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Liquidity, Business Cycles, and Monetary Policy

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This paper presents a model of a monetary economy where there are differences in liquidity across assets. Money circulates because it is more liquid than other assets, not because it has any special function. The model is used to investigate how aggregate activity and asset prices fluctuate with shocks to productivity and liquidity and to examine what role government policy might have through open-market operations that change the mix of assets held by the private sector.

I. Introduction

This paper presents a model of a monetary economy where there are differences in liquidity across assets. Our aim is to study how aggregate activity and asset prices fluctuate with shocks to productivity and liquidity. In doing

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so, we examine what role government policy might have through open-market operations that change the mix of assets held by the private sector.

Part of our purpose is to construct a workhorse model of money and liquidity that does not stray too far from the other workhorse of modern macroeconomics, the real business cycle model. We thus maintain the assumption of competitive markets. In a standard competitive framework, money has no role unless endowed with a special function—for example, that the purchase of goods requires cash in advance. In our model, the reason why money can improve resource allocation is not because money has a special function but because, crucially, we assume that other assets are partially illiquid, less liquid than money. Ours might be thought of as a liquidity-in-advance framework.

Illiquidity has to do with some impediment to the resale of assets. With this in mind, we construct a model in which the resale of assets is a central feature of the economy. We consider a group of entrepreneurs who each uses his or her own capital stock and skill to produce output from labor (which is supplied by workers). Capital depreciates and is restocked through investment, but the investment technology for producing new capital from output is not commonly available: in each period only some of the entrepreneurs are able to invest, and the arrival of investment opportunities is randomly distributed across entrepreneurs through time. Hence, in each period there is a need to channel funds from those entrepreneurs who do not have an investment opportunity (that period's savers) to those who do (that period's investors).

To acquire funds for the production of new capital, an investing entrepreneur issues equity claims to the capital's future returns. However, we assume that because the investing entrepreneur's skill will be needed to produce these future returns and he cannot precommit to work, at the time of investment he can credibly pledge only a fraction—say, θ—of the future returns from the new capital. Unless θ is high enough, he faces a borrowing constraint: he must finance part of the cost of investment from his available resources. The lower the θ, the tighter the borrowing constraint and the larger the down payment per unit of investment that he must make out of his own funds.

He will typically have on his balance sheet two kinds of asset that can be resold to raise funds. He may have money, and he may have equity previously issued by other entrepreneurs. Both of these will have been acquired by him at some point in the past, when he himself was a saver.

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Crucially, we suppose that equity is less liquid than money. We parameterize the degree to which equity is illiquid by making a stylized assumption: in each period only a proportion—say, ϕ—of an agent's equity holding can be resold. Although the entrepreneur with an investment opportunity this period can readily divest ϕ of his equity holding, to divest any more he will have to wait until next period, by which time the opportunity may have disappeared. The lower the ϕ, the tighter the resalability constraint. Unlike his equity holding, the entrepreneur's money holding is perfectly liquid: it can all be used to buy goods straightaway.

In practice, of course, there are wide differences in resalability across different kinds of equity: compare the stock of publicly traded companies with shares in privately held businesses. Indeed, there are many financial assets that are hardly any less liquid than money—for example, government bonds. Thus, in our stylized model, "money" should be interpreted very broadly to include all financial assets that are essentially as liquid as money. Under the heading of "equity" come all financial assets that are less than perfectly liquid. By assumption, all these nonmonetary assets are subject to the common resalability constraint parameterized by ϕ.

To understand how fiat money can lubricate this economy, notice that the task of channeling funds from those entrepreneurs who do not have an investment opportunity into the hands of those who do is thwarted by the fact that investing entrepreneurs are unable to offer savers adequate compensation: the borrowing constraint (θ) means that new capital investment cannot be entirely self-financed by issuing new equity, and the resalability constraint (ϕ) means that enough of the old equity cannot change hands quickly. Fiat money can help alleviate this problem. Our analysis shows that if θ and ϕ are not high enough—if (and only if) a particular combination of θ and ϕ lies below a certain threshold—then the circulation of fiat money, passing each period from investors to savers in exchange for goods, serves to boost aggregate activity. Whenever fiat money plays this essential role, we say that the economy is a monetary economy. Whether agents use fiat money—whether the economy is monetary—is determined endogenously.

We show that in a monetary economy, the expected rate of return on money is low, less than the expected rate of return on equity. (The steady state of an economy where the stock of fiat money is fixed would necessarily have a zero net return on money.) Nevertheless, a saver chooses to hold some money in his portfolio, because in the event that he has an opportunity to invest in the future he will be liquidity constrained, and money is more liquid than equity. The gap between the return on money and the return on equity is a liquidity premium.

We also show that both of the returns on equity and money are lower than the rate of time preference. This is because borrowing constraints starve the economy of means of saving—too little equity can be credibly pledged—which raises asset prices and lowers yields. As a consequence, agents who never have investment opportunities, such as the workers, choose to hold neither equity nor money. Assuming workers cannot borrow against their future labor income, we show that they simply consume their wage, period by period. This may help explain why certain households neither save nor participate in asset markets. It is not that they do not have access to those markets or that they are particularly impatient but rather that the return on assets is not enough to attract them.[1]

In our θ-ϕ framework, θ and ϕ are exogenous parameters. Although the borrowing constraint (θ) and the resalability constraint (ϕ) might both be thought of as varieties of liquidity constraint,[2] in this paper we are especially concerned with the effects of shocks to ϕ, which we identify as liquidity shocks. We are motivated here by the fact that in the recent financial turmoil many assets—such as asset-backed securities and auction-rate securities—that used to be highly liquid became much less resalable.[3] Even though we focus on shocks to ϕ, it is important to recognize that θ is an essential component of the model. Were θ to be sufficiently close to one, then new capital investment could be self-financed by issuing new equity and there would be no need for old equity to circulate (reminiscent of the idea that in the Arrow-Debreu framework markets need open only once, at an initial date); liquidity shocks—shocks to ϕ—would have no effect.

The mechanism by which liquidity shocks affect our monetary economy is absent from most real business cycle models. In our model, there are critical feedbacks from asset prices to aggregate activity. Consider a persistent liquidity shock: suppose ϕ falls and is anticipated to recover only slowly. The impact of this fall in resalability is to shrink the funds available to investors to use as down payment. Further, anticipating lower future resalability, the price of equity falls relative to the value of money—think of this as a "flight to liquidity"—which tends to raise the size of the required down payment per unit of investment. All in all, via these feedback mechanisms, investment falls as ϕ falls. Asset prices and aggregate activity are vulnerable to liquidity shocks, unlike in a standard general equilibrium asset pricing model.

[1] The model can be extended to show that if workers face idiosyncratic shocks to spending needs, then they may save but only use money to do so.

[2] Brunnermeier and Pedersen (2009) use "funding liquidity" to refer to the borrowing constraint and "market liquidity" to refer to the resalability constraint.

[3] In our first presentations of this research (see, e.g., Kiyotaki and Moore [2001]), although we separately identified the borrowing and resalability constraints, for analytical convenience we set $\phi = \theta$. However, it helps to keep ϕ distinct from θ, as we do in the current paper, because we are thus able to pin down the effects of shocks to ϕ and identify a monetary policy that can be used in response. We made use of the θ-ϕ framework in other papers, though sometimes with different notation (Kiyotaki and Moore 2002, 2003, 2005a, 2005b).

Our basic model, presented in Sections II and III, has a fixed stock of fiat money. Government is introduced in the full model of Section IV, which examines monetary policy. How might government, through interventions by the central bank, ameliorate the effects of liquidity shocks? Specifically, how might policy change behavior in the private economy?

