamma)$ to the players.

We have kept the model as parsimonious as possible, to facilitate its exposition. In the Appendix, we state and prove our results for a more general set-up, where both parties can invest and the bargaining powers (that is, the probabilities of being chosen to make an offer) can be asymmetric.

3 The implementability of efficient investment

3.1 Efficiency under an arbitrary contract

Naturally, we are interested in the existence of an efficient SPNE, where investment occurs in the first period. More precisely, we say that – given a scenario and a continuation probability – contract $\gamma$ implements investment if investment happens in the first period in some SPNE of the game induced by $\gamma$.\textsuperscript{15}

An important step of the analysis is to evaluate the continuation payoffs conditional on an investment level. To this end, suppose first that party 1 has invested. Then, since there is no further investment opportunity, the game becomes a pure bargaining game. In this case, a standard argument shows that the continuation payoffs are uniquely determined:

Lemma 1. For any $\gamma \in \Gamma$, in any subgame given investment the SPNE (continuation) payoffs are the NBS payoffs: $U_i^0(1; \gamma)$, $i = 1, 2$.

\textsuperscript{14}It is worth pointing out that this is not equivalent to discounting by a factor $\delta$. That equivalence only holds when the disagreement payoffs are normalized to zero. At the cost of added complexity and loss of focus, it would be straightforward to consider impatient players in addition to the probability of exogenous breakdown.

\textsuperscript{15}We do not require unique implementation.
It is remarkable that the parties’ equilibrium payoffs following (efficient) investment coincide with the static payoffs, irrespective of the probability of continuation. This clearly shows that the exposure to hold-up is not affected by the introduction of investment dynamics. As will be seen, however, this irrelevance result does not carry over to the incentives for investment. The next theorem characterizes the precise conditions under which a contract implements investment. It is useful to define, for any $\delta \in [0, 1)$ and contract $\gamma \in \Gamma$, a payoff function$^{16}$ for party 1:

$$U_1^\delta(I; \gamma) := (1 - \delta)d_1(I; \gamma) + \frac{R(I) - (1 - \delta)D(I; \gamma)}{2}.$$  

Notice that this coincides with party 1’s NBS payoff, $U_1^0(I; \gamma)$, when $\delta = 0$.

**Theorem 1.** For any scenario $(R(\cdot), c)$, contract $\gamma$ implements the efficient investment if and only if$^{17}$

\( (P) \quad U_1^0(1; \gamma) - c \geq d_1(0; \gamma) \)

and

\( (IC) \quad U_1^\delta(1; \gamma) - c \geq U_1^\delta(0; \gamma) - \frac{\delta c}{2}. \)

Theorem 1 identifies two conditions that are necessary and sufficient for efficient investment under contract $\gamma$. Condition $(P)$ states that party 1’s equilibrium payoff from investment (as identified in Lemma 1) must be no less than her contract payoff – which she could collect by not investing and disagreeing indefinitely thereafter. In other words, it is her Participation constraint for trade (with investment) to occur. This condition is implicit, but never binding, in the standard model of hold-up, corresponding to $\delta = 0$.

Condition $(IC)$, our Incentive Compatibility constraint, captures the need for a “marginal” benefit associated with investment on the equilibrium path: party 1 must not gain from under-investing and bargaining efficiently. It is based on the off-path behavior where, in the continuation equilibrium following under-investment, the under-investor makes up the shortfall in the next period (if the game does not end before). The derivation of condition $(IC)$ is relegated to the Appendix. Its interpretation is easier if we rearrange it as a

$^{16}$These are not actual equilibrium payoffs, but they correspond to the hypothetical payoffs that would result in the static game with the disagreement payoffs being $(1 - \delta)d_i$ (instead of $d_i$).

$^{17}$Note that the non-investor’s participation constraint is built into the bargaining payoff $U_2^0(1; \gamma)$ and it is satisfied by Assumption 2 (parts a and c). He clearly has no incentive constraint.
non-negativity constraint:

\[(IC') \quad \Delta V^\delta(\gamma) := \frac{R(1) - R(0) - (2 - \delta)c + (1 - \delta)(\Delta d_1(\gamma) - \Delta d_2(\gamma))}{2} \geq 0. \]

Differentiating with respect to \(\delta\) we obtain

\[
\frac{d\Delta V^\delta(\gamma)}{d\delta} = \frac{c - \Delta d_1(\gamma) + \Delta d_2(\gamma)}{2} > 0,
\]

where the inequality follows by Assumptions 2-a and 2-b. In words, the \((IC')\) constraint is the easier to satisfy the larger the continuation probability is.

In particular, for any given contract there are stronger incentives for investment in our dynamic model with \(\delta > 0\) than in the static model \((\delta = 0)\). Where do the extra incentives come from? As noted following Lemma 1, on the equilibrium path the parties share the returns to investment in precisely the same way as when \(\delta = 0\). The new incentives come from the equilibrium dynamics, which influences the way in which the parties split the surplus off-the-equilibrium path, following underinvestment. In particular, if following her deviation to underinvestment party 1 is expected to invest in the next period (unless there is a breakdown), then the continuation payoffs are no longer \((d_1(0; \gamma), d_2(0; \gamma))\), what would result when \(\delta = 0\). From \((IC')\) we can see that there are two new effects (the ones multiplied by \(\delta\)). The first is the determinant one: if they don’t agree today then – with probability \(\delta\) – the deviator would incur the cost of investment she is expected to make tomorrow. As a result, when her partner is chosen to make the offer today, he charges for this saving, leading to the extra term of \(\delta c/2\), relaxing the constraint.\(^{18}\) Additionally, both continuation payoffs are expected to improve with the forthcoming investment. The (expected) net impact on the agreement is captured by the term \(\delta (\Delta d_2(\gamma) - \Delta d_1(\gamma))/2\) and it can either diminish or increase the investment incentives. As a result, this second effect need not relax the constraint, but – by Assumption 2 – the sum of the two effects is always positive, as shown above.

The above discussion presupposed that the parties agree immediately following underinvestment. However, when \(\delta > \delta^N(\gamma) := \frac{R(0) - D(0; \gamma)}{R(1) - c - D(0; \gamma)},\)\(^{19}\) the return following underinvestment is less than the sum of the parties’ continuation values (assuming investment in the next period). In that case, the incentive constraint is no longer given by \((IC')\). Not investing today simply delays investment to tomorrow, what is unprofitable if and only if

\(^{18}\)When the investor makes the offer, she will not share the saving with her partner.

\(^{19}\)Note that our assumptions guarantee that \(\delta^N(\gamma) \in (0, 1)\).
the risk of breakdown is costly – that is, the disagreement payoff (following no investment) is less than the net payoff following investment

$$U_1^0(1; \gamma) - c \geq \delta \left( U_1^0(1; \gamma) - c \right) + (1 - \delta)d_1(0; \gamma),$$

what is equivalent to $(P)$.

The relative importance of conditions $(P)$ and $(IC)$ varies as $\delta$ changes. When $\delta = 0$, the left-hand sides of the two conditions are identical. Thus, since $U_1^0(0; \gamma) = d_1(0; \gamma) + \frac{R(0) - D(0; \gamma)}{2} \geq d_1(0; \gamma)$, $(IC)$ implies $(P)$. This is what explains that the literature has only been concerned about $(IC)$. Since $(P)$ does not depend on $\delta$, the monotonicity of $(IC)$ means that there is a value of $\delta$ at which the two constraints become equivalent: $\delta^T(\gamma) := \frac{R(0) - D(0; \gamma)}{\Delta d_2(\gamma) + c - \Delta d_1(\gamma)}$. Below this value $(IC)$ is the stricter constraint, while for higher values of $\delta$ $(P)$ is the stricter constraint.

In sum, for low probability of continuation $(IC)$ is sufficient (as it is stricter than $(P)$ and it is the relevant incentive constraint) and for high probability of continuation $(P)$ is sufficient (as it, rather than $(IC)$, is the relevant incentive constraint) for implementation. The latter is the situation where the investor’s dilemma – when considering a deviation – is whether to trade or not to, as in the title of the paper. In order to determine where the cutoff is, note that $(P)$ is sufficient for $\delta \geq \min\{\delta^N(\gamma), \delta^T(\gamma)\}$, since for those continuation probabilities it is either the relevant incentive constraint or it implies the incentive constraint; while $(IC)$ is sufficient for $\delta \leq \min\{\delta^N(\gamma), \delta^T(\gamma)\}$, since for those continuation probabilities it is the relevant incentive constraint and it implies the participation constraint. Let us state this result as a corollary:

**Corollary 1.** For any scenario $(R(\cdot), c)$, contract $\gamma$ implements the efficient investment if

when \[
\begin{align*}
\delta &\geq \min\{\delta^N(\gamma), \delta^T(\gamma)\}, \quad (P) \\
\delta &\leq \min\{\delta^N(\gamma), \delta^T(\gamma)\}, \quad (IC)
\end{align*}
\]

Next, we investigate how Corollary 1 affects the comparison of alternative contracts.

### 3.2 The effects of contracts

The key factor in static models is the extent to which a contract enables a party to appropriate her investment returns, or equivalently, to reduce her exposure to hold-up “at the (investment) margin”. In our model, this effect is captured by the terms $\Delta d_j(\gamma)$, for investor $j = 1$ and non-investor $j = 2$. Specifically, $\Delta d_1(\gamma)$ represents the marginal return of investment that investor 1 appropriates according to contract $\gamma$, and $\Delta d_2(\gamma)$ reflects the
marginal return of 1’s investment “leaked” to the non-investor, 2. The former protects the investor from, while the latter exposes her to, the hold-up problem at the margin. Indeed, as we have seen from \((IC')\), condition \((IC)\) can be written using these marginal terms and it conforms to the standard insight that \(\Delta d_1(\gamma)\) relaxes, while \(\Delta d_2(\gamma)\) tightens the constraint.

