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Topologically Massive Gravity and Galilean Conformal Algebra: A Study of Correlation Functions

Arjun Bagchi

*School of Mathematics,
University of Edinburgh
Kings Buildings, Edinburgh EH9 3JZ
United Kingdom*

`arjun.bagchi@ed.ac.uk`

ABSTRACT: The Galilean Conformal Algebra (GCA) arises from the conformal algebra in the non-relativistic limit. In two dimensions, one can view it as a limit of linear combinations of the two copies Virasoro algebra. Recently, it has been argued that Topologically Massive Gravity (TMG) realizes the quantum 2d GCA in a particular scaling limit of the gravitational Chern-Simons term. To add strength to this claim, we demonstrate a matching of correlation functions on both sides of this correspondence. A priori looking for spatially dependent correlators seems to force us to deal with high spin operators in the bulk. We get around this difficulty by constructing the non-relativistic Energy-Momentum tensor and considering its correlation functions. On the gravity side, our analysis makes heavy use of recent results of Holographic Renormalization in Topologically Massive Gravity.

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1. Introduction

The AdS/CFT conjecture [1] has been the cornerstone in the research in string theory over the past decade. Recently, there has been a flurry of activity in the community in applying the correspondence to potential real life condensed matter systems. Relatedly, there has been a growing interest in non-relativistic versions of AdS/CFT. The most studied of these versions has been the ones with Schrodinger symmetry algebra, the largest symmetry algebra of the free Schrodinger equations [2, 3, 4]. This symmetry is known to be realized in cold atoms at unitarity [5]. A gravity dual for these systems was proposed in [6, 7], following which quite a body of literature has built up. We refer the reader to the excellent review [8] and the references therein for a flavour of the activity in this direction.

In this paper, we would focus on the version of non-relativistic AdS/CFT which can be best motivated as the true non-relativistic limit of the conjecture [12, 15, 16]. The relativistic conformal algebra on the boundary of AdS is systematically reduced to what we call the Galilean Conformal Algebra by a process of parametric group contraction. This algebra is surprising in many aspects, the most intriguing of which is the fact that it can be given an infinite lift in any space-time dimensions. The GCA is also important to non-relativistic hydrodynamics. The finite algebra turns out to be the symmetry algebra of the incompressible Euler equations and a part of the full infinite algebra is also realized as its symmetries.

Given the surprising infinite dimensional lift of the GCA for all spacetime dimensions, it is natural to first figure out what the story is in two dimensions. As is well known, the case of $D=2$ is special because here the relativistic conformal algebra gets enhanced to two copies of the infinite dimensional Virasoro algebra. One would expect the infinite GCA to be related to these two copies of the relativistic Virasoro algebra and this expectation was borne out by our analysis in [16]. The GCA emerges by taking simple linear combinations of the two copies of the Virasoro algebra and then looking at the non-relativistic limit. One can also look to the quantum aspects of the GCA in two dimensions in the same spirit as Virasoro algebra. The central charges for the GCA are asymmetric, they are linear combinations of the parent relativistic central charges.

The initial bulk description of the GCA was given in terms of a novel Newton-Cartan like $AdS_2 \times R^d$ in [12]. The bulk metric degenerates in this limit and one can formulate the gravity theory in terms of dynamical Christoffel symbols in a geometrized version of Newtonian gravity. Unlike in the usual Newton-Cartan formulation of flat space where one had a specially selected time direction, in AdS both the radial and temporal directions survive the scaling giving a fibre bundle structure where the base is an AdS_2 made out of the surviving radial and time directions and there are flat spatial fibres over this base.

For the bulk theory dual to the two-dimensional boundary GCA, one would need to consider such a Newton-Cartan like $AdS_2 \times R$ emerging from an AdS_3 ¹. The natural candidate for trying to model the asymmetric central charges would be Cosmological Topologically Massive gravity. This contains a vacuum AdS_3 solution and also an excited state of a BTZ black hole. This has been recently looked at in [18]. The authors found that the GCA emerged as the asymptotic symmetry of this structure in the non-relativistic limit. The asymmetrical central charges for the GCA was realized by making the coefficient of the gravitational Chern-Simons term very large. The authors also found a non-relativistic generalization of Cardy's entropy formula for the BTZ blackhole in by considering this limit.

In this paper, we find further evidence that this connection is true. We look at correlation functions on both sides of the duality. At first sight this is a complicated problem. The construction of the correlation functions of primary operators of the boundary GCA in [15] emphasized that if we are to look for non-trivial spatial dependence, then we would need to focus on the high spin sector. In the dual gravity side, computing correlators for arbitrarily high spins is at best cumbersome. We get around this apparently insurmountable problem by making an observation about energy-momentum tensors which leads to a gross simplification. The calculation on the field theory side follows from the systematic theory of limits established in [16] and the corresponding gravity calculation makes heavy use of recent work in holographic renormalization of CTMG in [27].

The rest of the paper is organized as follows. In Sec 2, we revisit the Galilean Conformal Algebra in general dimensions and then concentrate on the results in two dimensions. In Sec 3, we focus our attention on the construction of the non-relativistic energy momentum tensors and describe a few of their properties. In Sec 4, we start discussing the bulk theory. We revisit the simplified case where usual holography works for correlation function [14] and then emphasize why we need to look at Topologically Massive Gravity, briefly describing the construction of [18]. In

¹Recently, in [20], we constructed metrics which realized the global 2d GCA as the isometry-algebra. For the algebra in two spacetime dimensions, the metrics in four and five dimensions turned out to have exotic signatures. This further suggests that the Newton-Cartan structure is the best framework for describing the non-relativistic bulk.

Sec 5, after a brief summary of holographic renormalization, the results of [27] are discussed. We then apply these results to reproduce the calculations in the boundary theory and add support to the claim of [18].