The central bank can buy and hold private equity—presuming that the central bank does not violate the private sector resalability constraint. An open-market operation to purchase equity by issuing fiat money shifts up the ratio of the values of money to equity held by the private sector (see Metzler 1951). Investing entrepreneurs are in a position to invest more when their portfolios are more liquid. In effect, the government improves liquidity in the private economy by taking relatively illiquid assets onto its own books, thereby boosting aggregate activity. This unconventional form of monetary policy has been employed by central banks around the world in recent years to ease the global financial crisis and appears to have met with some success; see, for example, Del Negro et. al. (2017). Interventions by the central bank have real effects in our economy because they operate across a liquidity margin—the difference in liquidity between money and equity. With its emphasis on liquidity rather than sticky prices, our framework harks back to an earlier interpretation of Keynes (1936), following Tobin (1969).

Before we come to this policy analysis, it helps to start with the basic model without government. We relate our paper to the literature and make some final remarks in Section V. Proofs are contained in the appendix.

II. Basic Model without Government

Consider an infinite-horizon, discrete-time economy with four objects traded: a nondurable output, labor, equity, and fiat money. Fiat money is intrinsically useless and is in fixed supply M in the basic model of this and the next section.

There are two populations of agents, entrepreneurs and workers, each with unit measure. Let us start with the entrepreneurs, who are the central actors in the drama. At date t, a typical entrepreneur has expected discounted utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \tag{1}$$

of consumption path $\{c_t, c_{t+1}, c_{t+2}, \ldots\}$, where $u(c) = \log c$ and $0 < \beta < 1$. He has no labor endowment. All entrepreneurs have access to a constant-returns-to-scale technology for producing output from capital and labor. An entrepreneur holding k_t capital at the start of period t can employ ℓ_t labor to produce

$$y_t = A_t k_t^{\gamma} \ell_t^{1-\gamma} \tag{2}$$

output, where $0 < \gamma < 1$. Production is completed within the period t, during which time capital depreciates to λk_t, $0 < \lambda < 1$. We assume that the productivity parameter $A_t > 0$, which is common to all entrepreneurs, follows a stationary stochastic process. Given that each entrepreneur can employ labor at a competitive real wage rate, w_t, gross profit is proportional to the capital stock:

$$y_t - w_t \ell_t = r_t k_t, \tag{3}$$

where, as we will see, gross profit per unit of capital, r_t, depends on productivity, aggregate capital stock, and labor supply.

The entrepreneur may also have an opportunity to produce new capital. Specifically, at each date t, with probability π he has access to a constant-returns technology that produces i_t units of capital from i_t units of output. The arrival of such an investment opportunity is independently distributed across entrepreneurs and through time and is independent of aggregate shocks. Again, investment is completed within the period t—although newly produced capital does not become available as an input to the production of output until the following period, $t + 1$:

$$k_{t+1} = \lambda k_t + i_t. \tag{4}$$

We assume there is no insurance market against having an investment opportunity.[4] We also make a regularity assumption that the subjective discount factor is larger than the fraction of capital left after production (one minus the depreciation rate):

ASSUMPTION 1. $\beta > \lambda$.

This mild restriction is not essential but will make the distribution of capital and asset holdings across individual entrepreneurs well behaved.

To finance the cost of investment, the entrepreneur who has an investment opportunity can issue equity claims to the future returns from newly produced capital. Normalize one unit of equity at date t to be a claim to the future returns from one unit of investment at date t: it pays r_{t+1} output at date $t + 1$, λr_{t+2} at date $t + 2$, $\lambda^2 r_{t+3}$ at date $t + 3$, and so on.

We make two critical assumptions. First, the entrepreneur who produces new capital cannot fully precommit to work with it, even though his specific skills will be needed for it to produce output. To capture this lack of commitment power in a simple way, we assume that an investing

[4] This assumption can be justified in a variety of ways. For example, it may not be possible to verify that someone has an investment opportunity, or verification may take so long that the opportunity has gone by the time the claim is paid out. A long-term insurance contract based on self-reporting will not fully work if people are able to save covertly. Each of these justifications warrants formal modeling. But we are reasonably confident that even if partial insurance were possible, our broad conclusions would still hold. So rather than clutter up the model, we simply assume that no insurance scheme is feasible.

entrepreneur can credibly pledge at most a fraction $\theta < 1$ of the future returns.[5] Loosely put, we are assuming that only a fraction θ of the new capital can be mortgaged.

We take θ to be an exogenous parameter: the fraction of new capital returns that can be issued as equity at the time of investment. The smaller the θ, the tighter the borrowing constraint that an investing entrepreneur faces. To meet the cost of investment, he has to use any money that he may hold and raise further funds by—as far as possible—reselling any holding of other entrepreneurs' equity that he may have accumulated through past purchases.

The second critical assumption is that entrepreneurs cannot dispose of their equity holdings as quickly as money. Again, to capture this idea in a simple way, we assume that before the investment opportunity disappears, the investing entrepreneur can resell only a fraction $\phi_t < 1$ of his holding of other entrepreneurs' equity. (He can use all of his own money.) This is tantamount to assuming a peculiar transaction cost per period: zero for the first fraction ϕ_t of equity sold and then infinity.[6]

Like θ, we take ϕ_t to be an exogenous parameter: the fraction of equity holdings that can be resold in each period. The smaller the ϕ_t, the less liquid the equity and the tighter the resalability constraint.

We suppose that the aggregate productivity A_t and the liquidity of equity ϕ_t jointly follow a stationary Markov process in the neighborhood of the constant unconditional mean (A, ϕ). A shock to A_t is a productivity shock, and a shock to ϕ_t is a liquidity shock. (We do not shock θ, which is why it does not have a subscript.)

In general, an entrepreneur has three kinds of asset in his portfolio: money, his holding of other entrepreneurs' equity, and the uncommitted fraction, $1 - \theta$, of the returns from his own capital, which might loosely be termed "unmortgaged capital stock"—own capital stock minus own equity issued.

Balance sheet	
Money holding	Own equity issued
Holding of other entrepreneurs' equity	Net worth
Own capital stock	

It turns out to be generally hard to analyze aggregate fluctuations of the economy with these three assets, because there is a complex dynamic

[5] Compare with Hart and Moore (1994), where the borrowing constraint is shown to be a consequence of the fact that the human capital of the agent who is raising funds—here the investing entrepreneur—is inalienable.

[6] One way to endogenize ϕ_t is to make use of a search and matching framework. See Cui and Radde (2016).

interaction between the distribution of asset holdings across the entrepreneurs and their choices of consumption, investment, and portfolio. Thus, we make a simplifying assumption: in every period, we suppose that an entrepreneur can issue new equity against a fraction ϕ_t of any uncommitted returns from his old capital—in loose terms, he can mortgage a fraction ϕ_t of any as yet unmortaged capital stock.[7] Think of mortgaging old capital stock—or reselling equity—as akin to peeling an onion slowly, layer by layer, a fraction ϕ_t in each period t.

The upshot of this assumption is that an entrepreneur's holding of others' equity and his unmortgaged capital stock are perfect substitutes as means of saving for him: both pay the same return stream per unit (r_{t+1} at date $t+1$, λr_{t+2} at date $t+2$, $\lambda^2 r_{t+3}$ at date $t+3$, etc.), and up to a fraction ϕ_t of both can be resold or mortgaged per period. In effect, by making the simplifying assumption, we have reduced the number of assets that we need to keep track of to two: besides money, the holdings of other entrepreneurs' equity (outside equity) and the unmortgaged capital stock (inside equity) can be lumped together as simply "equity."

Let n_t be the equity and m_t the money held by an individual entrepreneur at the start of period t. He faces two liquidity constraints:

$$n_{t+1} \geq (1-\theta)i_t + (1-\phi_t)\lambda n_t \tag{5}$$

and

$$m_{t+1} \geq 0. \tag{6}$$

During the period, the entrepreneur who invests i_t can issue at most θi_t equity against the new capital, and he can dispose of at most a fraction ϕ_t of his equity holding after depreciation. Inequality (5) brings these constraints together: his equity holding at the start of period $t+1$ must be at least $1-\theta$ times investment plus $1-\phi_t$ times depreciated equity. Inequality (6) says that his money holding cannot be negative.