The marginal returns are neither the only drivers of the dependence of \((P)\) on the contract in place, nor do they affect it in the same way as \((IC)\). To see this, rewrite \((P)\) as:

\[
(P') \quad \frac{R(1) - D(1; \gamma)}{2} + \Delta d_1(\gamma) \geq c.
\]

The first term is the proportion retained from the (efficient) trade surplus \(R(1) - D(1; \gamma)\) – the degree to which a party is exposed to hold-up in the absolute sense (that is, taking investment as given). The added term is the investor’s marginal return, \(\Delta d_1(\gamma)\), which relaxes her participation constraint – just as in \((IC)\). However, the “leakage” of her investment returns, \(\Delta d_2(\gamma)\), does not affect the participation constraint. Due to these differences, \((P')\) yields a distinct new insight about the role of contracts – whenever it is the sufficient constraint.

To see the main implication of a contract’s role being to relax the participation constraint, it is useful to start with the extreme case, where the investment is totally relationship specific so that a failure to consummate trade leads to no change in the payoffs due to investment: \(\Delta d_i(\gamma) \equiv 0, i = 1, 2\). Condition \((IC')\) is the same for all such contracts: they are indistinguishable with regard to this condition, for any \(\delta\). Yet, they are not same with respect to \((P')\). In particular, the extent to which contracts expose the parties to hold-up conditional on investment – \(R(1) - D(1; \gamma)\) – is affected by the contract. Strikingly, \((P')\) tells us that – independently of how the aggregate disagreement payoff is shared – a contract maximizing the parties’ (aggregate) exposure to hold-up (minimizing their aggregate disagreement payoff, \(D(1; \gamma)\)) is the one that relaxes the constraint the most. The same insight carries over even when the investment is not fully relationship specific, as long as the marginal effects of two compared contracts are identical.

In order to present our formal results, we need to specify at which point we are comparing the contracts. One possibility is the \textit{ex post} view, when the scenario and the breakdown probability are (already) known. This is the least demanding way in which a contract can be superior to another, at the price of the relation applying only in a specific situation.

\textbf{Definition 1.} \textit{Given a scenario \((R(.), c)\) and continuation probability \(\delta\)
(i) **contract** \(\gamma'\) **trumps** contract \(\gamma\) **if it implements investment while** \(\gamma\) **does not;**

(ii) **contract** \(\gamma'\) **ties** contract \(\gamma\) **if either both contracts implement investment or neither of them do.**

The other extreme is the *ex ante* view, when we require a contract to be superior behind the veil of ignorance: for all possible scenarios.\(^{20}\) This approach is useful, in particular, when the contract covers different, perhaps even *ex ante* unknown, investment/trade opportunities.

Let \(\mathcal{D}\) denote an arbitrary set of continuation probabilities, and \(S(\gamma, \gamma')\) the set of scenarios satisfying Assumptions 1 and 2 given contracts \(\gamma\) and \(\gamma'.\)\(^{21}\)

**Definition 2.** (i) **Contract** \(\gamma'\) **weakly dominates** contract \(\gamma\) **for** \(\delta \in \mathcal{D},\) if \(\gamma'\) **implements investment for all values of** \(\delta \in \mathcal{D},\) \((R(,), c) \in S(\gamma, \gamma')\) **such that** \(\gamma\) **implements it** \((\gamma'\) **either trumps or ties** \(\gamma\) **for all scenarios in** \(S(\gamma, \gamma')\) **and** \(\delta \in \mathcal{D}\).**

(ii) **Contract** \(\gamma'\) **is optimal in** \(\Gamma\) **for** \(\delta \in \mathcal{D},\) **if it weakly dominates all** \(\gamma \in \Gamma\) **for** \(\delta \in \mathcal{D}.\)

(iii) **Contract** \(\gamma'\) **dominates** contract \(\gamma\) **for** \(\delta \in \mathcal{D},\) if \(\gamma'\) **weakly dominates** \(\gamma\) **but** \(\gamma\) **does not weakly dominate** \(\gamma'\) \((\gamma'\) **trumps** \(\gamma\) **for some scenario and ties it for the rest in** \(S(\gamma, \gamma')\), **for** \(\delta \in \mathcal{D}\).**

(iv) **Contracts** \(\gamma\) **and** \(\gamma'\) **are equivalent for** \(\delta \in \mathcal{D},\) **if** \(\gamma'\) **implements investment if and only if** \(\gamma\) **implements it** \((\text{they tie for all scenarios in } S(\gamma, \gamma'), \text{ for } \delta \in \mathcal{D}).\)

We can now state our starkest result formally:

**Proposition 1. (No-trade Payoff Minimization Principle)** Suppose for a pair of contracts \(\gamma', \gamma,\) \(\Delta d_i(\gamma') = \Delta d_i(\gamma), i = 1, 2.\) If\(^{22}\)

\[
D(1; \gamma') < D(1; \gamma),
\]

---

\(^{20}\)If we required the relation to hold for all scenarios and all continuation probabilities, then improving over the optimal contracts for the static model would be trivially impossible.

\(^{21}\)As Assumption 1 is independent of the contract in place, while Assumption 2 is about specificity, what makes investment harder to implement, in keeping with the literature, we compare contracts only for scenarios where both contracts satisfy the assumptions. A contract that reduces specificity is clearly beneficial.

\(^{22}\)Given the supposition, the aggregate disagreement payoffs are ordered the same way with or without investment.
then there exists $\delta^* \in [0, 1)$

i) $\gamma'$ and $\gamma$ are equivalent for $\delta \leq \delta^*$,

ii) $\gamma'$ dominates $\gamma$ for $\delta > \delta^*$.

Thus, among contracts that provide the same marginal protection against hold-up it is a good policy to reduce the aggregate disagreement payoff whenever the continuation probability is sufficiently high (and it does not hurt otherwise). Consequently, even if $\delta$ were unknown, lowering the no-trade payoff would be advisable.

The result follows from three observations. First, since $(IC)$ stays constant across the contracts, while $(P)$ is relaxed, $\gamma'$ weakly dominates $\gamma$. Second, for some scenarios $(P)$ will be satisfied under the new contract when originally it was not, and thus if $(P)$ is sufficient at some of those scenarios, $\gamma'$ dominates $\gamma$. Third, by Corollary 1, if $\gamma'$ dominates $\gamma$ for $\delta$, it also dominates it for any higher $\delta$, since for none of these scenarios will a higher $\delta$ make $(IC)$ the sufficient constraint if it was not before. Finally, to see that $\delta^* < 1$, just note that, by Assumption 1, $(IC)$ is always satisfied for $\delta$ close enough to 1, as it is strictly satisfied in the hypothetical case that $\delta = 1$.

For $\delta$ high enough, the same effects behind the No-trade Payoff Minimization Principle continue to work even when the marginal effects might act in the opposite direction. For low $\delta$, the standard intuition applies:

**Proposition 2.** Consider any pair of contracts $\gamma', \gamma$. There exist $\delta'(\varepsilon, K) < \delta'' \in [0, 1)$, such that contract $\gamma'$ dominates contract $\gamma$

a) for any $\delta < \delta'$, if $\Delta d_1(\gamma') - \Delta d_2(\gamma') > \Delta d_1(\gamma) - \Delta d_2(\gamma)$;

b) for any $\delta > \delta''$, if $d_1(1; \gamma') - d_2(1; \gamma') - 2d_1(0; \gamma') > d_1(1; \gamma) - d_2(1; \gamma) - 2d_1(0; \gamma)$.

Thus, we find the standard insight to be somewhat robust to introducing a small probability of continuation: for a small $\delta$, the extent to which contracts influence the investors’ exposure to hold-up “at the margin” explains the relative performance of contracts well: we wish to increase the marginal protection of the investor from hold-up ($\Delta d_1(\gamma)$), while decreasing the leakage of investment returns ($\Delta d_2(\gamma)$). The same cannot be said, however, when $\delta$ is large. In this case, a more complex version of the No-trade Payoff Minimization Principle applies: we wish to decrease the no-trade payoffs of the investor following

---

23 Recall that we do not allow for zero probability of breakdown, in which case contracts would (almost) never matter.

24 Allowing for all scenarios the robustness disappears, but with domain restrictions it can be substantial.
no investment, while decreasing that of her partner following investment (in addition to increasing the investor’s no-trade payoff following investment). Given Corollary 1, the logic of this result is straightforward: since we are dealing with a participation constraint, it is beneficial to reduce the outside option. This involves no investment (due to relationship specificity), that is why the investor’s disagreement payoffs are to be minimized following no investment. The rest of the no-trade payoffs are affecting the share of the surplus following investment, but only at “half intensity” as the bargaining power is shared. Since the investor’s payoff is increasing in her own and decreasing in her partner’s no-trade payoffs, the result follows.

It is interesting to observe that $d_1(1; \gamma) - d_2(1; \gamma) - 2d_1(0; \gamma) \equiv \Delta d_1(\gamma) - \Delta d_2(\gamma) - D(0; \gamma)$. That is, the sufficient statistic for a contract for high $\delta$ involves the same difference of the marginal effects as when $\delta$ is low, “corrected” by the aggregate disagreement payoff (without investment).

Another insightful way to rewrite the expression is $d_1(1; \gamma) - d_2(1; \gamma) - 2d_1(0; \gamma) \equiv 2\Delta d_1(\gamma) - D(1; \gamma)$, what leads to the observation above that the leakage of investment returns has no effect for high values of $\delta$. Due to its relevance, we state it as a separate corollary.

**Corollary 2. (Irrelevance of Leakage)** Suppose for a pair of contracts $\gamma'$ and $\gamma$

$$\Delta d_1(\gamma) = \Delta d_1(\gamma') \text{ and } D(1; \gamma') = D(1; \gamma),$$

while

$$\Delta d_2(\gamma) < \Delta d_2(\gamma').$$

Then there exists $\delta''(\varepsilon, K) \in [0,1)$ such that$^{25}$

i) $\gamma$ dominates $\gamma'$ when $\delta < \delta''$, and 

ii) $\gamma$ and $\gamma'$ are equivalent when $\delta \geq \delta''$.