2. A Review of the GCA

2.1 GCA in arbitrary dimensions

The maximal set of conformal isometries of Galilean spacetime generates the infinite dimensional Galilean Conformal Algebra [12]. The notion of Galilean spacetime is a little subtle since the spacetime metric degenerates into a spatial part and a temporal piece. Nevertheless there is a definite limiting sense (of the relativistic spacetime) in which one can define the conformal isometries (see [13]) of the nonrelativistic geometry. Algebraically, the set of vector fields generating these symmetries are given by

$$\begin{aligned} L^{(n)} &= -(n+1)t^n x_i \partial_i - t^{n+1} \partial_t, \\ M_i^{(n)} &= t^{n+1} \partial_i, \\ J_a^{(n)} \equiv J_{ij}^{(n)} &= -t^n (x_i \partial_j - x_j \partial_i), \end{aligned} \quad (2.1)$$

for integer values of n . Here $i = 1 \dots (d-1)$ range over the spatial directions. These vector fields obey the algebra

$$\begin{aligned} [L^{(m)}, L^{(n)}] &= (m-n)L^{(m+n)}, & [L^{(m)}, J_a^{(n)}] &= -nJ_a^{(m+n)}, \\ [J_a^{(n)}, J_b^{(m)}] &= f_{abc}J_c^{(n+m)}, & [L^{(m)}, M_i^{(n)}] &= (m-n)M_i^{(m+n)}. \end{aligned} \quad (2.2)$$

There is a finite dimensional subalgebra of the GCA (also sometimes referred to as the GCA) which consists of taking $n = 0, \pm 1$ for the $L^{(n)}, M_i^{(n)}$ together with $J_a^{(0)}$. This algebra is obtained by considering the nonrelativistic contraction of the usual (finite dimensional) global conformal algebra $SO(d, 2)$ (in $d > 2$ spacetime dimensions).

2.2 GCA in 2d

In two spacetime dimensions, as is well known, the situation is special. The relativistic conformal algebra is infinite dimensional and consists of two copies of the Virasoro algebra. One expects this to be related to the infinite dimensional GCA algebra [16]. Indeed in two dimensions the non-trivial generators in (2.2) are the L_n and the M_n :

$$L_n = -(n+1)t^n x \partial_x - t^{n+1} \partial_t, \quad M_n = t^{n+1} \partial_x, \quad (2.3)$$

which obey

$$[L_m, L_n] = (m-n)L_{m+n}, \quad [M_m, M_n] = 0, \quad [L_m, M_n] = (m-n)M_{m+n} \quad (2.4)$$

These generators in (2.3) arise precisely from a nonrelativistic contraction of the two copies of the Virasoro algebra. To see this, let us remember that the non-relativistic contraction consists of taking the scaling

$$t \rightarrow t, \quad x \rightarrow \epsilon x, \quad (2.5)$$

with $\epsilon \rightarrow 0$. This is equivalent to taking the velocities $v \sim \epsilon$ to zero (in units where $c = 1$). Consider the vector fields which generate the centre-less Virasoro Algebra in two dimensions :

$$\mathcal{L}_n = -z^{n+1}\partial_z, \quad \bar{\mathcal{L}}_n = -\bar{z}^{n+1}\partial_{\bar{z}}. \quad (2.6)$$

In terms of space and time coordinates, $z = t + x$, $\bar{z} = t - x$. Expressing $\mathcal{L}_n, \bar{\mathcal{L}}_n$ in terms of t, x and taking the above scaling (2.5) reveals that in the limit the combinations

$$\mathcal{L}_n + \bar{\mathcal{L}}_n = -t^{n+1}\partial_t - (n+1)t^n x \partial_x + \mathcal{O}(\epsilon^2); \quad \mathcal{L}_n - \bar{\mathcal{L}}_n = -\frac{1}{\epsilon}t^{n+1}\partial_x + \mathcal{O}(\epsilon). \quad (2.7)$$

Therefore we see that as $\epsilon \rightarrow 0$

$$\mathcal{L}_n + \bar{\mathcal{L}}_n \longrightarrow L_n, \quad \epsilon(\mathcal{L}_n - \bar{\mathcal{L}}_n) \longrightarrow -M_n. \quad (2.8)$$

At the quantum level the two copies of the Virasoro get respective central extensions

$$\begin{aligned} [\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}, \\ [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] &= (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0}. \end{aligned} \quad (2.9)$$

Considering the linear combinations (2.7) which give rise to the GCA generators as in (2.8), we find

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + C_1 m(m^2-1)\delta_{m+n,0}, \\ [L_m, M_n] &= (m-n)M_{m+n} + C_2 m(m^2-1)\delta_{m+n,0}, \\ [M_m, M_n] &= 0. \end{aligned} \quad (2.10)$$

This is the centrally extended GCA in 2d. Note that the relation between central charges is

$$C_1 = \frac{c + \bar{c}}{12}, \quad \frac{C_2}{\epsilon} = \frac{\bar{c} - c}{12}. \quad (2.11)$$

Thus, for a non-zero C_2 in the limit $\epsilon \rightarrow 0$ we see that we need $\bar{c} - c \propto \mathcal{O}(\frac{1}{\epsilon})$. At the same time requiring C_1 to be finite we find that $c + \bar{c}$ should be $\mathcal{O}(1)$. Thus (2.11) can hold only if c and $\bar{c}$ are large (in the limit $\epsilon \rightarrow 0$) and opposite in sign. This immediately implies that the original 2d CFT on which we take the non-relativistic limit cannot be unitary. This is, of course, not a problem since there are many statistical mechanical models which are described at a fixed point by non-unitary CFTs.

2.3 Representations of the 2d GCA

We will construct the representations of the GCA by considering the states having definite scaling dimensions [15, 16]:

$$L_0|\Delta\rangle = \Delta|\Delta\rangle. \quad (2.12)$$

Using the commutation relations (2.10), we obtain

$$L_0 L_n |\Delta\rangle = (\Delta - n) L_n |\Delta\rangle, \quad L_0 M_n |\Delta\rangle = (\Delta - n) M_n |\Delta\rangle. \quad (2.13)$$