Let q_t be the price of equity in terms of output, the numeraire; q_t is also equal to Tobin's q: the ratio of the market value of capital to the replacement cost. Let p_t be the price of money. (Warning—p_t is customarily defined as the inverse: the price of output in terms of money. But a priori money may not have value, so it is better not to make it the numeraire.) The entrepreneur's flow-of-funds constraint at date t is then given by

$$c_t + i_t + q_t(n_{t+1} - i_t - \lambda n_t) + p_t(m_{t+1} - m_t) = r_t n_t. \tag{7}$$

The left-hand side is his expenditure on consumption, investment, and net purchases of equity and money. The right-hand side is his dividend

[7] One reason may be that with age, capital becomes less specific to the producing entrepreneur so that he can credibly commit to pay more of the output from older capital.

income, which is proportional to his holding of equity at the start of this period.

Turn now to the workers. Because there is no heterogeneity among workers and the population of workers is unity, we consider a representative worker. At date t, the representative worker has expected discounted utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left[C_s^w - \frac{\omega}{1+\nu}(L_s)^{1+\nu}\right] \tag{8}$$

of paths of consumption $\{C_t^w, C_{t+1}^w, C_{t+2}^w, \ldots\}$ and labor supply $\{L_t, L_{t+1}, L_{t+2}, \ldots\}$, where $\omega > 0$, $\nu > 0$, and $U[\cdot]$ is increasing and strictly concave. If the worker starts date t holding N_t^w equity and M_t^w money, her flow-of-funds constraint is

$$C_t^w + q_t(N_{t+1}^w - \lambda N_t^w) + p_t(M_{t+1}^w - M_t^w) = w_t L_t + r_t N_t^w. \tag{9}$$

The consumption expenditure and net purchase of equity and money in the left-hand side are financed by wage and dividend income in the right-hand side. Workers, who do not have investment opportunities, face the same resalability constraints as entrepreneurs and cannot borrow against their future labor income:

$$N_{t+1}^w \geq (1-\phi_t)\lambda N_t^w \geq 0 \text{ and } M_{t+1}^w \geq 0. \tag{10}$$

An equilibrium process of prices $\{p_t, q_t, w_t\}$ is such that entrepreneurs choose labor demand l_t to maximize gross profit (3) subject to the production function (2) for a given start-of-period capital stock and choose consumption, investment, capital stock, and start-of-next-period equity and money holdings $\{c_t, i_t, k_{t+1}, n_{t+1}, m_{t+1}\}$ to maximize (1) subject to (4)–(7); workers choose consumption, labor supply, equity, and money holding $\{C_t^w, L_t, N_{t+1}^w, M_{t+1}^w\}$ to maximize (8) subject to (9) and (10); and the markets for output, labor, equity, and money all clear.

Before we characterize equilibrium, it helps to clear the decks a little by suppressing reference to the workers. Given that their population has unit measure, it follows from (8) and (9) that their aggregate labor supply equals $(w_t/\omega)^{1/\nu}$. Maximizing the gross profit of a typical entrepreneur with capital k_t, we find his labor demand, $k_t[(1-\gamma)A_t/w_t]^{1/\gamma}$, which is proportional to k_t. So if the aggregate stock of capital at the start of date t is K_t, labor market clearing requires that

$$(w_t/\omega)^{1/\nu} = K_t[(1-\gamma)A_t/w_t]^{1/\gamma}.$$

Substituting the equilibrium wage w_t back into the left-hand side of (3), we find that the individual entrepreneur's maximized gross profit equals $r_t k_t$, where

$$r_t = a_t(K_t)^{\alpha-1}, \tag{11}$$

and the parameters a_t and α are derived from A_t, γ, ω, and ν:

$$\begin{aligned} a_t &= \gamma\left(\frac{1-\gamma}{\omega}\right)^{(1-\gamma)/(\gamma+\nu)}(A_t)^{(1+\nu)/(\gamma+\nu)}, \\ \alpha &= \frac{\gamma(1+\nu)}{\gamma+\nu}. \end{aligned} \tag{12}$$

Note from (12) that α lies between 0 and 1, so that r_t—which is parametric for the individual entrepreneur—declines with the aggregate stock of capital K_t, because the wage increases with K_t. But for the entrepreneurial sector as a whole, gross profit r_tK_t increases with K_t. Also note from (12) that r_t is increasing in the productivity parameter A_t through a_t. Below, we show that in the neighborhood of the steady-state monetary equilibrium, the worker will choose to hold neither money nor equity. That is, in aggregate, workers simply consume their labor income at each date:

$$C_t^w = w_tL_t = \frac{1-\gamma}{\gamma}a_t(K_t)^\alpha. \tag{13}$$

The ratio of total wage income to capital income is $1-\gamma:\gamma$, given the Cobb-Douglas production function.

We are now in a position to characterize the equilibrium behavior of the entrepreneurs. Consider an entrepreneur holding equity n_t and money m_t at the start of period t. First, suppose he has an investment opportunity: let this be denoted by a superscript i on his choice of consumption and start-of-next-period equity and money holdings, $(c_t^i, n_{t+1}^i, m_{t+1}^i)$. He has two ways of acquiring equity n_{t+1}^i: either produce it at unit cost 1 or buy it in the market at price q_t. (See the left-hand side of the flow-of-funds constraint [7], where, recall, i_t corresponds to investment.) If q_t is less than 1, the agent will not invest. If q_t equals 1, he will be indifferent. If q_t is greater than 1, he will invest by selling as much equity as he can subject to constraint (5). The entrepreneur's production choice is similar to Tobin's q theory of investment.

Consider first the economy without aggregate uncertainty, to inquire under what conditions the first best is achieved. (All proofs are in the appendix.)

Claim 1. Suppose $(A_t, \phi_t) = (A, \phi)$ for all t. Suppose further that θ and ϕ satisfy condition 1:

Condition 1. $(1-\lambda)\theta + \pi\lambda\phi > (1-\lambda)(1-\pi)$.

Then there exists a deterministic steady state in which all the aggregate variables are constant and

a. the allocation of resources is first best;
b. Tobin's q is equal to unity: $q = 1$;

c. money has no value: $p = 0$; and

d. the gross profit rate equals the time preference rate plus the depreciation rate: $r = [(1/\beta) - 1] + (1 - \lambda) = (1/\beta) - \lambda$.

The intuition behind claim 1 is that if the investing entrepreneurs can issue new equity relatively freely and/or existing equity is relatively liquid—if condition 1 is satisfied—then the equity market is able to transfer enough resources from the savers to the investing entrepreneurs to achieve the first-best allocation.[8] There is no advantage to having investment opportunity; Tobin's q is equal to 1 (the market value of capital is equal to the replacement cost), and both investing entrepreneurs and savers earn the same net rate of return on equity, equal to the time preference rate. Because the economy achieves the first-best allocation without money, money has no value.

We now consider the economy with aggregate uncertainty: the aggregate productivity and the liquidity of equity (A_t, ϕ_t) follow a stochastic process in the neighborhood of constant (A, ϕ). Under condition 1, a continuity argument could be used to show that there is a recursive competitive equilibrium in the neighborhood of this first-best deterministic steady state, in which Tobin's q equals unity $(q_t = 1)$ and money has no value $(p_t = 0)$. Since our primary interest is in monetary equilibria, we omit the details.