### 3.2.1 Illustration

To illustrate the new insight, consider a simple example in which party 1 has a fully relationship specific investment decision. Depending on whether investment is made, the

$^{25}$It is easy to see that – for every scenario – the cut-off is $\text{min}\{\delta^N(\gamma'), \delta^T(\gamma')\}$. Note that, given the hypothesis, $D(0; \gamma) = D(1; \gamma) - \Delta d_1(\gamma) - \Delta d_2(\gamma) > D(0; \gamma')$, so since, by Assumption 1, $\delta^N$ is decreasing in $D(0), \delta^N(\gamma) < \delta^N(\gamma')$. $\delta''$ must then be in between these two values, where the incentive and participation constraints (for contract $\gamma$) cross (if they do).
aggregate returns and the contract payoffs under three contracts – $\gamma$, $\gamma'$ and $\gamma''$ – are as follows:

<table>
<thead>
<tr>
<th>payoffs</th>
<th>$R$</th>
<th>$d_1(\gamma), d_2(\gamma)$</th>
<th>$d_1(\gamma'), d_2(\gamma')$</th>
<th>$d_1(\gamma''), d_2(\gamma'')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>not investing</td>
<td>2</td>
<td>1, 1</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>0, 0</td>
</tr>
<tr>
<td>investing</td>
<td>5</td>
<td>1, 1</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
<td>$\frac{1}{2}, \frac{1}{2}$</td>
</tr>
</tbody>
</table>

Assume the cost $c$ of investing is less than 3, the aggregate gain from investment, so it is socially desirable for party 1 to invest. The standard hold-up model would predict the three contracts to perform equally. None of the contracts lower the investor’s exposure to hold-up at the “margin” (c.f. Proposition 1). Specifically, under contracts $\gamma$ and $\gamma'$, investing never improves that party’s no-trade payoff. Under contract $\gamma''$, investing improves her no-trade payoff by $\frac{1}{2}$, but this is “offset” by the concomitant improvement in her opponent’s no-trade payoff (i.e., the leakage of the investment return). Consequently, since the investor (party 1) internalizes only half of the return $\frac{1}{4}(5 - 2) = 1.5$, all three contracts will implement the efficient outcome in the static model if and only if $c \leq 1.5$.

In our dynamic model two changes arise. First, the same contracts may implement investment for higher costs. Second, these improvements are not homogeneous across contacts. A graphical representation of the parameter configurations that implement efficient investment is provided below.

Let us start with our “bogey man” contract, $\gamma$, which allows for no trade surplus in the absence of investment. In this case, the participation constraint is the same as the incentive constraint with $\delta = 0$. Therefore, the dynamics is useless in incentivizing investment given this contract.

Now, consider $\gamma'$. A shift from $\gamma$ to $\gamma'$ relaxes the participation constraint since $\gamma'$ has a lower joint no-trade payoff (c.f. Proposition 1). It is straightforward to verify that $\delta^N(\gamma') = \frac{1}{4 - c}$, while $\delta^T(\gamma') = \frac{1}{c}$. The former is the lower value whenever $c \leq 2$, which is exactly the new participation constraint. Thus, by Corollary 1, for $\delta \leq \frac{1}{4 - c}$ the incentive constraint is necessary and sufficient, so the upper bound on the implementable cost (in this area) is relaxed to $c \leq \frac{3}{2 - 3}$. When $\delta > \frac{1}{4 - c}$, the participation constraint binds instead.

---

26 This is not a coincidence. It can be shown in general that $\delta^N \leq \delta^T$ if and only if the participation constraint is satisfied. See the proof of Corollary 1.

27 The LHS of $(IC'')$ becomes $$(5 - 2) - (2 - \delta)c,$$
which is nonnegative if $\delta \geq 2 - \frac{3}{c}$, which is equivalent to $c \leq \frac{3}{2 - 3}$.  

17
leading to $c \leq 2$.

Finally, let us turn to $\gamma''$. A shift from $\gamma'$ to $\gamma''$ further relaxes both constraints since $\gamma''$ reduces the investor’s exposure to hold-up at the margin (c.f. Proposition 2) and the concomitant leakage has no effect on $(P')$ (c.f. Corollary 2). $(P')$ now becomes $c \leq \frac{5-1}{2} + .5 = 2.5$. It is straightforward to verify that $\delta^N(\gamma'') = \frac{2}{5-c}$, which is less than $\delta^T(\gamma'') = \frac{2}{c}$ for $c \leq 2.5$. Thus, by Corollary 1, for $\delta \leq \frac{2}{5-c}$ we have the incentive constraint binding – which is unchanged, as the marginal effects of investment on the two parties cancel out – so the upper bound on the implementable cost continues at $c \leq \frac{3}{2-\delta}$. When $\delta \geq \frac{2}{5-c}$, the participation constraint binds, leading to $c \leq 2.5$.

4 Applications

In what follows, we explore how the new insights apply to some well-known contract types. In particular, we illustrate how investment dynamics influences the prescriptions for organizational design.

4.1 The Grossman-Hart-Moore (GHM) model of asset ownership

Suppose there are two assets – in principle, one “operated” by each party – $A = \{a_1, a_2\}$. An ownership structure, $\gamma$, is represented by a pair, $(A^*_{i}, A^\gamma_{i})$, where $A^*_{i} \subset A, i = 1, 2$, stands for the asset(s) party $i$ owns. There are four alternative structures: (1) separate ownership
or non-integration: \( \gamma_N := (\{a_1\}, \{a_2\}) \);28 (2) common ownership (or integration) by party 1: \( \gamma_1 := (\{a_1, a_2\}, \emptyset) \); (3) common ownership (or integration) by party 2: \( \gamma_2 := (\emptyset, \{a_1, a_2\}) \); and (4) joint ownership, where neither party has the residual rights (they need the partner’s agreement) \( \gamma_J := (\emptyset, \emptyset) \).

In the GHM theory, a party’s contract payoff, \( d_i(I; \gamma) \), represents the revenue that she can generate by exercising her residual property rights in the event of disagreement. This set-up easily lends itself to analysis in our dynamic model in which, given some ownership structure, the parties play our investment-trading game. Following Hart (1995), we make a few assumptions. First, a party’s contract payoff depends only on the asset(s) he owns: i.e., \( d_i(I; \gamma) = d_i(I; \gamma') \) if \( A_i^\gamma = A_i^{\gamma'} \), for all \( I \). Second, owning more assets can only raise one’s contract payoff: \( d_i(I, \gamma) \leq d_i(I, \gamma') \) if \( A_i^\gamma \subseteq A_i^{\gamma'} \), for all \( I \). Third, the investments are interpreted as acquisition of human capital not embodied in the assets, so it is reasonable to assume that one’s contract payoff does not depend on his partner’s investment: \( \Delta d_2(\gamma) \equiv 0 \). That is, there is no leakage of investment returns.

The crucial element in the GHM theory is the extent to which each ownership structure determines one’s exposure to hold-up at the margin. Consequently, it is convenient for alternative ownership structures to be well ordered in this respect, which is accomplished by requiring that additional assets owned reduce this exposure (recall that we have assumed away “leakage”):

Assumption 3. \( \Delta d_1(\gamma) \leq \Delta d_1(\gamma') \) if \( A_i^\gamma \subset A_i^{\gamma'} \).

Given this assumption, there are two salient ways that the ownership structures can be ordered:

Definition 3. The assets are marginal substitutes if
\[
\Delta d_1(\gamma_1) = \Delta d_1(\gamma_N) > \Delta d_1(\gamma_2) = \Delta d_1(\gamma_J).
\]

The assets are marginal complements if
\[
\Delta d_1(\gamma_1) > \Delta d_1(\gamma_N) = \Delta d_1(\gamma_2) = \Delta d_1(\gamma_J).
\]

To highlight our new insight relative to the existing one, we define two salient cases of interest, in terms of how asset ownership affects the parties’ absolute aggregate exposure to hold-up.

28 We ignore \((\{a_2\}, \{a_1\})\), for simplicity and consistency with GHM. Actually, “cross ownership” might be an additional safeguard against opportunism. See, for example, Heide and John (1988).
Definition 4. The assets are **substitutes** if

\[ D(0; \gamma_N) > D(0; \gamma_1) = D(0; \gamma_2) = D(0; \gamma_J). \]

The assets are **complements** if

\[ D(0; \gamma_1) = D(0; \gamma_2) > D(0; \gamma_N) = D(0; \gamma_J). \]

When assets are substitutes, then, starting from separate ownership (and efficient investment), if a party gains an asset, his disagreement payoff does not rise as much as the other party’s disagreement payoff declines. In this sense, the assets are more valuable (outside the relationship) when owned separately than when owned under a common ownership. In the same sense, complementary assets are more valuable (outside the relationship) when owned by the same party than when they are owned separately. Joint ownership always leads to the worst aggregate disagreement payoff, as neither party has residual control.

Invoking Proposition 1 we have a stark result when the investor’s marginal protection is constant across contracts:

**Proposition 3.** Suppose that in addition to Assumption 3, \( \Delta d_1(\gamma) = \Delta d_1(\gamma') \). Then there exists \( 0 \leq \tilde{\delta} < 1 \) such that, when \( \delta \leq \tilde{\delta} \) all ownership structures are equivalent. When \( \delta > \tilde{\delta} \)

a) If the assets are complements, then joint ownership is equivalent to separate ownership and they dominate either form of integration;

b) If the assets are substitutes, then joint ownership is equivalent to either form of integration and they dominate separate ownership.

When the investor’s marginal protection is affected by the contract, the results are less clear-cut but we can invoke Proposition 2, to make a series of observations.

**Proposition 4.** Suppose Assumption 3 holds. Then there exist \( 0 \leq \tilde{\delta}(\varepsilon) < \tilde{\delta} < 1 \) such that

i) if \( \delta < \tilde{\delta} \), then

a) if the assets are marginal substitutes, then separate ownership is equivalent to common ownership by the investor, both of which dominate common ownership by the non-investor, which is equivalent to joint ownership;

---

29. Of course, a cardinal ranking of the disagreement payoffs would lead to a full ranking of ownership structures
b) if the assets are marginal complements, then common ownership by the investor dominates all other ownership structures (which are equivalent);

ii) if $\delta > \delta^*$, then

c) if the assets are substitutes, then common ownership by the investor dominates separate ownership;

d) if the assets are complements, but marginal substitutes, then separate ownership dominates common ownership by either party.