Then the L_n, M_n with $n > 0$ lower the value of the scaling dimension, while those with $n < 0$ raise it. If we demand that the dimension of the states be bounded from below then we are led to defining primary states in the theory having the following properties :

$$L_n |\Delta\rangle = 0, \quad M_n |\Delta\rangle = 0, \quad (2.14)$$

for all $n > 0$. Since the conditions (2.14) are compatible with M_0 in the sense

$$L_n M_0 |\Delta\rangle = 0, \quad M_n M_0 |\Delta\rangle = 0, \quad (2.15)$$

and also since L_0 and M_0 commute, we may introduce an additional label, which we will call “rapidity” ξ :

$$M_0 |\Delta, \xi\rangle = \xi |\Delta, \xi\rangle. \quad (2.16)$$

Starting with a primary state $|\Delta, \xi\rangle$, one can build up a tower of operators by the action of L_{-n} and M_{-n} with $n > 0$. The above construction is quite analogous to that of the relativistic 2d CFT. In fact, from the viewpoint of the limit (2.8) we see that the two labels Δ and ξ are related to the conformal weights in the 2d CFT as

$$\Delta = \lim_{\epsilon \rightarrow 0} (h + \bar{h}), \quad \xi = \lim_{\epsilon \rightarrow 0} \epsilon (\bar{h} - h), \quad (2.17)$$

where h and $\bar{h}$ are the eigenvalues of $\mathcal{L}_0$ and $\bar{\mathcal{L}}_0$, respectively. We will assume that the operator state correspondence in the 2d CFT gives a similar correspondence between the states and the operators in the GCA:

$$\mathcal{O}(t, x) \leftrightarrow \mathcal{O}(0)|0\rangle, \quad (2.18)$$

where $|0\rangle$ would be the vacuum state which is invariant under the generators $L_0, L_{\pm 1}, M_0, M_{\pm 1}$.

2.4 Two Function of the GCA

The constraints from the Ward identities for the global transformations $L_{0,\pm 1}, M_{0,\pm 1}$ apply to primary GCA operators.

Therefore consider the two point function of primary operators $\mathcal{O}_1(t_1, x_1)$ and $\mathcal{O}_2(t_2, x_2)$ of conformal and rapidity weights (Δ_1, ξ_1) and (Δ_2, ξ_2) respectively.

$$G_{\text{GCA}}^{(2)}(t_1, x_1, t_2, x_2) = \langle \mathcal{O}_1(t_1, x_1) \mathcal{O}_2(t_2, x_2) \rangle. \quad (2.19)$$

The correlation functions only depend on differences of the coordinates $t_{12} = t_1 - t_2$ and $x_{12} = x_1 - x_2$ because of the translation symmetries L_{-1} and M_{-1} . The remaining symmetries give four more differential equations which constrain the answer to be [15]

$$G_{\text{GCA}}^{(2)}(\{t_i, x_i\}) = C_{12} \delta_{\Delta_1, \Delta_2} \delta_{\xi_1, \xi_2} t_{12}^{-2\Delta_1} \exp\left(\frac{2\xi_1 x_{12}}{t_{12}}\right). \quad (2.20)$$

Here C_{12} is an arbitrary constant, which we can always take to be one by choosing the normalization of the operators. We can similarly construct the three point function which again is fixed by the symmetries upto an overall constant.

The GCA two and three point functions can also be obtained by taking an appropriate scaling limit of the usual 2d CFT answers. This limit requires scaling the quantum numbers of the operators as (2.17), along with the non-relativistic limit for the coordinates (2.5).

Let us study the scaling limit of the two point correlator.

$$\begin{aligned} G_{\text{2d CFT}}^{(2)} &= \delta_{h_1, h_2} \delta_{\bar{h}_1, \bar{h}_2} z_{12}^{-2h_1} \bar{z}_{12}^{-2\bar{h}_1} \\ &= \delta_{h_1, h_2} \delta_{\bar{h}_1, \bar{h}_2} t_{12}^{-2h_1} \left(1 + \epsilon \frac{x_{12}}{t_{12}}\right)^{-2h_1} t_{12}^{-2\bar{h}_1} \left(1 - \epsilon \frac{x_{12}}{t_{12}}\right)^{-2\bar{h}_1} \\ &= \delta_{h_1, h_2} \delta_{\bar{h}_1, \bar{h}_2} t_{12}^{-2(h_1 + \bar{h}_1)} \exp\left(-2(h_1 - \bar{h}_1)\left(\epsilon \frac{x_{12}}{t_{12}} + \mathcal{O}(\epsilon^2)\right)\right). \end{aligned} \quad (2.21)$$

Now by taking the scaling limit as (2.17), we obtain the GCA two point function

$$\lim_{\epsilon \rightarrow 0} G_{2\text{d CFT}}^{(2)} = \delta_{\Delta_1, \Delta_2} \delta_{\xi_1, \xi_2} t_{12}^{-2\Delta_1} \exp\left(\frac{2\xi_1 x_{12}}{t_{12}}\right) = G_{\text{GCA}}^{(2)}. \quad (2.22)$$

A similar analysis yields the three point function of the GCA from the relativistic three point function.

3. The Non-relativistic Energy-Momentum Tensor

From the above, we saw that we have non-trivial correlation functions (non-trivial in the sense of spatial dependence) only when we look at high values of the spin $h - \bar{h}$ (in order to get a non-zero ξ). Our ultimate aim in this paper is to construct a matching of non-trivial correlation functions from the boundary and the bulk. It seems naively, that one needs to consider fields of arbitrary high spin propagating in the bulk if one wants to recover some spatially dependent answers. This is a complicated exercise. We also wish to probe the quantum aspects of the GCA. There is the additional drawback that the two and three point correlation function of the above described quasi-primary fields would be independent of the central charge. So it seems that one would need to look at four or higher point functions of the primary operators with arbitrarily high spin to address the GCA correlation functions in their full generality.

The dependence on central charges has an obvious answer: looking at the energy-momentum tensor. It turns out that if one constructs the non-relativistic EM tensor properly, one gets spatially dependent correlation functions. The focal point of the argument is that one must remember that in this non-relativistic limit, we also scale the central charges of the relativistic theory (see (2.11)). If we focus in the energy-momentum tensor, as we would do now, one can easily show that there are spatial dependent terms in the correlation functions, even though we are not looking at a high spin field.