To ensure that q_t is strictly greater than 1 and money has value in equilibrium, we assume that θ and ϕ satisfy assumption 2:

Assumption 2. $0 < \Phi(\theta, \phi)$, where

$$\begin{aligned}\Phi(\theta,\phi) \equiv{} & \pi\lambda\beta^2(1-\pi)(1-\phi)[(1-\lambda)(1-\pi)-(1-\lambda)\theta-\pi\lambda\phi] \\ & + [(\beta-\lambda)(1-\pi)-(1-\lambda)\theta-\pi\lambda\phi][(1-\lambda)(1-\theta)+\pi\lambda(1-\phi)] \\ & \times [\lambda(1-\beta)(1-\pi)+(1-\lambda)\theta+\lambda(\beta+\pi-\pi\beta)\phi].\end{aligned}$$

Observe that all the brackets in the right-hand side are positive, except for the terms $(1-\lambda)(1-\pi)-(1-\lambda)\theta-\pi\lambda\phi$ and $(\beta-\lambda)(1-\pi)-(1-\lambda)\theta-\pi\lambda\phi$. Thus, a sufficient condition for assumption 2 is

$$(1-\lambda)\theta + \pi\lambda\phi < (\beta-\lambda)(1-\pi),$$

[8] In steady state, aggregate saving (which equals aggregate investment) is equal to the depreciation of capital. The right-hand side of condition 1 is the ratio of the aggregate saving of the (fraction $1-\pi$) noninvesting entrepreneurs to the aggregate capital stock in first best. The left-hand side is the ratio of the equity issued or resold by the investing entrepreneurs to the aggregate capital stock: $(1-\lambda)\theta$ corresponds to new equity issued, and $\pi\lambda\phi$ corresponds to old equity resold by the (fraction π) investing entrepreneurs. Thus, condition 1 says that the equity issued or resold by the investing entrepreneurs is enough to shift the aggregate saving of the noninvesting entrepreneurs.

and a necessary condition is

$$(1-\lambda)\theta + \pi\lambda\phi < (1-\lambda)(1-\pi).$$

Notice that if condition 1 in claim 1 were satisfied, then this necessary condition would not hold.

CLAIM 2. Under assumptions 1 and 2, there exists a deterministic steady-state equilibrium for constant (A, ϕ) in which money has value. In the neighborhood of such a steady-state equilibrium, there is a recursive equilibrium for stochastic (A_t, ϕ_t) such that

a. the price of money, p_t, is strictly positive;
b. the price of capital, q_t, is strictly greater than 1 but strictly less than $1/\theta$; and
c. an entrepreneur with an investment opportunity faces binding liquidity constraints and will choose not to hold money: $m_{t+1}^i = 0$.

We will be in a position to prove the claim once we have laid out the equilibrium conditions—we use a method of guess and verify in the following. For completeness, it should be pointed out that for intermediate values of θ and ϕ that satisfy neither assumption 2 nor condition 1, we can show that money has no value even though the liquidity constraint (5) still binds. To streamline the paper, we have chosen not to give an exhaustive account of the equilibria throughout the parameter space.

There is a caveat to claim 2*a*. Fiat money can be valuable to someone only if other people find it valuable; hence, there is always a nonmonetary equilibrium in which the price of fiat money is zero. When there is a monetary equilibrium in addition to the nonmonetary equilibrium, we restrict attention to the monetary equilibrium: $p_t > 0$.[9] Claim 2*c* says that the entrepreneur prefers investment with the maximum leverage to holding money, even though the return is in the form of equity that is less liquid than money at date $t + 1$. (Incidentally, even though the investing entrepreneurs do not want to hold money for liquidity purposes, the noninvesting entrepreneurs do—see below. This is why claim 2*a* holds.)

Thus, for an investing entrepreneur, the liquidity constraints (5) and (6) both bind. His flow-of-funds constraint (7) can be rewritten as

$$c_t^i + (1-\theta q_t)i_t = (r_t + \lambda\phi_t q_t)n_t + p_t m_t. \tag{14}$$

To finance investment i_t, the entrepreneur issues equity θi_t at price q_t. Thus, the second term in the left-hand side is the investment cost that has to be financed internally: the down payment for investment. The left-hand side equals the total liquidity needs of the investing entrepreneur.

[9] This is connected to the literature on rational bubbles; e.g., Santos and Woodford (1997).

The right-hand side corresponds to the maximum liquidity supplied from dividends, sales of the resalable fraction of equity after depreciation, and the value of money. Solving this flow-of-funds constraint with respect to the equity of the next period, we obtain

$$c_t^i + q_t^R n_{t+1}^i = r_t n_t + [\phi_t q_t + (1 - \phi_t) q_t^R] \lambda n_t + p_t m_t, \tag{15}$$

where

$$q_t^R \equiv \frac{1 - \theta q_t}{1 - \theta} < 1, \text{ as } q_t > 1. \tag{16}$$

The value of q_t^R is the effective replacement cost of equity to the investing entrepreneur: because he needs a down payment $1 - \theta q_t$ for every unit of investment of which he retains $1 - \theta$ inside equity, he needs $(1 - \theta q_t)/(1 - \theta)$ to acquire one unit of inside equity. The right-hand side of (15) is his net worth: gross dividend plus the value of his depreciated equity λn_t—of which the resalable fraction ϕ_t is valued at market price and the nonresalable fraction $1 - \phi_t$ is valued by the effective replacement cost—plus the value of money.

Given the discounted logarithmic preferences (1), the entrepreneur saves a fraction β of his net worth and consumes a fraction $1 - \beta$:[10]

$$c_t^i = (1 - \beta)\{r_t n_t + [\phi_t q_t + (1 - \phi_t) q_t^R] \lambda n_t + p_t m_t\}. \tag{17}$$

And so, from (14) we obtain an expression for his investment in period t:

$$i_t = \frac{(r_t + \lambda \phi_t q_t) n_t + p_t m_t - c_t^i}{1 - \theta q_t}. \tag{18}$$

Investment is equal to the ratio of liquidity available after consumption to the required down payment per unit of investment.

Next, suppose that the entrepreneur does not have an investment opportunity; denote this by a superscript s to stand for a saver. The flow-of-funds constraint (7) reduces to

$$c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s = r_t n_t + q_t \lambda n_t + p_t m_t. \tag{19}$$

For the moment, let us assume that constraints (5) and (6) do not bind for savers. Then the right-hand side of (19) corresponds to the saver's net worth. It is the same as the right-hand side of (15), except that now his depreciated equity is valued at the market price q_t. From this net worth, he consumes a fraction $1 - \beta$:

$$c_t^s = (1 - \beta)(r_t n_t + q_t \lambda n_t + p_t m_t). \tag{20}$$

[10] Compare (1) to a Cobb-Douglas utility function, where the expenditure share of present consumption out of total wealth is constant and equal to $1/(1 + \beta + \beta^2 + \ldots) = 1 - \beta$.

Note that consumption of a saver is larger than consumption of an investing entrepreneur if both hold the same equity and money at the start of the period. For the saver, his remaining funds are split across a portfolio of m^s_{t+1} and n^s_{t+1}.

To determine the optimal portfolio, consider the choice of sacrificing one unit of consumption c_t to purchase either $1/p_t$ units of money or $1/q_t$ units of equity, which are then used to augment consumption at date $t+1$. The first-order condition is

$$\begin{aligned} u'(c_t) &= E_t\left\{\frac{p_{t+1}}{p_t}\beta[(1-\pi)u'(c^s_{t+1}) + \pi u'(c^i_{t+1})]\right\} \\ &= (1-\pi)E_t\left\{\frac{r_{t+1} + \lambda q_{t+1}}{q_t}\beta u'(c^s_{t+1})\right\} \qquad (21) \\ &\quad + \pi E_t\left\{\frac{r_{t+1} + \lambda\phi_{t+1}q_{t+1} + \lambda(1-\phi_{t+1})q^R_{t+1}}{q_t}\beta u'(c^i_{t+1})\right\}. \end{aligned}$$

The right-hand side of the first line of (21) is the expected gain from holding $1/p_t$ additional units of money at date $t+1$: money always yields p_{t+1}, which will proportionately increase utility by $u'(c^s_{t+1})$ when he does not have a date $t+1$ investment opportunity (probability $1-\pi$) and by $u'(c^i_{t+1})$ when he does (probability π). The second line is the expected gain from holding $1/q_t$ additional units of equity at date $t+1$. Per unit, this additional equity yields r_{t+1} dividend plus its depreciated value. With probability $1-\pi$ that the entrepreneur does not have a date $t+1$ investment opportunity, the depreciated equity is valued at the market price q_{t+1}, and these yields increase utility in proportion to $u'(c^s_{t+1})$. With probability π, the entrepreneur does have an investment opportunity at date $t+1$, in which case he will value depreciated equity by the market price q_{t+1} for the resalable fraction and by the effective replacement cost q^R_{t+1} for the nonresalable fraction, and these yields increase utility in proportion to $u'(c^i_{t+1})$.