Proposition 4-i)a) and -i)b) find the robustness of the GHM prescription that marginal substitutes should be owned separately and marginal complements should be owned together, to introducing a small probability of continuation of bargaining and possible investment dynamics. On the other hand, parts ii)c) and ii)d) show results of the “opposite flavor” to hold, if $\delta$ is sufficiently large. In this case, the comparison between ownership structures is a horse race between the marginal protection to hold-up ($2\Delta d_1(.)$) and the aggregate disagreement payoff ($D(0;.)$). When assets are substitutes, then irrespective of the marginal complementarity/substitutability, common ownership by the investor – what provides the lower aggregate disagreement payoff – dominates separate ownership. Complementary assets, on the other hand, should be owned separately if they are marginal substitutes. If they are marginal complements, then the result depends on the net effect, and cannot be stated in general. In a similar way, if the disagreement payoff effect dominates that of marginal protection, joint ownership may be optimal.

As the predictions of the static and dynamic models are so starkly different, it is worth relating them to the actually observed organizational structure. Hart (1995) alludes to the evidence – Joskow (1985), Stuckey (1983) – that complementary assets are usually vertically integrated as proof in favor of the superiority of asset integration. We believe that this evidence is subject to interpretation. In these examples it is unclear whether within the vertically integrated firm we still have two parties making relationship-specific, non-contractible investments. In fact, a principal-agent relationship is more likely. In the cases where the GHM paradigm clearly applies, vertical integration is often better described as a merger and thus corresponds to a joint ownership structure.30

30See Whinston (2003) for a discussion on the little guidance that the empirical literature can give us regarding the applicability of the GHM paradigm.
4.2 Exclusive Dealing

An agreement to deal with a partner excluding all others has been the subject of much debate. Antitrust authorities have either banned or held in suspicion any exclusive practices that may foreclose competition. Others suggested that the voluntary nature of such agreements may reflect some efficiency benefits they might bring. One such hypothesis is that the security of the trading relationship such an agreement brings can motivate the partners to make relationship-specific investments: in other words, exclusivity may protect from hold-up.

Whether this hypothesis holds true can be studied within the framework of our model. There are four possibilities, as exclusivity may be granted to either party, to both parties, or to neither of them. Let $X_i$ denote an agreement for party $i = 1, 2$ not to engage in external trade, $X_b$ the agreement for both parties not to trade externally, and $NX$ the absence of such agreement. Thus $\Gamma = \{X_1, X_2, X_b, NX\}$. It is reasonable to assume that the opportunity to trade externally is valuable:

$$D(\cdot; NX) > D(\cdot; X_i) > D(\cdot; X_b), i = 1, 2.$$  

The returns on investment transferable outside the relationship are by definition zero for some of the contracts: $\Delta d_1(X_1) = \Delta d_2(X_2) = \Delta d_1(X_b) = \Delta d_2(X_b) = 0$, that is the very point of an exclusivity deal. For the rest of the contract payoffs, there are three cases of special interest. The first is one where the specific investments are fully relationship specific. For instance, a specialized investment tailored to one’s partner may be lost when one changes his partner. This implies that $\Delta d_j(\gamma) = 0$ for all $\gamma \in \Gamma$ and $j = 1, 2$. Segal and Whinston (2000) found that an exclusivity agreement is of no value in promoting investments in this situation.

Next, is the case where investment is transferable but in a way that benefits only the investor, in the sense that $\Delta d_1(NX) = \Delta d_1(X_2) > 0 = \Delta d_2(\cdot)$. That is, there is no leakage of investment returns to the partner. This is a reasonable assumption in most trading relationships. The last focal case is one where the investment benefits only the trading partner: $\Delta d_1(NX) = \Delta d_1(X_2) = 0 < \Delta d_2(X_1) = \Delta d_2(NX)$. Such “leakage” of investment is an issue for sports clubs and entertainment agencies, which often discover, train and groom their talents, only to see them switching to different teams or different agencies, taking with them the human capital and marketing assets cultivated by the original partner. Invoking Propositions 1 and 2, the following series of results hold.
Proposition 5. a) Suppose the investments are non-transferable in the sense that $\Delta d_j(\gamma) = 0$ for all $\gamma \in \Gamma$ and $j = 1, 2$. Then there exist $0 \leq \delta(\varepsilon) < \delta < 1$ such that, for $\delta \leq \delta$, all arrangements in $\Gamma$ are equivalent but, for $\delta > \delta$, $X_b$ dominates $X_i$, $i = 1, 2$, which in turn dominate $NX$.

b) Suppose $\Delta d_1(NX) = \Delta d_1(X_2) > 0 = \Delta d_2(X_1) = \Delta d_2(NX)$. Then there exist $0 \leq \delta(\varepsilon) < \delta < 1$ such that, for $\delta \leq \delta$, $NX$ is equivalent to $X_2$ which in turn dominates $X_b$ that is equivalent to $X_1$, but for any $\delta > \delta$, $X_1$ is dominated by $X_b$, while $X_2$ dominates $NX$. When $\Delta d_1(X_2) \leq D(0; X_2) - D(0; X_b)$, $X_b$ is optimal.

c) Suppose $\Delta d_1(NX) = \Delta d_1(X_2) = 0 < \Delta d_2(X_1) = \Delta d_2(NX)$. Then, $X_b$ is optimal.

Part a) contrasts the differences between the cases with small $\delta$ and large $\delta$. In the former case, exclusive dealing has no effect on the investment incentives, as it was found by Segal and Whinston (2000), since when investments are fully relationship specific, exclusivity affects the scope of the hold-up problem only in absolute terms. With a large $\delta$, this latter effect matters, however, and exclusivity does promote investment, the more so with exclusivity imposed on both parties than just on one party.

Part b) deals with the case in which the investment is transferable to external trade for the investor. In this case, the possibility of external trade actually improves the incentives at the margin, so with small probability of continuation, exclusivity offered by the investor is strictly undesirable. Exclusivity offered by the investor’s partner is irrelevant. With large $\delta$, however, by the no-trade payoff minimization principle, it is a good idea to require exclusivity by the non-investor, irrespective of whether the investor is committed. Exclusivity offered by both players is optimal as long as the extent of the transferability is small.

Finally, when investment returns “leak”, exclusivity offered by both parties promotes investments regardless of $\delta$. Leakage of investment returns undermines the incentives, which can be avoided by removing the partner’s access to external trade. Granting exclusivity to party 1, which prohibits party 2’s external trade, promotes the former’s investment for $\delta$ small (exclusivity to party 2 does not matter). Otherwise, the leakage by itself does not pose a problem, but the parties’ aggregate exposure to the holdup becomes important. Exclusivity increases the exposure, which increases their ability to punish, and thus improves the incentives even of those who grant the exclusivity clause.

De Meza and Selvaggi (2007) also find that exclusive dealing may promote specific investment, but in a markedly different model. In addition to a buyer and a seller, they
explicitly model a third party (another buyer), who makes no investment but can either trade with the seller directly or can buy the good off the other buyer – when this is efficient. If the investing buyer has exclusivity protection, the second buyer can only participate in an eventual resale. Their result and ours complement each other towards establishing a positive role exclusivity may play in promoting specific investments.

5 Extension to trade contracts

In the main part of the analysis we have assumed that the contracts are only invoked if trade does not take place. In other words, trade only happened conditional on agreement, leading to the dilemma highlighted in the title of the paper. It is nonetheless interesting to consider the situation where the contracts continue being the disagreement payoffs in the (re)negotiation, but they need not preclude – the “imposition” of – trade. In other words, the contract can specify default terms of trade, and the parties (re)negotiate over the terms of trade to (potentially) replace the default ones. In this section we extend our analysis to deal with this scenario.

The inability to contract on investment can often be overcome indirectly, through contracting on the price and quantity of \textit{ex post} trade. However, a number of scenarios have been identified in which a trade contract does not deliver full efficiency. We will discuss two of these. Both scenarios recognize the renegotiability of contracts as an important ingredient to obtain this result. In addition, they require some assumptions about the nature of specific investments: either cooperativeness or unpredictability of investment benefit, which will be described more fully below. Again, our set-up is well suited to subsume these scenarios.

Example 3. \textit{(Trade contracts with cooperative investment)} The parties may contract \textit{ex ante} on the terms of trade, which may later be renegotiated. Many authors have analyzed the effects of such contracts on the incentives for specific investments (Edlin and Reichelstein, 1996; Che and Hausch, 1999, and Segal and Whinston, 2002, among others). Of particular interest is the case in which investments are cooperative in the sense that investors do not directly benefit from their investments, as such investments have been found particularly difficult to motivate via \textit{ex ante} trade contracts (Che and Hausch, 1999). These models and the related questions can be reexamined in our dynamic context, when the disagreement payoffs are allowed to depend on the terms of trade initially agreed upon.
Example 4. *(Contracting in a complex environment)* Often parties to an *ex ante* contract trade in a complex environment, which makes it difficult for them to forecast the type of trade that will best harness their specific investments. Segal (1999) and Hart and Moore (1999) consider a model in which a seller and a buyer can trade one many different types of “widgets.” One of the types becomes *ex post* optimal to trade, and the parties’ investments only raise the value of trading the special type of widget. They find *ex ante* trade contracts to be of little value when it is nearly impossible to predict the special type of widget. This model again lends itself to our dynamic setup, with the disagreement payoffs allowed to depend on the type of widget that the parties may initially agree to trade.

5.1 Cooperative Investments

Two parties, a seller (party 1) and a buyer (party 2), have an opportunity to trade \( q \in Q \subset \mathbb{R}_+ \) units of a good, which would cost party 1 \( w(q, I) \) and would provide a value of \( v(q, I) \) to party 2, given investment (by party 1) \( I \).\(^{31}\) The parties can initially sign a contract \((\hat{q}, \hat{t}) \in \Gamma \subset Q \times \mathbb{R}\) which obligates them to trade \( \hat{q} \) units in exchange for payment \( \hat{t} \), unless they renegotiate. The set \( \Gamma \) includes the possibility of the null contract, \((0, 0)\), with an associated outcome \( v(0, \cdot) = w(0, \cdot) = 0 \).\(^{32}\) Given the trade contract, the parties renegotiate according to the extensive form specified in Section 2. In particular, if the bargaining breaks down they trade \( \hat{q} \) and collect gross payoffs, \( d_1(I; \hat{q}, \hat{t}) = \hat{t} - w(\hat{q}, I) \) and \( d_2(I; \hat{q}, \hat{t}) = v(\hat{q}, I) - \hat{t} \), respectively.