Let us for the moment restrict our attention to the GCAs which are obtainable as a limit from relativistic CFTs. Our results would be obtainable from the non-relativistic algebra independently as well. Below, we would also present another method of deriving the results from mode expansions, which in some sense is more intrinsically non-relativistic. We define the non-relativistic EM tensors in the following way:

$$T_{(1)} = T(z) + \bar{T}(\bar{z}) \quad T_{(2)} = \epsilon [T(z) - \bar{T}(\bar{z})] \quad (3.1)$$

This definition is a natural choice because of the particular linear combinations of the relativistic Virasoro algebra which yields the GCA in the non-relativistic limit (2.8). Now, as is well known, the OPEs among the relativistic EM tensors are of the form

$$\begin{aligned} T(z)T(0) &\sim \frac{c/2}{z^4} + \frac{2T(0)}{z^2} + \frac{\partial T(0)}{z} \\ \bar{T}(\bar{z})\bar{T}(0) &\sim \frac{\bar{c}/2}{\bar{z}^4} + \frac{2\bar{T}(0)}{\bar{z}^2} + \frac{\partial \bar{T}(0)}{\bar{z}} \\ T(z)\bar{T}(0) &\sim \text{finite.} \end{aligned} \quad (3.2)$$

which leads to the two-point functions

$$\langle T(z)T(0) \rangle = \frac{c}{2z^4}, \quad \langle \bar{T}(\bar{z})\bar{T}(0) \rangle = \frac{\bar{c}}{2\bar{z}^4}, \quad \langle T(z)\bar{T}(0) \rangle = 0. \quad (3.3)$$

To find the non-relativistic answers we use the above results (3.3) and perform a calculation similar to the limiting calculation of the generic two-point function. We keep in mind $z = t + x$, $\bar{z} = t - x$ and $c, \bar{c} = 6(C_1 \pm C_2/\epsilon)$.

$$\begin{aligned}\langle T_{(1)}(t, x)T_{(1)}(0, 0) \rangle &= \lim_{\epsilon \rightarrow 0} 6t^{-4} \left[(C_1 + \frac{C_2}{\epsilon})(1 + 4\frac{\epsilon x}{t} + \dots) + (C_1 - \frac{C_2}{\epsilon})(1 - 4\frac{\epsilon x}{t} + \dots) \right] \\ \langle T_{(1)}(t, x)T_{(2)}(0, 0) \rangle &= \lim_{\epsilon \rightarrow 0} 6\epsilon t^{-4} \left[(C_1 + \frac{C_2}{\epsilon})(1 + 4\frac{\epsilon x}{t} + \dots) - (C_1 - \frac{C_2}{\epsilon})(1 - 4\frac{\epsilon x}{t} + \dots) \right] \\ \langle T_{(2)}(t, x)T_{(2)}(0, 0) \rangle &= \lim_{\epsilon \rightarrow 0} 6\epsilon^2 t^{-4} \left[(C_1 + \frac{C_2}{\epsilon})(1 + 4\frac{\epsilon x}{t} + \dots) + (C_1 - \frac{C_2}{\epsilon})(1 - 4\frac{\epsilon x}{t} + \dots) \right]\end{aligned}$$

The answers after taking the non-relativistic limit turn out to be

$$\langle T_{(1)}(t, x)T_{(1)}(0, 0) \rangle = 6t^{-4}(C_1 + 4\frac{C_2 x}{t}) \quad (3.4)$$

$$\langle T_{(1)}(t, x)T_{(2)}(0, 0) \rangle = 6C_2 t^{-4} \quad (3.5)$$

$$\langle T_{(2)}(t, x)T_{(2)}(0, 0) \rangle = 0 \quad (3.6)$$

So, here we see that the energy-momentum tensor correlation functions will have non-trivial pieces with spatial dependence which survive the scaling limit. Dealing with these would be relatively simpler compared to the arbitrarily high spin states in the dual gravity picture.

Let us re-derive the previous answers in a different way and thereby gain a better understanding of the energy momentum tensor of the GCA. We turn to mode expansions of the relativistic E-M tensor in terms of the Virasoro algebra generators. As is well known, we can expand the holomorphic and anti-holomorphic energy-momentum tensors as follows:

$$T(z) = \sum_n \mathcal{L}_n z^{-n-2}, \quad \bar{T}(\bar{z}) = \sum_n \bar{\mathcal{L}}_n \bar{z}^{-n-2} \quad (3.7)$$

So for the linear combinations which yield the non-relativistic versions of the energy-momentum tensors, these expressions become

$$T_{(1)} = \sum_n \mathcal{L}_n z^{-n-2} + \sum_n \bar{\mathcal{L}}_n \bar{z}^{-n-2}, \quad T_{(2)} = \epsilon(\sum_n \mathcal{L}_n z^{-n-2} - \sum_n \bar{\mathcal{L}}_n \bar{z}^{-n-2}) \quad (3.8)$$

Expanding in terms of t, x , we get

$$\begin{aligned}T_{(1)} &= \sum_n t^{-n-2} \left((\mathcal{L}_n + \bar{\mathcal{L}}_n) - (n+2)\frac{\epsilon x}{t}(\mathcal{L}_n - \bar{\mathcal{L}}_n) + \frac{(n+2)(n+3)}{2}(\frac{\epsilon x}{t})^2(\mathcal{L}_n + \bar{\mathcal{L}}_n) + \dots \right) \\ T_{(2)} &= \epsilon \sum_n t^{-n-2} \left((\mathcal{L}_n - \bar{\mathcal{L}}_n) - (n+2)\frac{\epsilon x}{t}(\mathcal{L}_n + \bar{\mathcal{L}}_n) + \dots \right)\end{aligned} \quad (3.9)$$

Taking the $\epsilon \rightarrow 0$ limit, one gets the mode expansion of the non-relativistic EM tensor in terms of the GCA generators.

$$T_{(1)} = \sum_n t^{-n-2} \left[L_n + (n+2)\frac{x}{t}M_n \right], \quad T_{(2)} = \sum_n M_n t^{-n-2} \quad (3.10)$$

Now, we can use (3.10) to re-calculated the correlation functions of the energy-momentum tensors. Using (2.10), it is straight-forward to reproduce (3.4). Below, we give the details of one such

computation.