Notice that because the effective replacement cost is lower than the market price, the effective return on equity is lower just when the entrepreneur is more in need of funds, namely, when an investment opportunity arises and his marginal utility of consumption is higher ($c^i_{t+1} < c^s_{t+1}$). That is, over and above aggregate risk, equity carries an idiosyncratic risk: its effective return is negatively correlated with the idiosyncratic variations in marginal utility that stem from the stochastic investment opportunities. Money is free from such idiosyncratic risk.

We are now in a position to consider the aggregate economy. The great merit of the expressions for an investing entrepreneur's consumption and investment choices, c^i_t and i_t, and a noninvesting entrepreneur's consumption and savings choices, c^s_t, n^s_{t+1}, and m^s_{t+1}, is that they are all linear in start-of-period equity and money holdings n_t and m_t.[11] Hence, aggregation is easy: we do not need to keep track of the distributions. Notice that because workers do not choose to save, the aggregate holdings of equity and money of the entrepreneurs are equal to the aggregate capital stock K_t and money supply M. At the start of date t, a fraction π of K_t and M is held by entrepreneurs who have an investment opportunity. From (18), total investment I_t in new capital therefore satisfies

$$(1-\theta q_t)I_t = \pi\{\beta[(r_t+\lambda\phi_t q_t)K_t + p_t M] - (1-\beta)(1-\phi_t)\lambda q_t^R K_t\}. \quad (22)$$

Goods market clearing requires that total output (net of labor costs, which equal the consumption of workers), $r_t K_t$, equals investment plus the consumption of entrepreneurs. Using (17) and (20), we therefore have

$$\begin{aligned} r_t K_t = a_t K_t^{\alpha} = I_t + (1-\beta) \\ \times\{[r_t + (1-\pi+\pi\phi_t)\lambda q_t + \pi(1-\phi_t)\lambda q_t^R]K_t + p_t M\}. \end{aligned} \quad (23)$$

We still need to find the aggregate counterpart to portfolio equation (21). During period t, the investing entrepreneurs sell a fraction θ of their investment I_t, together with a fraction ϕ_t of their depreciated equity holdings $\pi\lambda K_t$, to the noninvesting entrepreneurs, so the stock of equity held by the group of noninvesting entrepreneurs at the end of the period is given by $\theta I_t + \phi_t\pi\lambda K_t + (1-\pi)\lambda K_t \equiv N_{t+1}^s$. And by claim 2$c$ we know that this group also holds all the money stock, M. The group's savings portfolio (N_{t+1}^s, M) satisfies (21), which leads to

$$\begin{aligned} (1-\pi)E_t\left[\frac{(r_{t+1}+\lambda q_{t+1})/q_t - p_{t+1}/p_t}{(r_{t+1}+q_{t+1}\lambda)N_{t+1}^s + p_{t+1}M}\right] \\ = \pi E_t\left[\frac{p_{t+1}/p_t - [r_{t+1}+\phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q_{t+1}^R]/q_t}{[r_{t+1}+\phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q_{t+1}^R]N_{t+1}^s + p_{t+1}M}\right]. \end{aligned} \quad (24)$$

Equation (24) lies at the heart of the model. When there is no investment opportunity at date $t+1$, so that the partial liquidity of equity does not matter, the return on equity, $(r_{t+1}+\lambda q_{t+1})/q_t$, exceeds the return on money, p_{t+1}/p_t: the left-hand side of (24) is positive. However, when there is an investment opportunity, the effective return on equity, $[r_{t+1}+\phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q_{t+1}^R]/q_t$, is less than the return on money: the right-hand side of (24) is positive. These return differentials have to be weighted by the respective probabilities and marginal utilities. Note that because of the impact of idiosyncratic risk on the right-hand side, the liquidity premium of

[11] From (19) and (20), the value of savings, $q_t n_{t+1}^s + p_t m_{t+1}^s$, is linear in n_t and m_t, and the reciprocal of the right-hand portfolio eq. (21) is homogeneous in (n_{t+1}^s, m_{t+1}^s)—noting that $u'(c) = 1/c$ and (17) and (20) hold at $t+1$. See the appendix for further details in the context of our full model.

equity over money in the left-hand side may be substantial and may vary through time.

Aside from the liquidity shock ϕ_t and the technology parameter A_t, which follow an exogenous stationary Markov process, the only state variable in this system is K_t, which evolves according to

$$K_{t+1} = \lambda K_t + I_t. \tag{25}$$

Restricting attention to a stationary price process, we can define the competitive equilibrium recursively as a function $(r_t, I_t, p_t, q_t, K_{t+1})$ of the aggregate state (K_t, A_t, ϕ_t) that satisfies (11) and (22)–(25), together with the law of motion of A_t and ϕ_t.

From these equations it can be seen that there are rich interactions between quantities (I_t, K_{t+1}) and asset prices (p_t, q_t). In this sense, our economy is similar to Keynes (1936) and Tobin (1969).[12]

In steady state, when $a_t = a$ (the right-hand side of [12] with $A_t = A$) and $\phi_t = \phi$, capital stock K, investment I, and prices p and q satisfy $I = (1 - \lambda)K$ and

$$r = 1 - \lambda + (1 - \beta)[r + \lambda(1 - \pi + \pi\phi)q + \lambda\pi(1 - \phi)q^R + b], \tag{26}$$

$$(1 - \lambda)(1 - \theta q) = \pi[\beta(r + \lambda\phi q) - \lambda(1 - \beta)(1 - \phi)q^R + \beta b], \tag{27}$$

$$(1 - \pi)\frac{(r/q) + \lambda - 1}{(r + \lambda q)\chi + b} = \pi\frac{1 - [r + \lambda\phi q + \lambda(1 - \phi)q^R/q]}{[r + \lambda\phi q + \lambda(1 - \phi)q^R]\chi + b}, \tag{28}$$

where $r = aK^{\alpha-1}$, $b = pM/K$, and $\chi \equiv \theta(1 - \lambda) + (1 - \pi + \pi\phi)\lambda$ (the steady-state fraction of equity held by noninvesting entrepreneurs at the end of a period).

Equations (26)–(28) can be viewed as a simultaneous system in three unknowns: the price of capital, q; the gross profit rate on capital, r; and the ratio of real money balances to capital stock, b. Equations (26) and (27) can be solved for r and b, each as affine functions of q, which, when substituted into (28), yield a quadratic equation in q with a unique positive solution. Assumption 2 is sufficient to ensure that this solution lies strictly above 1 (but below $1/\theta$). Assumption 2 is the necessary and sufficient condition for money to have value: $p > 0$.

As a prelude to the dynamic analysis that we undertake later on, notice that the technology parameter A affects the steady-state system only through the gross profit term $r = aK^{\alpha-1}$. That is, a rise in the steady-state value of A increases the capital stock K but does not affect q, the price of capital. The price of money p increases to leave $b = pM/K$ unchanged.

[12] Following the tradition of Hicks (1937), we see (23) and (24) as akin to the investment-saving (IS) and liquidity preference–money supply (LM) equations—though we derived our equations from the optimal choices of forward-looking agents who face financing constraints.