Of particular interest are purely cooperative investments, such that the investor does not directly benefit from her investment:

\[
\Delta d_1(\gamma) = w(\hat{q}, 0) - w(\hat{q}, 1) = 0 < \Delta d_2(\gamma) = v(\hat{q}, 1) - v(\hat{q}, 0)
\]

for any \( \gamma = (\hat{q}, \hat{t}) \) with \( \hat{q} > 0 \). In other words, any non-trivial contract increases one’s exposure to hold-up at the margin. Accordingly, Che and Hausch (1999) find that in the static model the null contract dominates any non-trivial trade contract.

Whether this conclusion holds true in the current dynamic model depends on whether there exists an (excessive) trade level\(^{33}\) – conditional on investment – that will generate a gross aggregate loss (and therefore a lower aggregate contract payoff than the null contract).

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\(^{31}\)This can be reconciled with the previous notation by setting \( R(I) = \max_q(v(q, I) - w(q, I)) \).

\(^{32}\)Of course, for \( \gamma \in \Gamma \), we also require Assumption 2 to hold.

\(^{33}\)This is basically a question of what levels of trade a judge is willing to enforce.
Proposition 6. Suppose investments are purely cooperative. Then there exist $0 \leq \delta(\varepsilon) < \hat{\delta} < 1$ such that,

a) The null contract is optimal, for $\delta \leq \hat{\delta}$.

b) If $v(\hat{q}^-, 1) - w(\hat{q}^-, 1) \geq 0$, then the null contract is optimal, regardless of $\delta$.

c) If $v(\hat{q}^-, 1) - w(\hat{q}^-, 1) < 0$, then a contract to trade $\hat{q}^-$ is optimal, for $\delta > \hat{\delta}$.

Thus, the optimality of the null contract found by Che and Hausch may extend to our dynamic model even with high probability of continuation, but it requires that any feasible trade lead to a surplus. Otherwise, contracting to realize a trade loss can incentivize investment and renegotiation.

5.2 Complexity (the “widget model”)

Suppose a seller (party 1) and a buyer (party 2) can trade one of $n$ different types of “widgets.” Let $q \in Q^n$ represent a particular type of widget, with $|Q^n| = n + 1$. $q = 0 \in Q^n$ represents no trade. After party 1 has decided whether to invest, the parties learn one of the widget types to be special, in that it generates higher joint surplus. Each widget has equal chance of becoming special. The special widget, regardless of its type, costs party 1 $w(I)$ and and yields the surplus of $v(I)$ to party 2. If widget $q$ is ordinary, then it costs $w_q$ to party 1 and yields the surplus of $v_q$ to party 2, with $v_0 = w_0 = 0$. Note that investment does not affect either the cost or the benefit of trading an ordinary widget. We assume $R(I) := v(I) - w(I) > v_q - w_q, I \in \{0, 1\}$ and $\forall$ (ordinary) $q \in Q^n$, so that it is efficient for the parties to trade the special widget.

As the investment’s value is realized only when the special good is traded, it entails the same sort of difficulties with ex ante contracts in generating incentives as cooperative investment. Specifically, the parties may sign a contract that requires them to trade a particular type $\hat{q} \in Q^n$ of widget for some transfer payment $\hat{t}$. The disagreement payoffs for parties 1 and 2 are random, since the type $\hat{q}$ becomes special with probability $\frac{1}{n}$ and ordinary with the remaining probability. Segal (1999) and Hart and Moore (1999) considered such
a model. Of special interest is the limiting case in which the environment gets complex in
the sense that \( n \to \infty \). Let \( Q = \lim_{n \to \infty} Q^n \). We consider this limiting case. Assume \( Q \) is
compact.

Suppose the parties contract to trade a particular type \( \hat{q} \). There is zero probability that
that type will be special, so the contract payoffs are \( d_1(I; \hat{q}, \hat{t}) = \hat{t} - w_\hat{q} \) and \( d_2(I; \hat{q}, \hat{t}) = v_\hat{q} - \hat{t} \).
Notice that these payoffs do not depend on the investments at all. Hence, \( \Delta d_j(\gamma) = 0 \), for
all \( \gamma \in \Gamma := Q \times \mathbb{R}, j = 1, 2 \). Again, it is useful to define the type of widget,

\[
\hat{q}^* \in \arg \min_{q \in Q} [v_q - w_q],
\]

that would lead to the worst joint payoff unless renegotiated. We obtain a result similar to
that with the cooperative investment.

**Proposition 7.** *It is optimal for the parties to contract to trade \( \hat{q}^* \).*

The intuition behind this result is clear. Due to the full relationship specificity of
investment, all contracts are equivalent with respect to \( (IC) \). Hence, alternative contracts
can be only be differentiated by \( (P) \). Proposition 1 then implies that the contract to trade
the “worst” type of widget is optimal. Of course, such a contract may boil down to the
null contract.

In sum, our results from both cases suggest that the foundations of some incomplete
contracts can sometimes be justified in the dynamic setting but only if the null contract
minimizes the gains from trade.

### 6 Related Literature

Several papers have developed somewhat similar insights, though in differing modelling
contexts. Of special note are repeated games and relational contracting. For example,
Baker et al. (2001, 2002) demonstrate that the absolute payoff levels can affect the efficiency
ranking of different ownership structures in a repeated trade setting. This is because the
set of self-enforcing contracts that are sustainable depend on the (absolute) payoffs one
gets from breaching the contract, which in turn vary with the ownership structure. Thus,

\[\text{This is a bit awkward: there are no marginal effects of investment on the disagreement payoffs, which is the standard definition of relationship specificity, but the disagreement payoffs are also realized within the relationship.}\]
in order to make the best relational contract possible, a specific way of allocating the assets may be superior.

The repeated trade opportunities assumed in these papers make the folk theorem of repeated games applicable, which implies that an efficient outcome is sustainable as $\delta \to 1$, irrespective of the underlying organizational arrangements. In this sense, the organizational issues become irrelevant for a sufficiently large $\delta$. By contrast, the parties have a single trading opportunity in our model (just as in the standard hold-up problem), which makes the folk theorem inapplicable. Indeed, contract design remains relevant even when $\delta \approx 1$ in our model. At the same time, these games\(^{35}\) have the feature that the worse the worst aggregate equilibrium payoff is, the better it incentivizes the most efficient outcomes, resembling the No-trade Payoff Minimization Principle. However, the underlying intuition is completely different. These equilibria are constructed by the threat of a switch to the “bad” equilibrium, that is why a powerful threat is useful.\(^{36}\) Our model does not exploit the multiplicity of equilibria.

Evans (2008) maintains the standard assumption that investments must be sunk before negotiation, but allows for the seller to delay production and for the buyer to delay delivery. He shows that giving one of the parties the right to propose (ex post) an option contract – that the other can freely exercise – together with the posting of a bond, can combine to make efficient investment happen in a wide range of scenarios. While the bond is similar to the No-trade Payoff Minimization Principle, the main intuition is the exploitation of the existence of multiple continuation equilibria following a deviation to penalize the deviator, similarly to repeated games.

Matouschek (2004) studies the effects of ex ante contracts on the ex post trading (in)efficiencies when the parties have two-sided asymmetric information à la Myerson and Satterthwaite (1983), but have no opportunity to invest. Similarly to our results, contracts inducing low disagreement payoffs increase the probability that agreement is reached when it is efficient. However, they prove more costly when agreement fails to obtain. As a result, whether – a version of – the No-trade Payoff Minimization Principle applies in his set-up depends on the resolution of this probabilistic trade-off. In our model, the Principle applies whenever a participation constraint is binding – and it does no harm otherwise.

\(^{35}\)In fact, a more general class of stochastic games, see Goldlucke and Kranz (2018).
\(^{36}\)In the spirit of this discussion, but for intermediate values of the players’ discount factor, $\delta$, Halonen (2002) shows in a repeated-game model that joint ownership of an asset may strictly dominate single ownership as the former can make the repeated game punishment more severe.
In the context of incentive contracts, Baker et al. (1994) suggest that when some performance measure is non-verifiable it may be advantageous to make also some of the verifiable measures implicit in the contract. While on the face of it, this looks very similar to – some of – what we are proposing, the underlying logic is completely different. In their case, not making the verifiable part explicit confers a power of retaliation on the principal, in case the agent shirks on the non-verifiable part.

Anderlini et al. (2011) study the usefulness of courts refusing to enforce certain clauses of a contract in a situation where parties are asymmetrically informed at the point of signing a contract. They find that some of the inefficiencies resulting from asymmetric information can be avoided if the courts make the set of contracts coarser than necessary. While the context is different, their conclusion resonates well with our result that joint ownership may be optimal, for example.

Finally, this paper is related to Hart and Moore (2008), who develop the idea that contracts may serve as reference points. Their contracting parties have a sense of entitlement fixed by the – extremes of a loose – *ex ante* contract, which will lead them to ex post non-Coasian bargaining *within* the limits of the old contract, if they feel shortchanged relative to what they feel entitled to. The inefficiency could be just subjective, but they assume that it is realized via a shading of the quality of *ex post* performance.\(^{37}\) The current paper can be seen as developing a “dual” view. Like theirs, our point of departure is that contracts fix the expectations of the parties. But instead of these acting as the source of efficiency loss, they become a source of incentives in our model. Also, while the sense of entitlement in their model is psychological, it is rational in our model, supported by the equilibrium strategies.

### 7 Concluding remarks

We have shown that allowing for a simple and plausible investment dynamics in a hold-up model produces new implications for the design of contracts and organizations. The novel prediction is that the incentives for investment depend not just on how a contract affects the investor’s exposure to hold-up *at the margin* – the focus of the existing con-

\(^{37}\)This approach is in line with the one taken by Kreps (1997), Benabou and Tirole (2003), Besley and Ghatak (2005) and others, where some *intrinsic* motivation of the workers to perform well is incorporated into a principal-agent model. The aggrievement of Hart and Moore (2008) can be directly associated with a loss of such an intrinsic motivation.
tract/organization literature – but, sometimes more importantly, on how it affects that exposure in *absolute* terms. This has not been a concern in the static models – the incentive compatibility constraint is always the stricter one there – but it is an important consideration in our dynamic model, since the steeper incentives provided by investment dynamics may cause the participation constraint to be binding.