$$\begin{aligned}
\langle T_{(1)}(t_1, x_1) T_{(1)}(t_2, x_2) \rangle &= \sum_{m,n} \langle 0 | (L_m + (m+2) \frac{x_1}{t_1} M_m) t_1^{-m-2} (L_n + (n+2) \frac{x_2}{t_2} M_n) t_2^{-n-2} | 0 \rangle \\
&= \sum_{m>1, n<-1} t_1^{-m-2} t_2^{-n-2} \left(\langle 0 | L_n L_m + [L_m, L_n] | 0 \rangle + (m+2) \frac{x_1}{t_1} \langle 0 | L_n M_m + [M_m, L_n] | 0 \rangle \right. \\
&\quad \left. + (n+2) \frac{x_2}{t_2} \langle 0 | M_n L_m + [L_m, M_n] | 0 \rangle \right) \\
&= \sum_{m>1, n<-1} \delta_{m+n,0} (m^3 - m) t_1^{-m-2} t_2^{-n-2} \left(C_1 + C_2 \left\{ (m+2) \frac{x_1}{t_1} + (n+2) \frac{x_2}{t_2} \right\} \right) \\
&= C_1 \sum_{n=0}^{\infty} \{ (n+2)^3 - (n+2) \} t_1^{-4} \left(\frac{t_2}{t_1} \right)^n + C_2 \sum_{n=0}^{\infty} (n+4) \{ (n+2)^3 - (n+2) \} x_1 t_1^{-5} \left(\frac{t_2}{t_1} \right)^n \\
&\quad + C_2 \sum_{p=-1}^{\infty} (p+1) \{ (p+3)^3 - (p+3) \} x_2 t_1^{-5} \left(\frac{t_2}{t_1} \right)^p \\
&= \sum_{n=0}^{\infty} \left((n+1)(n+2)(n+3) C_1 t_1^{-4} \left(\frac{t_2}{t_1} \right)^n + (n+1)(n+2)(n+3)(n+4) C_2 (x_1 - x_2) t_1^{-5} \left(\frac{t_2}{t_1} \right)^n \right) \\
&\Rightarrow \langle T_{(1)}(t_1, x_1) T_{(1)}(t_2, x_2) \rangle = 6(t_1 - t_2)^{-4} \left(C_1 + 4C_2 \frac{(x_1 - x_2)}{(t_1 - t_2)} \right) \tag{3.11}
\end{aligned}$$

In (3.4), we have used $x_1 = x, x_2 = 0, t_1 = t, t_2 = 0$. Similarly, the rest of the answers of (3.4) can be reproduced.

4. The Gravity Dual to the 2D GCA

In [12], the gravity dual of the GCA was proposed to be a novel Newton-Cartan like $AdS_2 \times R^d$ with a degenerate metric for the whole of the spacetime and dynamical Christoffel symbols. This was a natural nonrelativistic extension of the geometrized version of the Newtonian theory in flat spacetimes. For the two dimensional GCA, the dual candidate is thus a limit of AdS_3 which is a Newton-Cartan like $AdS_2 \times R$. The Brown-Henneaux analysis [21] of AdS_3 gives rise to two copies of the Virasoro algebra which emerge as the asymptotic symmetries of the AdS_3 . The classical GCA (without the central charges) arises on taking the appropriate contraction of these bulk isometries. Now, for the quantum GCA with both central charges C_1, C_2 turned on, there is more to this story. First, let us consider $C_2 = 0$. From the Brown-Henneaux analysis in AdS_3 , central charges for both the Virasoros are generated and $c_L = c_R = \frac{3l}{2G}$, where l, G are the radius of AdS_3 and the Newton constant in 3d respectively. We see that in the GCA limit, this is enough to generate $C_1 = \frac{l}{4G}$. As we can see, one would need to look beyond the usual AdS_3 if one has to realize a non-zero value of C_2 . But before we venture into this, let us remind ourselves of a set-up where the usual holographic dictionary works for the non-relativistic theory described by the GCA.

4.1 Trivial Holography

We have seen above that the general two point functions of the non-relativistic field theory depend on time and space directions. Let us concentrate for the moment on the sub-sector where we have set all rapidity eigenvalues $\xi = 0$. The correlation functions then become ultra time dependent and the structure is reminiscent of a one-dimensional CFT. One expects that in this case, the spatial

fibres in the bulk Newton-Cartan structure would not play any role in the holographic description and all the information would come from the AdS_2 base. This expectation is indeed correct and following [14], one can consider, e.g. a scalar field in this background whose equation of motion is given by

$$\frac{1}{\sqrt{G}}\partial_M\left(\sqrt{G}G^{MN}\partial_N\phi(t,z,x_i)\right)-m^2\phi(t,z,x_i)=0, \quad (4.1)$$

where G_{MN} is the metric of AdS_3 in the Poincaré coordinates. The non-relativistic scaling in the bulk is $\{x \rightarrow \epsilon x; t, z \rightarrow t, z\}$ where the radial direction being a measure of the energy of the boundary theory, scales like time. Denoting g_{ab} as the metric of AdS_2 , (4.1) becomes

$$\left[\frac{1}{z^3}\partial_a\left(z^3g^{ab}\partial_b\phi(t,z,x)\right)-m^2\phi(t,z,x)\right]+\frac{z^2}{\epsilon^2}\partial_x^2\phi(t,z,x)=0. \quad (4.2)$$

We wish to have a well behaved equation as $\epsilon \rightarrow 0$ and hence

$$\frac{1}{z^3}\partial_a\left(z^3g^{ab}\partial_b\phi(t,z,x)\right)-m^2\phi(t,z,x)=0, \quad \partial_x^2\phi(t,z,x)=0. \quad (4.3)$$

We could also in principle have a mass term in the x -equation if the 3d mass had a $\frac{1}{\epsilon}$ piece. This does not change the results that follow. The first equation may be obtained from a two dimensional action given by

$$I = \int dt dz \sqrt{G} \frac{1}{2} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right), \quad (4.4)$$

while the second equation may be treated as a constraint. As is well known, this can be solved in terms of Bessel functions and the most general solution is

$$\phi(t,z) = ze^{-i\omega t} (AI_\alpha(\omega z) + BK_\alpha(\omega z)), \quad (4.5)$$

where $\alpha = \sqrt{m^2 + 1}$. Now one can use the usual prescription [30, 31] of AdS/CFT now for the case of three bulk dimensions and find the bulk solution by given a boundary value

$$\phi(t,z) = c\delta^{\Delta-2} \int dt' \phi_\delta(t') \left(\frac{z}{z^2 + |t-t'|^2} \right)^\Delta, \quad (4.6)$$

where $\Delta = \alpha + \frac{1}{2}$ and ϕ_δ denotes the Dirichlet boundary value at $z = \delta$. This can be used to read the two point function of the boundary GCA:

$$\langle \mathcal{O}(t_1) \mathcal{O}(t_2) \rangle \sim (t_1 - t_2)^{-2\Delta}. \quad (4.7)$$