It is interesting to compare our economy, in which the liquidity constraints (5) and (6) bind for investing entrepreneurs, to an economy without such constraints. Consider steady states. Without the liquidity constraints, the economy would achieve first best: the price of capital would equal its cost, 1, and the capital stock—say, K^*—would equate the return on capital, $aK^{\alpha-1} + \lambda$, to the agents' common subjective return, $1/\beta$. (See claim 1.) We show below that in our constrained economy, the level of activity—measured by the capital stock K—is strictly below K^*. Because of the borrowing constraint and the partial liquidity of equity, the economy fails to transfer enough resources to the investing entrepreneurs to achieve the first-best level of investment.

On account of the liquidity constraint, there is a wedge between the marginal product of capital and the expected rate(s) of return on equity. It turns out that the expected rate(s) of return on equity and the rate of return on money all lie below the time preference. Intuitively, because the rates of return on assets to savers are below their time preference rate, they do not save enough to escape the liquidity constraint that they will face when they have an opportunity to invest in the future.

Claim 3. Under assumptions 1 and 2, in the neighborhood of the steady-state monetary economy,

a. the stock of capital K_{t+1} is less than in first best:

$$K_{t+1} < K^* \Leftrightarrow E_t\left(a_{t+1}K_{t+1}^{\alpha-1} + \lambda\right) > \frac{1}{\beta};$$

b. the expected rate of return on equity, contingent on not having an investment opportunity in the next period, is lower than the time preference rate:

$$E_t \frac{a_{t+1}K_{t+1}^{\alpha-1} + \lambda q_{t+1}}{q_t} < \frac{1}{\beta};$$

c. the expected rate of return on money is yet lower:

$$E_t \frac{p_{t+1}}{p_t} < E_t \frac{a_{t+1}K_{t+1}^{\alpha-1} + \lambda q_{t+1}}{q_t}; \text{and}$$

d. the expected rate of return on equity, contingent on having an investment opportunity in the next period, is lower still:

$$E_t \frac{a_{t+1}K_{t+1}^{\alpha-1} + \phi_{t+1}\lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R}{q_t} < E_t \frac{p_{t+1}}{p_t}.$$

Claim $3c$ and $3d$ can be understood in terms of (28), given that in steady state $q > 1 > q^R$: the numerators in (28) are both positive. The difference between the expected return on equity and money in claim $3c$, reflecting the liquidity premium, equals the nominal interest rate on equity.[13]

In our monetary economy, there is a spectrum of interest rates. They are, in descending order: the expected marginal product of capital, the time preference rate, the expected rate of return on equity (contingent on the saver not having an investment opportunity in the next period), the expected rate of return on money, and the expected rate of return on equity contingent on the saver having an investment opportunity in the next period. Thus, in our economy, the impact of asset markets on aggregate production cannot be summarized by a single real interest rate. Equally, it would be misleading to use the rates of returns on money or equity to calibrate the time preference rate.

The fact that the expected rates of return on equity and money are both lower than the time preference rate justifies our earlier assertion that workers will not choose to save by holding equity or money.[14] (Of course, if workers could borrow against their future labor income, they would do so. But we have ruled this out.) In steady state, workers enjoy a constant consumption equal to their wages.

The reason why an entrepreneur saves and workers do not is because the entrepreneur is preparing for his next investment opportunity. And the entrepreneur saves using money as well as equity, despite money's particularly low return, because he anticipates that he will be liquidity constrained at the time of investment. Along a typical time path, he experiences episodes without investment, during which he consumes part of his saving. As the return on saving—on both equity and money—is less than his time preference rate, the value of his net worth gradually shrinks, as does his consumption. He expands again only at the time of investment. In the aggregate picture, we do not see all this fine grain. But it is important to realize that, even in steady state, the economy is made up of a myriad of such individual histories.

[13] By the Fisher equation, the nominal interest rate on equity equals the net real return on equity plus the inflation rate. But minus the inflation rate equals the net real return on money. Hence, the nominal interest rate on equity equals the real return on equity minus the return on money, i.e., the liquidity premium. Because our money is broad money (all assets that are as liquid as fiat money), our nominal interest rate is akin to the interest rate in Keynes (1936): the difference in the rate of return on partially liquid assets vs. that on fully liquid assets.

[14] Workers may save if they face their own investment opportunity shocks. Suppose, e.g., that each worker randomly faces a "health shock" that entails immediately spending some fixed amount ζ to maintain her human capital. (Health insurance may cover some of the cost, but the patient has to make a copayment from her own pocket.) Then, if the resalability of equity is low, a worker may choose to save entirely in money enough to cover the amount ζ. The point is that even though the rate of return on equity is higher than that on money, on account of the resalability constraint she would need to save more in equity than in money, which may be less attractive given that the rate of return on equity is lower than her time preference rate. See Kiyotaki and Moore (2005b) for details.

III. Dynamics and Numerical Examples

To examine the dynamics of our economy, we present numerical examples specifying a law of motion for productivity and liquidity (A_t, ϕ_t). Suppose that (A_t, ϕ_t) follow independent AR(1) processes such that $a_t = \gamma[(1-\gamma)/\omega]^{(1-\gamma)/(\gamma+\nu)}(A_t)^{(1+\nu)/(\gamma+\nu)}$ (from [12]) and ϕ_t satisfy

$$a_t - a = \rho_a(a_{t-1} - a) + \varepsilon_{at}, \tag{29}$$

$$\phi_t - \phi = \rho_\phi(\phi_{t-1} - \phi) + \varepsilon_{\phi t}, \tag{30}$$

where ρ_a and $\rho_\phi \in (0, 1)$. For calibration, we set $\rho_a = \rho_\phi = 0.95$. The variables ε_{at} and $\varepsilon_{\phi t}$ are independently and identically distributed innovations of the levels of productivity and liquidity, which have mean zero and are mutually independent. We present our numerical examples to illustrate the qualitative features of our model rather than to be a precise calibration. We consider one period to be one quarter and choose standard parameters that are broadly consistent with the existing literature: $\beta = 0.99$ (subjective discount factor), $\nu = 1$ (inverse of the elasticity of labor supply), $\lambda = 0.97$ (one minus depreciation rate), $\gamma = 0.4$ (share of capital), and $\pi = 0.05$ (arrival rate of investment opportunity). For the parameters of the borrowing and resalability constraints, we choose $\theta = 0.3$ and $\phi = 0.2$, so that the spread of the rates of return between equity and money equals 3.1 percent annually and the ratio of real balances to annual output equals one-third in the deterministic steady state.[15] Table 1 shows values in the deterministic steady state.

Figure 1 shows the impulse response function to a 1 percent increase in A_t, which increases a_t by $(1+\nu)/(\gamma+\nu) = 1.43$ percent.

Because capital stock is predetermined and the labor market clears, output increases by 1.43 percent (the same proportion as a_t). Then, from the goods market equilibrium condition (23) in conjunction with (22), we see that asset prices (p_t, q_t) have to increase with productivity to raise consumption and investment in line with output. Although investment is more sensitive to the asset prices and thus increases proportionately more than consumption, the aggregate consumption of both entrepreneurs and workers increases substantially (especially since workers' consumption is equal to their wage income). This is different from a first-best allocation in which consumption would be much smoother than investment because, without the binding liquidity constraints, consumption would depend on permanent rather than current income. Also, in a first-best

[15] Note that in steady state, the rate of return on equity (contingent on the saver not having an investment opportunity in the next period) is between 0 percent (the rate of return on money) and 4 percent (the time preference rate). We choose the share of capital and depreciation rate of capital to be a little higher than usual to emphasize the financing need of capital investment. See Ajello (2016) and Del Negro et al. (2017) for alternative calibration strategies. The former relies on firm-level panel data, and the latter relies on Krishnamurthy and Vissing-Jorgensen (2012) and financial market data.