A shift of emphasis from marginal to absolute exposure directly takes us back to the original “transaction cost analysis” (TCA) authors (Klein et al., 1978; Williamson 1979, 1985, 1996). They were largely concerned about the absolute level of hold-up parties are exposed to as the rationale for organizational interventions. Despite sharing this view, our predictions differ from those of these authors as well. Our theory predicts that contracts that would exacerbate the parties’ vulnerability to hold-up – rather than those protecting them from it (as proposed by the TCA authors), can be desirable. As discussed above, this view throws a more positive light on a variety of “hostage taking” or “hands-tying” arrangements such as exclusivity agreements, joint ownership of assets, and trade contracts compelling parties to trade excessive amounts.

Real-world contracts are full of vague language and difficult-to-verify terms. Terms such as “good faith,” “best efforts,” and “commercially reasonable” are commonplace in any commercial contract. Scott and Triantis (2006) argue that the use of “unnecessarily” vague terms in a contract may be the rational outcome of resolving the trade-off between front-end (contract writing) and back-end (litigation, evidence production) costs. The No-trade Payoff Minimization Principle reinforces and complements their findings. By including vague terms, the parties increase the back-end costs, thereby providing the necessary penalty in case litigation indeed occurs.

Finally, the fact that our predictions are largely based on the absolute level of quasi-rents could also make them more empirically testable. As Whinston (2003) points out, the GHM

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38The TCA paradigm has often been criticized (c.f. Holmström and Roberts, 1998) about the fact that it ignored two empirically relevant features of an economic relationship: the cost of relationship-specific investments and the asymmetry of (bargaining) power between the parties. Our results (in the Appendix) remedy both of these weaknesses.

39This is consistent with the Hart and Moore (2008) argument. Signing a contract which is “loose” – and thus leads to less efficient bargaining within it – would lead to a low disagreement payoff, enhancing investment incentives. In a sense this is exactly what happens in the case of joint ownership: if the parties get into a row during renegotiation it is very difficult to sort the mess out if individual property rights are not clearly defined.
theory is difficult to test,\textsuperscript{40} since the (marginal) effects of investment on the disagreement payoffs are difficult to estimate, especially since most feasible levels of investment are not made in equilibrium. By contrast, hypotheses pertaining to the effects of absolute degree of asset specificities can be tested without observing payoff consequences of all investment choices, especially when investments are fully relationship-specific.

8 Appendix

In this appendix, we state and prove our results for two-sided investments. Taking the limit of the cost and effects of party 2’s investment to zero – while satisfying the assumptions below – it is easy to see that the arguments adapt readily for the results presented in the main body of the paper. Let us start by generalizing our notation for two-sided investment.

We denote the vector of (binary) investments by $\mathbf{I} = (I_1, I_2)$ and define $\mathbf{I}^* = (1, 1)$. We denote the aggregate disagreement payoff by $D(\mathbf{I}; \gamma) = d_1(\mathbf{I}; \gamma) + d_2(\mathbf{I}; \gamma)$. We let $\Delta d_i(\gamma) := d_i(1, 1; \gamma) - d_i(0, 1; \gamma)$ denote the changes in the contract payoffs of party $i = 1, 2$, that result from party 1’s investing, given that her partner, 2, invests.\textsuperscript{41} Similarly, $\Delta_2 d_i(\gamma) := d_i(1, 1; \gamma) - d_i(1, 0; \gamma)$. The generalizations of the assumptions are

**Assumption 1’ (\mathbf{I}^* is efficient)**

\[ R(1, 1) - c_1 - c_2 > \max \{ R(1, 0) - c_1, R(0, 1) - c_2, R(0, 0) \} . \]

**Assumption 2’ (Specificity) For each $\gamma \in \Gamma$**

(a) $\Delta_i d_j(\gamma) \geq 0$ for $i \neq j = 1, 2$;

(b) $0 \leq \Delta_i d_i(\gamma) < c_i$ for $i = 1, 2$;

(c) $0 \leq \varepsilon < R(\mathbf{I}) - D(\mathbf{I}; \gamma) < R(\mathbf{I}') - D(\mathbf{I}'; \gamma)$ for any $\mathbf{I} < \mathbf{I}'$.

We provide the proofs for generalized bargaining powers, $\beta_1$ and $\beta_2$, with $\beta_1 + \beta_2 = 1$. Then we have that, for $i = 1, 2$:

\[
U_i^0(\mathbf{I}; \gamma) = d_i(\mathbf{I}; \gamma) + \beta_i (R(\mathbf{I}) - D(\mathbf{I}; \gamma))
\]

\[
U_i^\delta(\mathbf{I}; \gamma) = (1 - \delta)d_i(\mathbf{I}; \gamma) + \beta_i (R(\mathbf{I}) - (1 - \delta)D(\mathbf{I}; \gamma)) \text{ and}
\]

\[
\Delta_i V^\delta(\gamma) = \beta_i [\Delta_i R - (1 - \delta)\Delta_i d_i(\gamma)] + \beta_i (1 - \delta)\Delta_i d_i(\gamma) - (1 - \beta_i (1 - \delta)c_i).
\]

\textsuperscript{40}Though see Baker and Hubbard (2004).

\textsuperscript{41}Given that we will focus on the implementation of the efficient investment profile, we avoid specifying the effects of investment when the partner does not invest.
Lemma 1' For any $\gamma \in \Gamma$, given efficient investment the SPNE (continuation) payoffs in any subgame are the (G)NBS payoffs: $U_i^0(I^*; \gamma)$, $i = 1, 2$.

Proof: (We drop the $\gamma$ for transparency) Let $\bar{v}_i$ and $v_i$ be the supremum and infimum payoffs of party $i = 1, 2$ attainable in any SPNE, given that the efficient investment has been made (and therefore no further investment is possible). In the resulting pure bargaining game, for $i = 1, 2$,

\begin{align}
\bar{v}_i &= \beta_i[R(I^*) - \delta \bar{v}_{-i} - (1 - \delta)d_{-i}(I^*)] + \beta_{-i} [\delta \bar{v}_i + (1 - \delta)d_i(I^*)], \\
v_i &= \beta_i[R(I^*) - \delta v_{-i} - (1 - \delta)d_{-i}(I^*)] + \beta_{-i} [\delta v_i + (1 - \delta)d_i(I^*)].
\end{align}

The supremum payoff $\bar{v}_i$ is explained as follows. Party $i$ is chosen with probability $\beta_i$ to propose a share to his partner $-i$. If the latter rejects that proposal, then, with probability $\delta$, the game continues with $I^*$ as the state next period, from which $-i$ can earn at least $\bar{v}_{-i}$ by definition; with probability $1 - \delta$, the negotiation breaks down, and $-i$ collects $d_{-i}(I^*)$. In other words, the least $-i$ can get from rejecting $i$’s offer is $\delta \bar{v}_{-i} + (1 - \delta)d_{-i}(I^*)$. Hence, the most $i$ can earn is $R(I^*)$, the maximum surplus that can be generated from trading with $-i$, minus this amount, unless this is less than $i$’s outside option, what is at best $\delta \bar{v}_i + (1 - \delta)d_i(I^*)$ – we show that this is not the case below. This explains the first term. With probability $\beta_{-i} = 1 - \beta_i$, party $i$ becomes the receiver of $-i$’s offer. In this case, the most she can earn by rejecting is $\delta v_i + (1 - \delta)d_i(I^*)$, so $-i$ will not offer her more, explaining the second term. The infimum payoff is explained analogously.

Solving this system of four equations\(^{42}\) for $\{\bar{v}_i, v_i\}_{i=1,2}$ yields $\bar{v}_i = v_i = U_i^0(I^*) = d_i(I^*) + \beta_i (R(I^*) - D(I^*))$ as required. Finally, given these values, it is straightforward to check, that by Assumption 2’c, the outside option is lower than the payoff displayed in the first term of (1).

Theorem 1' For any scenario $(R(\cdot), c)$, contract $\gamma$ implements the efficient investment if and only if

\begin{align}
(P_1'') \quad & U_1^0(1, 1; \gamma) - c_1 \geq d_1(0, 1; \gamma), \\
(P_2'') \quad & U_2^0(1, 1; \gamma) - c_2 \geq d_2(1, 0; \gamma),
\end{align}

\(^{42}\)In fact, there are two independent pairs of equations, one with $\bar{v}_1$ and $v_2$ and one with $v_2$ and $\bar{v}_1$. 32
and

\((IC_1')\)

\(U_1^s(1, 1; \gamma) - c_1 \geq U_1^s(0, 1; \gamma) - \beta_2 \delta c_1,\)

\((IC_2')\)

\(U_2^s(1, 1; \gamma) - c_2 \geq U_2^s(1, 0; \gamma) - \beta_1 \delta c_2.\)

**Proof:** (We drop the \(\gamma\) for transparency)

(Necessity) By Lemma 1’, in any SPNE with efficient investment, party \(i = 1, 2\) obtains a payoff of \(U_i^0(\Gamma^*) - c_i\). Suppose party 1 deviates by not investing and refusing to trade ever. Since \(\delta < 1\), this leads to an expected payoff of \(d_1(0, 1)\). As she can unilaterally guarantee this payoff, the efficient investment equilibrium can only be supported if

\[U_1^0(\Gamma^*) - c_1 \geq d_1(0, 1),\]

as required by \((P_1')\).