4.2 Topologically Massive Gravity

We have seen in the previous section how we can apply the usual techniques of holography for the non-relativistic limit of AdS/CFT for the simple case of zero rapidities ($\xi = 0$). The answers are reminiscent of a truncation to an AdS_2/CFT_1 correspondence. But for non-zero values of ξ , the correlators in the boundary have non-trivial spatial dependence and to obtain the answers from the bulk, one would need to look at correlation functions of bulk fields which carry high spins and fixed conformal dimensions. We also want to probe the quantum GCA in all its generality and as remarked before, that would mean looking beyond the usual AdS_3 . It is natural to look for corrections to the bulk theory by higher curvature invariants. In three dimensions, there is the unique choice of adding a gravitational Chern-Simons term to the action. 3d gravity in the absence

of any other higher derivative terms contains no dynamical degrees of freedom. However, adding the Chern-Simons term to the action has the effect of adding propagating gravitons to the system and the theory goes under the name of Topologically Massive Gravity (TMG) [23, 22]. The action for TMG is given by

$$\mathcal{I} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[R + \frac{2}{\ell^2} + \frac{1}{2\mu} \epsilon^{\mu\nu\rho} \left(\Gamma_{\mu\lambda}^\sigma \partial_\nu \Gamma_{\rho\sigma}^\lambda + \frac{2}{3} \Gamma_{\mu\lambda}^\sigma \Gamma_{\nu\theta}^\lambda \Gamma_{\rho\sigma}^\theta \right) \right] \quad (4.8)$$

where ℓ is the AdS radius. The equations of motion are given by:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} = 0, \quad (4.9)$$

where $\Lambda = -\frac{1}{\ell^2}$ and the Cotton tensor, $C_{\mu\nu}$ is given by

$$C_{\mu\nu} = \epsilon_\mu^{\alpha\beta} \nabla_\alpha (R_{\beta\nu} - \frac{1}{4} R g_{\beta\nu}).$$

TMG with its gravitational Chern-Simons term still allows AdS_3 as a solution. The vacuum state is the globally AdS_3

$$ds^2 = - \left(1 + \frac{r^2}{\ell^2} \right) dt^2 + \left(1 + \frac{r^2}{\ell^2} \right)^{-1} dr^2 + r^2 d\phi^2. \quad (4.10)$$

One also has the BTZ black hole as a solution in TMG. But we would not be interested in it in the present context. Since the AdS_3 solutions are not changed despite the modification of the equations of motion, one can carry out the analogue of the Brown-Henneaux analysis, now for the TMG. One again ends up with two copies of the Virasoro algebra [26].

$$\begin{aligned} \mathcal{L}_n &= -ie^{inx^+} \left(\partial_+ - \frac{n^2 \ell^2}{2r^2} \partial_- - \frac{inr}{2} \partial_r \right), \\ \bar{\mathcal{L}}_n &= -ie^{inx^-} \left(\partial_- - \frac{n^2 \ell^2}{2r^2} \partial_+ + \frac{inr}{2} \partial_r \right) \end{aligned} \quad (4.11)$$

where $x^\pm = t/\ell \pm \phi$ and $\partial_\pm = \frac{1}{2}(\ell \partial_t \pm \partial_\phi)$. The topological term however effects the central charges and the effect is an asymmetry between the left and right movers [24, 25, 26].

$$c = \frac{3\ell}{2G} \left(1 + \frac{1}{\mu\ell} \right), \quad \bar{c} = \frac{3\ell}{2G} \left(1 - \frac{1}{\mu\ell} \right) \quad (4.12)$$

Now that we have unequal relativistic central charges, the setting looks ideal to take a nonrelativistic limit [18]. In the three dimensional bulk, now in global co-ordinates, we implement this by taking

$$t \rightarrow t, \quad r \rightarrow r, \quad \phi \rightarrow \epsilon\phi \quad \text{with} \quad \epsilon \rightarrow 0. \quad (4.13)$$

It is clear what linear combinations one needs to look at and hence we define the GCA generators L_n and M_n as

$$\begin{aligned} L_n &= ie^{int/\ell} \left[\ell \left(1 - \frac{n^2 \ell^2}{2r^2} \right) \partial_t + in\phi \left(1 + \frac{n^2 \ell^2}{2r^2} \right) \partial_\phi - inr \partial_r \right], \\ M_n &= ie^{int/\ell} \left(1 + \frac{n^2 \ell^2}{2r^2} \right) \partial_\phi. \end{aligned} \quad (4.14)$$

It is easily to check that the generators above satisfy the center-less GCA algebra. To obtain both the nonrelativistic central charges, we need to keep in mind that the relativistic central charges need to have $\frac{1}{\epsilon}$ pieces in them which would survive the contraction. Hence one must also scale

$$\mu \rightarrow \epsilon\mu. \quad (4.15)$$

Recalling (2.11) and (4.12) we get

$$C_1 = \frac{\ell}{4G}, \quad C_2 = \frac{1}{4G\mu}. \quad (4.16)$$

5. TMG/CFT

We would like to add support to the above relation between the non-relativistic limit of TMG and the GCA, now by looking at correlation functions. As stressed earlier, one way of going about this would be to look for correlation functions of high spin operators in the bulk. These would have non-trivial spatial dependence. To make a connection with TMG, one would need to compute beyond three-point correlators to see the dependence on the asymmetric central charges. As suggested before, the way around this rather Herculean task is to look at the two-point correlation function of the energy-momentum tensor. Before we go into that, we would like to highlight a few points regarding boundary conditions in AdS/CFT which are often glossed over. Our discussion would be very close to related discussions in [27, 28].

5.1 Boundary Conditions and AdS/CFT

In AdS/CFT, the boundary fields parametrizing the bulk field boundary conditions source the dual operators in the conformal field theory. The onshell action, through the AdS/CFT correspondence, is the generating functional for the CFT correlation functions. One must keep in mind that to obtain the correlation functions of operators in the dual CFT one must functionally differentiate with respect to the sources that couple to the operators in question. These sources should thus be unconstrained and specifically the boundary conditions must not be fixed by hand. The leading boundary behaviour should be determined by these unconstrained fields and the equations of motion would determine the sub-leading behaviour dynamically.