TABLE 1
Steady State

C/Y	I/Y	$K/4Y$	$pM/4Y$	q	$(r/q) + \lambda - 1$
72%	28%	2.34	33%	1.14	3.1% annual

equilibrium, Tobin's q would always equal unity and the value of money would always equal zero, whereas in our monetary equilibrium with binding liquidity constraints, quantities and asset prices move together.

Now let us consider liquidity shocks. Figure 2 shows the impulse response of quantities and asset prices when the resalability of the equity drops from 0.2 to 0.06, a fall of 70 percent.

When the resalability of equity falls and only slowly recovers, the investing entrepreneurs are less able to finance down payment from selling their equity holdings, and so investment decreases substantially. Capital stock and output gradually decrease with persistently lower investment. Also, savers now find money more attractive than equity (holding their rates of return unchanged), given that they can resell a smaller fraction of their equity holding when future investment opportunities arise (ceteris paribus, the numerator in the right-hand side of [24] rises as ϕ_{t+1} falls). Thus, the value of money increases compared to the equity price to restore asset market equilibrium. This can be thought of as a flight to liquidity—a flight from equity to money.

Notwithstanding this flight from equity, the real equity price tends to rise with the fall in liquidity, even though the nominal equity price always falls. One way to understand why is to think of the gap between Tobin's q and unity as a measure of the tightness of the liquidity constraint, which increases when the resalability of equity falls. Another way is to observe that, because output is not initially affected (given full employment), consumption must increase to maintain equilibrium in the goods market, and consumption rises through the wealth effect of a rise in asset prices. This negative comovement between investment, consumption, and equity price is a shortcoming of our basic model—a shortcoming shared by many macroeconomic models with flexible prices.[16] We address this in the next section.

Note that, in contrast to our monetary equilibrium, a first-best allocation would not react to the liquidity shock, as the liquidity constraint would not be binding.

IV. Full Model with Storage and Government

We now present the full model. The negative comovement between investment, consumption, and equity price in the basic model can be remedied by augmenting the model to include an alternative liquid means of saving:

[16] Shi (2015) points out that in our basic model it is difficult for a liquidity shock to generate a positive comovement between aggregate investment and the price of equity.

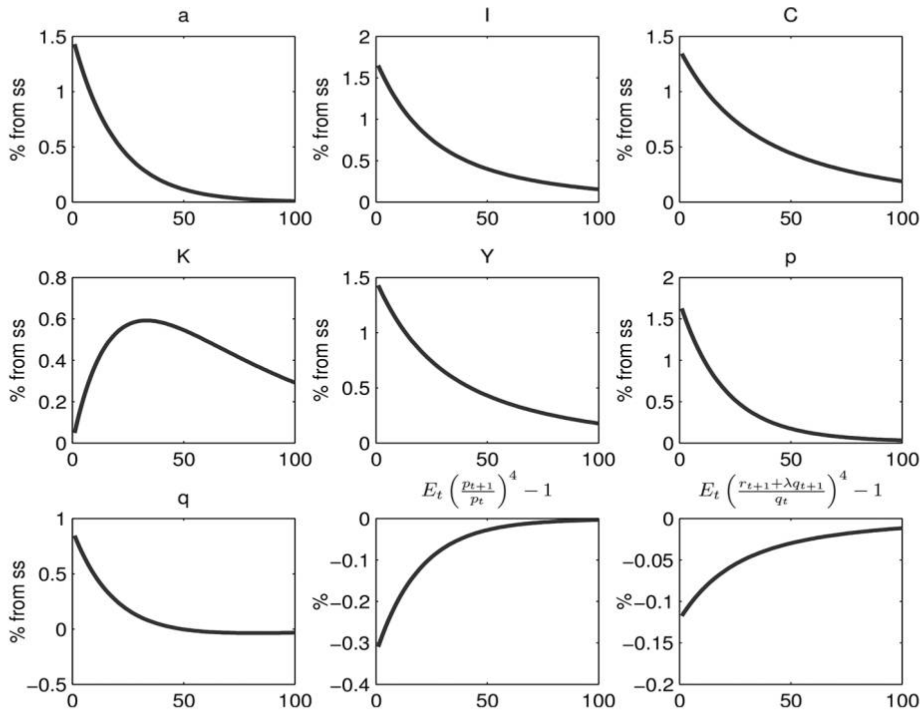

FIG. 1.—Impulse responses of basic economy to productivity shock; ss = steady state

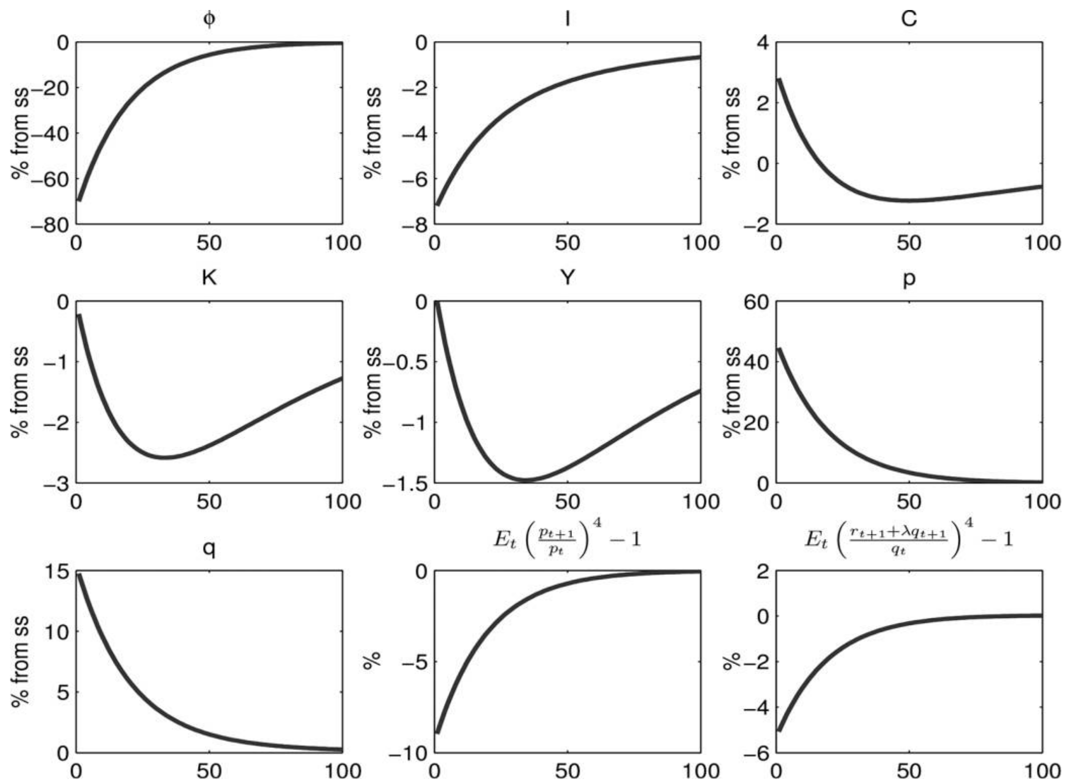

FIG. 2.—Impulse responses of basic economy to liquidity shock; ss = steady state

storage. Storage represents all the various means of short-term saving besides money. For example, storage might be the holding of foreign assets (though home citizens cannot borrow from foreigners or sell them equity). Formally, storage is an alternative liquid investment technology available to everyone, unlike the capital investment technology, which is available to only a select subset of entrepreneurs each period. We find that, having augmented our model to include storage, in response to a fall in the resalability of equity, resources flow out of capital investment into storage rather than into consumption. Loosely put, when financial markets are disrupted, capital investment by selected entrepreneurs (to whom the economy wants to funnel resources via financial markets) shrinks, whereas common storage investment expands. Interpreting storage as the holding of foreign assets, we might say that there is a "capital flight."