Consider next that party 1 deviates by not investing in the first period, but she intends to invest in the subsequent period if play gets there (that is, if there is neither agreement, nor breakdown).\(^{43}\) Now consider the bargaining in period 1. If party 2 gets to make the offer, by Lemma 1’ party 1 can guarantee herself the continuation payoff \(\delta(U_1^0(\Gamma^*) - c_1) + (1 - \delta)d_1(0, 1)\), whether or not 2 makes her this offer (if 2 offers more, even better). If 1 makes the offer, she can offer to 2 his continuation value (which he will accept). As we have not specified a complete strategy profile, we do not know what that is, but we can bound it from above: 2 cannot expect to obtain more than \(\delta U_2^0(\Gamma^*) + (1 - \delta)d_2(0, 1)\). The second term is obvious. To see that, in the absence of breakdown, 2 cannot hope for more than \(U_2^0(\Gamma^*)\) in any equilibrium continuation, note that there are only three possibilities: i) they never agree, in which case 2 earns \(d_2(0, 1) < U_2^0(\Gamma^*)\); ii) they agree following investment by 1, in which case 2 earns at most\(^{44}\) \(U_2^0(\Gamma^*)\); iii) they agree following no further investment, but unless this continuation value is exactly \(d_2(0, 1)\), in which case the claim holds anyway, it is always better to agree now rather than later (to avoid the possibility of breakdown), so this continuation cannot lead to the upper bound.

Putting everything together, party 1’s payoff from such a deviation can be at least

\[\beta_1[R(0, 1) - \delta U_2^0(\Gamma^*) - (1 - \delta)d_2(0, 1)] + \beta_2[\delta(U_1^0(\Gamma^*) - c_1) + (1 - \delta)d_1(0, 1)].\]

\(^{43}\)Conditional on investing ever, party 1 (at least) weakly prefers to do it in the subsequent period.

\(^{44}\)Depending on when party 1 invests.
Since such a deviation should not be profitable, we need
\[ U_1^0(\mathbf{I}^*) - c_1 \]
\[ \geq \beta_1 [R(0,1) - \delta U_2^0(\mathbf{I}^*) - (1 - \delta)d_2(0,1)] + \beta_2 \delta (U_1^0(\mathbf{I}^*) - c_1) + (1 - \delta)d_1(0,1) \]
\[ = \beta_1 [R(0,1) - \delta \{ d_2(\mathbf{I}^*) + \beta_2 (R(\mathbf{I}^*) - D(\mathbf{I}^*)) \}] - (1 - \delta)d_2(0,1) \]
\[ + \beta_2 \delta \{ d_1(\mathbf{I}^*) + \beta_1 (R(\mathbf{I}^*) - D(\mathbf{I}^*)) - c_1 \} + (1 - \delta)d_1(0,1) \]
\[ = \beta_1 [R(0,1) - \delta d_2(\mathbf{I}^*) - (1 - \delta)d_2(0,1)] + \]
\[ \beta_2 \delta [d_1(\mathbf{I}^*) - c_1] + (1 - \delta)d_1(0,1) \]
\[ = \beta_1 [R(0,1) - \delta d_2(\mathbf{I}^*) - (1 - \delta)d_2(0,1)] + \beta_2 \delta [d_1(\mathbf{I}^*) - c_1] + \]
\[ (1 - \delta)d_1(0,1) - (1 - \beta_2)(1 - \delta)d_1(0,1) \]
\[ = \beta_1 [R(0,1) - (1 - \delta)D(0,1)] - \beta_1 \delta d_2(\mathbf{I}^*) + \beta_2 \delta [d_1(\mathbf{I}^*) - c_1] + (1 - \delta)d_1(0,1) \]
\[ = U_1^\delta(0,1) - \beta_1 \delta d_2(\mathbf{I}^*) + (1 - \beta_1)\delta [d_1(\mathbf{I}^*) - c_1] \]
\[ = U_1^\delta(0,1) - \beta_1 \delta D(\mathbf{I}^*) + \delta d_1(\mathbf{I}^*) - \beta_2 \delta c_1 \]
\[ = U_1^\delta(0,1) + U_1^0(\mathbf{I}^*) - U_1^\delta(\mathbf{I}^*) - \beta_2 \delta c_1 \]
\[ \Leftrightarrow U_1^\delta(\mathbf{I}^*) - c_1 \geq U_1^0(0,1) - \beta_2 \delta c_1, \]

what is exactly \((IC_1^\nu)\).

The same arguments – considering party 2’s deviation – lead to the other two constraints.

**Sufficiency** To show that these conditions are also sufficient for the existence of a SPNE with efficient investment, consider the following simple investment strategy profile that clearly leads to the efficient investment: “Each party invests whenever (s)he has not invested previously.” Further, given \((P_1^\nu, P_2^\nu)\) and \((IC_1^\nu, IC_2^\nu)\), this strategy profile, along with the unique equilibrium bargaining outcome (given by Lemma 1’), form a SPNE. Following a unilateral deviation from investment, by party 1, say, the continuation equilibrium is a function of \(\delta\). If and only if \(\delta \leq \delta_1^N(\gamma) := \frac{R(0,1) - D(0,1,\gamma)}{R(1,1) - c_1 - D(0,1,\gamma)}\) then – given the investment strategy – the unique continuation is described by (2): since next period party 1 is expected to make up the shortfall in investment, she has to compensate her partner for forgoing waiting for it (if she makes the offer) by offering him \(\delta U_2^0(\mathbf{I}^*) + (1 - \delta)d_2(0,1)\), while she needs to take on the chin the discounting from her payoff of the saved investment cost if her opponent does, collecting \(\delta (U_1^0(\mathbf{I}^*) - c_1) + (1 - \delta)d_1(0,1)\). Given \(\delta \leq \delta_1^N(\gamma)\), these offers are always accepted. Finally, as we have seen in the necessity part of this proof,
(\(IC''_1\)) ensures that this deviation payoff is less than the equilibrium payoff, discouraging underinvestment.

When \(\delta > \delta^N_1\), the value in the first square brackets of (2) – party 1’s payoff if she ensures 2’s acceptance – is less than the one in the second (party 1’s “outside option”). In this case, there cannot be agreement following the deviation in investment. The equilibrium continuation is delay, investment and agreement according to Lemma 1’. The condition for party 1’s investment incentive becomes

\[ U^0_1(I^*) - c_1 \geq \delta(U^0_1(I^*) - c_1) + (1 - \delta)d_1(0,1), \]

what is equivalent to (\(P''_1\)). Thus, all four conditions together are sufficient for efficient investment. □

**Corollary 1’** For any scenario \((R(.), c)\), contract \(\gamma\) implements the efficient investment if

\[
\begin{align*}
\delta &\geq \min \left\{ \delta^N_1(\gamma) := \frac{R(0,1) - D(0,1;\gamma)}{R(1,1) - c_1 - D(0,1;\gamma)}, \delta^T_1(\gamma) := \frac{\beta_1(R(0,1) - D(0,1;\gamma))}{\beta_1\Delta_1d_2 + \beta_2(c_1 - \Delta_1d_1)} \right\}, (P''_1) \quad (IC''_1) \\
\delta &\geq \min \left\{ \delta^N_2(\gamma), \delta^T_2(\gamma) := \frac{\beta_2(R(1,0) - D(1,0;\gamma))}{\beta_2\Delta_2d_1 + \beta_1(c_2 - \Delta_2d_2)} \right\}, (P''_2) \quad \text{holds.}
\end{align*}
\]

**Proof:** From the proof of Theorem 1’ we already know that for \(\delta \geq \delta^N_1(\gamma)\), (\(P''_1\)) is sufficient. Next, we show that (\(P''\)) \(\Leftrightarrow\) \(\delta^T_1(\gamma) \geq \delta^N_1(\gamma)\), so indeed

\[
\min \{\delta^N_1(\gamma), \delta^T_1(\gamma)\} = \delta^N_1(\gamma)
\]

whenever (\(P''\)) is satisfied. Dividing across by the positive term, \((R(0,1) - D(0,1))\), we have that \(\delta^T_1 \geq \delta^N_1\)

\[
\Leftrightarrow \beta_1(R(1,1) - c_1 - D(0,1)) \geq \beta_1\Delta_1d_2 + \beta_2(c_1 - \Delta_1d_1)
\]

\[
\Leftrightarrow \beta_1(R(1,1) - D(0,1)) - c_1 \geq \beta_1\Delta_1d_2 - \beta_2\Delta_1d_1
\]

\[
\Leftrightarrow \beta_1(R(1,1) - D(1,1) + \Delta_1d_1 + \Delta_1d_2) - c_1 \geq \beta_1\Delta_1d_2 - \beta_2\Delta_1d_1
\]

\[
\Leftrightarrow \beta_1(R(1,1) - D(1,1) + \Delta_1d_1) - c_1 \geq -(1 - \beta_1)\Delta_1d_1
\]

\[
\Leftrightarrow \beta_1(R(1,1) - D(1,1)) - c_1 + \Delta_1d_1 \geq 0 \Leftrightarrow (P''_1).
\]

As the next step, we show that (\(IC''_1\)) is getting the laxer, the higher \(\delta\) is. As (\(IC''_1\)) is equivalent to \(\Delta_1V^\delta \geq 0\), all we need to show is that \(\frac{d\Delta_1V^\delta}{d\delta} \geq 0\). That is that \(\beta_1\Delta_1d_2 + \beta_2(c_1 - \Delta_1d_1) \geq 0\), what directly follows from Assumption 2’a-b.
Next, rewrite \((IC''')\) as \(U_1^\delta(1, 1) - U_1^\delta(0, 1) - (1 - \beta_2 \delta) c_1 \geq 0\) and \((P''\nu)\) as \(U_1^0(1, 1) - d_1(0, 1) - c_1 \geq 0\). The two constraints are equivalent, when the LHSs are equal:

\[
U_1^\delta(1, 1) - U_1^\delta(0, 1) + \beta_2 \delta c_1 = U_1^0(1, 1) - d_1(0, 1).
\]

Substituting into the LHS we have

\[
(1 - \delta)d_1(1, 1) + \beta_1 (R(1, 1) - (1 - \delta)D(1, 1)) - (1 - \delta)d_1(0, 1) + \beta_1 (R(0, 1) - (1 - \delta)D(0, 1)) + \beta_2 \delta c_1
\]

\[
= (1 - \delta)\Delta_1 d_1 + \beta_1 (R(1, 1) - R(0, 1)) + \beta_1 (1 - \delta) (D(0, 1) - D(1, 1)) + \beta_2 \delta c_1.
\]

Substituting into the RHS we have

\[
d_1(1, 1) + \beta_1 (R(1, 1) - D(1, 1)) - d_1(0, 1) = \Delta_1 d_1 + \beta_1 (R(1, 1) - D(1, 1)).
\]

Equating the two

\[-\delta \Delta_1 d_1 - \beta_1 R(0, 1) + \beta_1 ((1 - \delta) (D(0, 1) + \delta D(1, 1)) + \beta_2 \delta c_1 = 0\]

and solving for \(\delta\) we obtain

\[
\delta_1^T(\gamma) = \frac{\beta_1 (R(0, 1) - D(0, 1))}{-\Delta_1 d_1 + \beta_1 (D(1, 1) - D(0, 1)) + \beta_2 c_1} = \frac{\beta_1 (R(0, 1) - D(0, 1))}{\beta_1 \Delta_1 d_2 + \beta_2 (c_1 - \Delta_1 d_1)} > 0.
\]

The monotonicity proven above completes the proof that for \(\delta \leq \delta_1^T(\gamma)\), \((IC''')\) implies \((P''\nu)\). Finally, observe that \(\delta_1^T(\gamma) \geq \min \{\delta^N(\gamma), \delta_1^T(\gamma)\}\), so \(\delta \leq \min \{\delta_1^N(\gamma), \delta_1^T(\gamma)\}\) implies \(\delta \leq \delta_1^T(\gamma)\).