For the moment, let us concentrate on the bulk metric as that would be the field of interest for our later discussions. This should act as the source for the energy momentum tensor of the boundary field theory and hence the boundary conditions must be parametrized by an unconstrained metric. One needs to consider Asymptotically local AdS (AlAdS) spacetimes for this purpose. In the finite neighbourhood of the conformal boundary at $r = 0$, these spacetimes in Gaussian normal co-ordinates centred at the conformal boundary, admit the following metric

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x, r) dx^i dx^j, \quad (5.1)$$

where $g_{ij}(x, r) \rightarrow g_{(0)ij}(x)$ as $r \rightarrow 0$. ($g_{(0)ij}(x)$ is a non-degenerate metric). The precise form of $g_{ij}(x, r)$ is determined by solving the bulk equations asymptotically and for pure gravity in 3d, this yields

$$g_{ij}(x, r) = g_{(0)ij}(x) + r^2 g_{(2)ij}(x) + \dots \quad (5.2)$$

Now, the analysis of Brown and Henneaux [21] imposes the further criterion on the boundary conditions:

$$g_{(0)ij}(x) = \delta_{ij} \quad (5.3)$$

that implies that the metric should be asymptotically AdS. This, from the point of AdS/CFT is not general enough and is violated if one wishes to consider CFTs in non-trivial backgrounds or wishes, as we do, to calculate correlation functions of the stress-energy tensor.

5.2 Holographic Renormalization and Correlation functions

Correlation functions in the field theory suffer from ultraviolet (UV) divergences. These are expected to be related to the infrared (IR) divergences on the gravity side by the UV-IR correspondence in AdS/CFT. To get finite, sensible answers in the field theory, one needs to regularize and renormalize. Similarly, one would need to “holographically renormalize” in the bulk [32, 33]. The calculation outlined for the bulk dual of GCA, for scalar operators in the regime where $\xi = 0$ in Sec (4.1) is an example of implementation of the GKPW prescription [30, 31] in the bulk which should ideally be looked at as a relation between bare quantities. In principle, to get proper answers one should use the techniques of holographic renormalization. Holographic renormalization of the onshell gravity action depends crucially on the asymptotic form of the metric and hence the boundary conditions talked about earlier become important in this procedure. To holographically renormalize, one adds local boundary covariant counter-terms. For AlAdS spacetimes, the procedure yields the one-point function of the energy momentum tensor

$$\langle T_{ij} \rangle \sim g_{(d)ij} + X_{ij}[g_{(0)}] \quad (5.4)$$

where $X_{ij}[g_{(0)}]$ is a known function of $g_{(0)}$. One finds the higher point correlation functions of the energy-momentum tensor by taking derivatives of the above relation.

5.3 Holographic Renormalization and TMG

An analogous discussion to the above holds for the TMG and one can show that the Brown-Henneaux boundary conditions are not sufficiently general. In particular, at the “chiral” point $\mu\ell = 1$, where one of the central charges of the relativistic theory (see (4.12)) vanishes, there is the existence of extra logarithmic modes [34]. This invalidates the arguments of the chiral gravity conjecture put forward in [35]. We shall not dwell on these aspects in this paper.

We are looking to calculate correlation functions of the energy momentum tensor. For an honest calculation, one needs to adopt the techniques of holographic renormalization to this context. This has fortunately been dealt with in [27] in great details. Let us here point out a few new features on this set-up compared to the usual scenario in *AdS* spacetimes. The field equations for the TMG are third order in derivatives. This one can fix two pieces of boundary data: the metric and a component of the extrinsic curvature. The boundary metric sources the EM tensor while the boundary field parametrizing the extrinsic curvature sources a new operator. This new operator turns out to be the logarithmic partner of the energy momentum tensor in the chiral limit. The dual of the TMG at $\mu = 1$ is thus a logarithmic CFT with $c = 0$. We would however not have much to say about the role of this operator in the non-relativistic limit and will entirely focus on the energy momentum tensor.

5.4 TMG and GCA

For a general μ , the authors [27] compute the two point function of the energy-momentum tensor by using the techniques of holographic renormalization. We shall not go into the details of the derivation here and quote the results and proceed to use them in our context. The authors [27] use a linearized analysis and expand $g_{ij} = \eta_{ij} + h_{ij}$. They work in Poincare co-ordinates where the background metric is

$$G_{\mu\nu}dx^\mu dx^\nu = \frac{dr^2}{r^2} + \frac{1}{r^2}\eta_{ij}dx^i dx^j = \frac{dr^2}{r^2} + \frac{1}{r^2}dzd\bar{z} \quad (5.5)$$

where one first uses lightcone co-ordinates $v, u = t \pm x$ and then replaces $(v, u) \rightarrow (z, \bar{z})$, the complex boundary co-ordinates. The results for the two-point function of the energy-momentum tensor for a general μ is

$$\begin{aligned} \langle \bar{T}(z, \bar{z})\bar{T}(0) \rangle &= \frac{3}{2G_N} \frac{\mu+1}{2\mu} \frac{1}{\bar{z}^4} \\ \langle T(z, \bar{z})T(0) \rangle &= \frac{3}{2G_N} \frac{\mu-1}{2\mu} \frac{1}{z^4} \end{aligned} \quad (5.6)$$

To realize the non-relativistic results, we take linear combinations as before. This yields

$$\begin{aligned} \langle T_{(1)}(t, x)T_{(1)}(0) \rangle &= \langle [T(z, \bar{z}) + \bar{T}(z, \bar{z})][T(0) + \bar{T}(0)] \rangle \\ &= \frac{3}{2G_N} t^{-4} \left[\left(1 + 10\left(\frac{x}{t}\right)^2 + 35\left(\frac{x}{t}\right)^4 + \dots\right) + \frac{1}{\mu} \left(4\left(\frac{x}{t}\right) + 20\left(\frac{x}{t}\right)^3 + \dots\right) \right] \end{aligned} \quad (5.7)$$

Similarly,

$$\begin{aligned} \langle T_{(1)}(t, x)T_{(2)}(0) \rangle &= \epsilon \langle [T(z, \bar{z}) + \bar{T}(z, \bar{z})][T(0) - \bar{T}(0)] \rangle \\ &= \frac{3\epsilon}{2G_N} t^{-4} \left[\left(4\left(\frac{x}{t}\right) + 20\left(\frac{x}{t}\right)^3 + \dots\right) + \frac{1}{\mu} \left(1 + 10\left(\frac{x}{t}\right)^2 + 35\left(\frac{x}{t}\right)^4 + \dots\right) \right] \end{aligned} \quad (5.8)$$