Similar arguments work for party 2.]

**Proposition 1’ (No-trade Payoff Minimization Principle)** Suppose that for a pair of contracts \(\gamma', \gamma\), \(\Delta_i d_j(\gamma) = \Delta_i d_j(\gamma')\), \(\forall i, j\). Then, if\(^{45}\)

\[D(I^*; \gamma') < D(I^*; \gamma)\]

then there exists \(\delta^* \in [0, 1]\)

i) \(\gamma'\) and \(\gamma\) are equivalent for \(\delta \leq \delta^*\),

ii) \(\gamma'\) dominates \(\gamma\) for \(\delta > \delta^*\).

\(^{45}\)Given the supposition, the aggregate disagreement payoffs are ordered the same way if one party does not invest.
Proof: Given the hypothesis, for any scenario and continuation probability the two contracts lead to the same incentive compatibility constraints, while \( \gamma' \) has strictly laxer participation constraints. Consequently, \( \gamma' \) at least weakly dominates \( \gamma \). Denote the subset of scenarios in \( S(\gamma, \gamma') \) where \( (P^m) \) is not satisfied under \( \gamma \) but it is satisfied under \( \gamma' \) by \( V_1 \). This is clearly a non-empty set,\(^{46}\) which is independent of \( \delta \). Therefore, \( \gamma' \) dominates \( \gamma \), for party 1, unless \((IC_1')\) is the relevant incentive constraint – for either contract – and it is not satisfied in \( V_1 \) (in which case it is irrelevant whether the participation constraint is satisfied). That is, unless \( \delta < \inf_{(R,e) \in V_1} \min \{ \delta^N_1 (\gamma'), \delta^IC_1 \} \), where \( \delta^IC_1 \) is \((IC_1'')\) solved for \( \delta \).\(^{47}\) Putting the conditions for the two parties together, \( \delta^* = \min \{ \inf_{(R,e) \in V_1} \min \{ \delta^N_1 (\gamma'), \delta^IC_1 \}, \inf_{(R,e) \in V_2} \min \{ \delta^N_2 (\gamma'), \delta^IC_2 \} \} \).

Proposition 2’ Consider a pair of contracts \( \gamma', \gamma \). There exist \( \delta'(\varepsilon, K) \leq \delta'' \in [0,1) \), such that contract \( \gamma' \) weakly dominates contract \( \gamma \)

a) for any \( \delta < \delta' \), if and only if

\[
(3) \quad \beta_{-i} \Delta_i d_i (\gamma') - \beta_i \Delta_i d_{-i} (\gamma') \geq \beta_{-i} \Delta_i d_i (\gamma) - \beta_i \Delta_i d_{-i} (\gamma), i = 1, 2.
\]

If at least one inequality is strict, then \( \gamma' \) dominates \( \gamma \).

b) for any \( \delta > \delta'' \), if and only if

\[
(4) \quad \Delta_i d_i (\gamma') - \beta_i D(I^*; \gamma') \geq \Delta_i d_i (\gamma) - \beta_i D(I^*; \gamma), i = 1, 2.
\]

If at least one inequality is strict, then \( \gamma' \) dominates \( \gamma \).

Proof: a) Note that, if 
\( \delta \leq \inf_{(R,e) \in S} \{ \min \{ \delta^N_1 (\gamma), \delta^N_2 (\gamma), \delta^N_1 (\gamma'), \delta^N_2 (\gamma'), \delta^T_1 (\gamma), \delta^T_1 (\gamma'), \delta^T_2 (\gamma), \delta^T_2 (\gamma') \} \} \), by Corollary 1’, for any scenario \((R(.), e)\), \((IC_1'')\) and \((IC_2'')\) are sufficient for both \( \gamma \) and \( \gamma' \) to implement \( I^* \). Given (3), it follows that, for \( i = 1, 2 \),

\[
(5) \quad \frac{\Delta V_i^\delta (\gamma') - \Delta V_i^\delta (\gamma)}{1 - \delta} = (\beta_{-i} \Delta_i d_i (\gamma') - \beta_i \Delta_i d_{-i} (\gamma')) - (\beta_{-i} \Delta_i d_i (\gamma) - \beta_i \Delta_i d_{-i} (\gamma)) \geq 0.
\]

Suppose that \( \gamma \) implements \( I^* \) for some \( \delta \) and some scenario. Then, contract \( \gamma \) must satisfy \((IC_1'')\) and \((IC_2'')\), so \( \Delta V_i^\delta (\gamma) \geq 0 \), \( i = 1, 2 \). Therefore, by (5), we must have

\(^{46}\) Just take the subset of the scenarios in \( S(\gamma, \gamma') \) that satisfy the participation constraint under \( \gamma' \) with equality.

\(^{47}\) Explicitly, \( \delta^IC_1 = \frac{c_i - \beta_i (\Delta_i R - \Delta_i d_i) - \beta_{-i} \Delta_i d_i}{\beta_i \Delta_i d_2 + \beta_{-i} (c_i - \Delta_i d_i)} \). Note that it need not be in \([0,1] \).
\(\Delta V_i^\delta(\gamma') \geq 0, i = 1, 2\), so \(\gamma'\) satisfies \((IC_1''')\) and \((IC_2''')\). Since these are sufficient, \(\gamma'\) must also implement \(I^*\). Consequently, \(\gamma'\) weakly dominates \(\gamma\), proving the first statement. If one of the inequalities is strict, then the LHS of (5) is strictly positive for either \(i = 1\) or \(i = 2\). Therefore, since by Assumption 2'-a,b \(\beta_- \Delta_i d_i(.) - \beta_i \Delta_i d_{-i}(.) < \beta_- c_i\), we can always choose \(R\) and \(c_i\) in such a way that they satisfy Assumptions 1' and 2' and \(\Delta V_i^\delta(\gamma') > 0 > \Delta V_i^\delta(\gamma)\). That is, \(\gamma'\) trumps \(\gamma\), proving the second statement.

b) By Corollary 1', for any scenario \((R(.), c)\) and for any pair of contracts \(\gamma, \gamma'\) there exists \(\delta'' = \sup_{(R,c) \in S} \max \{ \min \{ \delta_1^R(\gamma), \delta_1^N(\gamma) \}, \min \{ \delta_2^R(\gamma), \delta_2^N(\gamma) \}, \min \{ \delta_1^T(\gamma'), \delta_1^N(\gamma') \}, \min \{ \delta_2^T(\gamma'), \delta_2^N(\gamma') \} \}\) 1 such that, for any \(\delta > \delta''\), the participation constraints are sufficient for a contract to implement \(I^*\). Given (4), \((P''_1)\) and \((P''_2)\) are weakly stricter for \(\gamma\) and thus whenever \(\gamma\) implements \(I^*\), so does \(\gamma'\). Thus, \(\gamma'\) weakly dominates \(\gamma\). If the inequality is strict for at least for one party, then \(\gamma'\) trumps \(\gamma\) in some scenarios in \(S(\gamma, \gamma')\). Therefore, \(\gamma'\) dominates \(\gamma\). \(\blacksquare\)

Corollary 2’ (Irrelevance of Leakage) Suppose for a pair of contracts \(\gamma'\) and \(\gamma\)

\[
\Delta_i d_i(\gamma') = \Delta_i d_i(\gamma), i = 1, 2, \text{ and } D(I^*; \gamma') = D(I^*; \gamma)
\]

while

\[
\Delta_i d_{-i}(\gamma') < \Delta_i d_{-i}(\gamma), \ i, j = 1, 2.
\]

Then there exists \(\hat{\delta}(\varepsilon, K) \in (0, 1)\) such that

i) \(\gamma'\) dominates \(\gamma\) when \(\delta < \hat{\delta}\), and

ii) \(\gamma'\) and \(\gamma\) are equivalent when \(\delta \geq \hat{\delta}\).

Proof: By Proposition 1’, given the hypothesis, for any scenario and continuation probability the two contracts are equivalent with respect to the participation constraints while \(\gamma'\) has a less strict incentive constraint. Consequently, \(\gamma'\) dominates \(\gamma\), as long as the incentive constraints are sufficient for both contracts (that is for \(\delta \leq \inf_{(R,c) \in S(\gamma, \gamma')} \min \{ \delta_1(\gamma'), \delta_2(\gamma') \}\)). For \(\delta \geq \sup_{(R,c) \in S(\gamma, \gamma')} \max \{ \min \{ \delta_1(\gamma), \delta_2(\gamma) \}, \min \{ \delta_1(\gamma'), \delta_2(\gamma) \} \}\), only the participation constraints matter but they are unchanged. In between the two thresholds define \(\hat{\delta}\) as the highest \(\delta\) such that for some scenarios and party the incentive constraint is sufficient, but it only holds under \(\gamma'\). \(\blacksquare\)

\[48\] Note that, under the hypothesis, both \(\delta_1^N(\gamma) > \delta_1^N(\gamma')\) and \(\delta_1^T(\gamma) > \delta_1^T(\gamma')\).
References