And,

$$\begin{aligned} \langle T_{(2)}(t, x)T_{(2)}(0) \rangle &= \epsilon^2 \langle [T(z, \bar{z}) - \bar{T}(z, \bar{z})][T(0) - \bar{T}(0)] \rangle \\ &= \frac{3\epsilon^2}{2G_N} t^{-4} \left[\left(1 + 10\left(\frac{x}{t}\right)^2 + 35\left(\frac{x}{t}\right)^4 + \dots\right) + \frac{1}{\mu} \left(4\left(\frac{x}{t}\right) + 20\left(\frac{x}{t}\right)^3 + \dots\right) \right] \end{aligned} \quad (5.9)$$

When we take the non-relativistic limit of $x \rightarrow \epsilon x, t \rightarrow t$ and $\epsilon \rightarrow 0$, we see that there is no non-trivial spatial dependence in (5.7) if we do not scale μ . To match (5.7) – (5.9) to the calculations performed purely in the non-relativistic boundary theory (3.4), we see that the coefficient of the gravitational Chern-Simons term μ must scale like $\mu \rightarrow \epsilon\mu$ as $\epsilon \rightarrow 0$, as proposed in (4.15) [18]. The calculation in [27] was performed in the range $0 < |\mu| < 2$. Later, these techniques have been extended to $\mu = 3$ which is of relevance to Schrodinger holography [29]. But there is no obstruction in extending these results to the limit where μ goes to zero. ²

$$\langle T_{(1)}(t, x)T_{(1)}(0) \rangle = \frac{3}{2G_N} t^{-4} \left[1 + \frac{4}{\mu} \left(\frac{x}{t}\right) \right]$$

²When μ takes integral values, in our case the strict $\mu = 0$ there are additional divergences, and additional contributions to one point functions, which are local in the sources. These terms contribute only contact terms to the two point functions. The identification of the central charges below continues to hold. We thank Marika Taylor for helpful correspondence on this issue.

$$\begin{aligned}\langle T_{(1)}(t, x)T_{(2)}(0) \rangle &= \frac{3}{2G_N\mu}t^{-4} \\ \langle T_{(2)}(t, x)T_{(2)}(0) \rangle &= 0.\end{aligned}\tag{5.10}$$

The identifications also lead to central terms of the 2d quantum GCA in terms of the bulk parameters as:

$$C_1 = \frac{1}{4G}, \quad C_2 = \frac{1}{4G\mu}\tag{5.11}$$

in agreement with the proposal of [18].

6. Conclusions and future directions

In this paper, we have reviewed our understanding of aspects of Galilean holography where the Galilean Conformal Algebra is the symmetry algebra of the boundary theory. We have specifically looked at the 2d GCA and pointed out novel spatial dependence that exists in the correlation functions of the non-relativistic energy momentum tensor. This is an instance where one does not need to consider a setting where the parent relativistic operators have arbitrarily high spins. The construction of the bulk calculation becomes much simpler as one does not have to look at correlation functions of arbitrary high spin fields in AdS. In the bulk calculation, we have stressed why we need to look beyond a Brown-Henneaux like boundary conditions when we are computing stress-energy correlators and why the framework of holographic renormalization is the correct set-up to use. We discussed why we need to go beyond AdS_3 and look at Topologically Massive 3d Gravity if we wanted to realize the 2d GCA in its full generality. Finally, we relied heavily on the techniques and results of [27] to show that in the non-relativistic limit, to obtain the non-trivial spatial dependence obtained in calculation in the nonrelativistic boundary theory. We found that one needs to scale the co-efficient of the gravitational Chern-Simons term, as proposed independently in [18]. Our analysis expectedly also yields the corrected central charges.

This is in some sense a first step towards understanding TMG/GCA holography. The process of holographic renormalization has been outlined in all generality for the TMG in [27]. It should, in principle, be possible to take the nonrelativistic limit at any stage and follow the steps of the argument to see if the procedure makes sense. We know for certain now that taking the limit at the end yields perfectly sensible answers which match with calculations purely in terms of the nonrelativistic boundary theory. It is plausible that following the results of limits taken at various points in the calculation, we would be able to learn more about the structure of the bulk theory. This is under current investigation.

There is also the rather intriguing aspect of the “chiral point” $\mu\ell = 1$ where one of the central charges of the relativistic CFT becomes zero. This has been shown to be a logarithmic CFT. From the point of view of the non-relativistic set-up, the chiral point could be viewed as a “double scaling” limit, where one takes $\mu \rightarrow \epsilon\mu$, $\ell \rightarrow \frac{1}{\epsilon}\ell$ keeping $\mu\ell = 1$. One knows that the aforementioned techniques of holographic renormalization break down for asymptotically flat spacetimes. But it is possible one might get away by keeping a very small cosmological constant in this case because we *do not* need to put $\mu = 0$ strictly anywhere. The result should be a logarithmic version of the GCA. We have not looked at the operator in the CFT which is sourced by the extrinsic curvature which one needs to turn on in TMG. This, as pointed out earlier, plays the role of the log-partner of the energy momentum tensor in the usual relativistic case. It is clear that this would also have

a vital role to play in the non-relativistic set-up. It is also curious to note that the $\ell \rightarrow \frac{1}{\epsilon}\ell$ limit of symmetries of AdS_3 leads to the BMS algebra which is isomorphic to the 2d GCA [19]. It would be interesting to understand how the whole story fits.

To conclude, let us point out something that we have glossed over in our discussions so far. It is well known that Topologically Massive Gravity is plagued with aspects of non-unitarity, coming from negative energy of the propagating gravitons. In the limit of large μ and the non-relativistic context, this problem persists and is reflected in the field theory in the generic non-unitarity of the GCA [16]. We should point out that since we are dealing with finite relativistic spins to start off with, in the non-relativistic sector we are in the $\xi = 0$ sector. But we are also in the $C_2 \neq 0$ sector. There exists null vectors in this GCA sector and interestingly, the operator M_0 has a block diagonal form reminiscent of logarithmic CFTs [16]. It would be of interest to explore the behaviour of the operator X_{ij} which sources the extrinsic curvature on the gravity side even from this perspective and understand the nature of the non-unitarity in the non-relativistic context.

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