Specifically, suppose that an agent can store $\sigma_t z_{t+1}$ units of goods at date t to obtain z_{t+1} units of goods at date $t+1$, where z_{t+1} must be nonnegative. Although the storage technology has constant returns to scale at the individual level, it has decreasing returns to scale in the aggregate:[17] σ_t is an increasing function of the aggregate quantity of storage Z_{t+1},

$$\sigma_t = \sigma(Z_{t+1}) = \left(\frac{Z_{t+1}}{\zeta_0}\right)^{\zeta}, \text{ where } \zeta_0, \zeta > 0.$$

The second change to our basic model is to introduce the government. Our goal here is merely to explore the effects on the economy of an exogenous government policy rule rather than to explain government behavior. At the start of date t, suppose that the government holds N_t^g equity. Unlike entrepreneurs, the government cannot produce new capital. However, it can engage in open-market operations to buy (resell) equity by issuing (taking in) money—it has sole access to a costless money-printing technology. When buying equity, the government does not violate the private sector's resalability constraints.[18] We assume that N_t^g is not so large that the private economy switches regimes—that is, we are still in an equilibrium where the liquidity constraints bind for investing entrepreneurs and money is valuable.

If M_t is the stock of money privately held by entrepreneurs at the start of date t, then the government's flow-of-funds constraint is given by

$$q_t(N_{t+1}^g - \lambda N_t^g) = r_t N_t^g + p_t(M_{t+1} - M_t) = r_t N_t^g + (\mu_t - 1)B_t, \quad (31)$$

where $B_t \equiv p_t M_t$ are real balances and $\mu_t \equiv M_{t+1}/M_t$ is the money supply growth rate. That is, equity purchases are met by the dividends from

[17] Instead of assuming decreasing returns in the aggregate, we could introduce another factor of production (such as labor) that is needed for storage besides the goods input. However, it simplifies the exposition not to do so.

[18] When reselling equity, the government is also subject to the resalability constraint: $N_{t+1}^g \geq (1 - \phi_t)\lambda N_t^g$.

its equity holdings plus seigniorage revenues. Since the government is a large agent, at least relative to each of the private agents, open-market operations will affect the prices p_t and q_t.

We will suppose that the government follows a rule for its open-market operations:

$$\frac{N^g_{t+1}}{K} = \psi_a \frac{a_t - a}{a} + \psi_\phi \frac{\phi_t - \phi}{\phi}, \tag{32}$$

where ψ_a and ψ_ϕ are policy parameters and K is the capital stock in steady state. This equation is the government's feedback rule: it chooses the size of its open-market operation (the ratio of its equity holding to the steady-state capital stock) as a function of the proportional deviations of productivity and liquidity from their steady-state levels.[19]

The earlier analysis carries through, with obvious modifications. See the appendix for details. The total supply of equity (which by construction is equal to the aggregate capital stock) equals the sum of the government's holding and the aggregate holding by entrepreneurs (denoted by N_{t+1}):

$$K_{t+1} = N^g_{t+1} + N_{t+1}. \tag{33}$$

Workers consume all their disposable income, and, given the form of their preferences in (8), government policy does not affect their labor supply. Equations (22)–(24) are modified to

$$(1-\theta q_t)I_t = \pi\{\beta[(r_t + \lambda\phi_t q_t)N_t + B_t + Z_t] - (1-\beta)(1-\phi_t)\lambda q^R_t N_t\}, \tag{34}$$

$$\begin{aligned} r_t K_t = a_t K_t^\alpha = I_t + \sigma(Z_{t+1})Z_{t+1} - Z_t + (1-\beta) \\ \times \{[r_t + (1-\pi+\pi\phi_t)\lambda q_t + \pi(1-\phi_t)\lambda q^R_t]N_t + B_t + Z_t\}, \end{aligned} \tag{35}$$

$$\begin{aligned} &(1-\pi)E_t\left[\frac{(r_{t+1}+\lambda q_{t+1})/q_t - B_{t+1}/(\mu_t B_t)}{(r_{t+1}+\lambda q_{t+1})N^s_{t+1} + B_{t+1} + Z_{t+1}}\right] \\ &= \pi E_t\left[\frac{B_{t+1}/(\mu_t B_t) - [r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q^R_{t+1}]/q_t}{[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q^R_{t+1}]N^s_{t+1} + B_{t+1} + Z_{t+1}}\right], \end{aligned} \tag{36}$$

$$\begin{aligned} &(1-\pi)E_t\left[\frac{(r_{t+1}+\lambda q_{t+1})/q_t - (1/\sigma(Z_{t+1}))}{(r_{t+1}+\lambda q_{t+1})N^s_{t+1} + B_{t+1} + Z_{t+1}}\right] \\ &= \pi E_t\left[\frac{(1/\sigma(Z_{t+1})) - [r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q^R_{t+1}]/q_t}{[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q^R_{t+1}]N^s_{t+1} + B_{t+1} + Z_{t+1}}\right], \end{aligned} \tag{37}$$

[19] For simplicity, we turn a blind eye to the fact that N^g_{t+1} may be negative. This could be avoided by assuming that the government has a sufficiently large holding of private equity in steady state (or by assuming that the government's feedback rule is subject to the nonnegativity constraint on N^g_{t+1}). The analysis and results that we report below would not be substantially different.

where $N^s_{t+1} = \theta I_t + \phi_t \pi \lambda N_t + (1 - \pi)\lambda N_t + \lambda N^g_t - N^g_{t+1}$. In investment equation (34), entrepreneurs use their money and storage and the resalable portion of their equity—net of their consumption—to finance the down payment. In goods market equilibrium (35), output (net of the workers' consumption) equals the sum of capital investment, storage investment, and the entrepreneurs' consumption. Portfolio equation (36) gives the trade-off between holding equity and money, and new portfolio equation (37) gives the trade-off between holding equity and storage.

Restricting attention to a stationary price process, we can define the competitive equilibrium recursively as a function $(r_t, I_t, B_t, q_t, Z_{t+1}, K_{t+1}, N_{t+1}, N^g_{t+1}, \mu_t)$ of the aggregate state $(K_t, Z_t, N^g_t, a_t, \phi_t)$ that satisfies (11), (25), and (31)–(37) together with the exogenous law of motion of (a_t, ϕ_t).[20]

How does the presence of storage, as an alternative means of liquid saving, alter the impulse responses? Figure 3 compares the impulse responses to a liquidity shock in the model with storage (solid lines) and the model without storage (dotted lines, taken from fig. 2). We choose a storage technology that has close to constant returns to scale ($\zeta = 0.0001$) and is such that the steady-state level of storage ($\zeta_0 = 0.5$) is modest (5 percent) compared to the steady-state capital stock ($K = 10.0$). A storage technology that has close to constant returns leads to volatile storage investment: this helps consumption to move with investment. However, we would not want to go all the way to constant returns because then the steady-state ratio of real balances to storage would be indeterminate. There is no change in the deterministic steady state (except that the liquidity is provided by both money and storage), because money and storage are perfect substitutes in steady state.

In response to a fall in the resalability of equity, storage increases sharply and investment falls more significantly than the economy without storage, leading to a more significant fall in output. Importantly, consumption can now also fall along with investment, as output is soaked up by the sharp rise in storage.

Also, money and storage are close substitutes, with expected rates of return close to unity, whereas the liquidity premium of equity has to be higher to compensate for its lower resalability. As a consequence, the flight to liquidity induces the equity price to fall somewhat, at least initially.

Taking these findings together, we see that the presence of an alternative liquid means of saving has overcome the shortcomings of our basic model. Quantities (investment and consumption) and stock price move together, as storage serves as a buffer stock to absorb output and stabilize the value of money.

[20] If there were a one-time lump sum transfer of money to the entrepreneurs (a helicopter drop), then aggregate quantities would not change in our economy given that prices and wages are flexible. The consumption and investment of individual entrepreneurs would be affected, however, because there would be some redistribution.

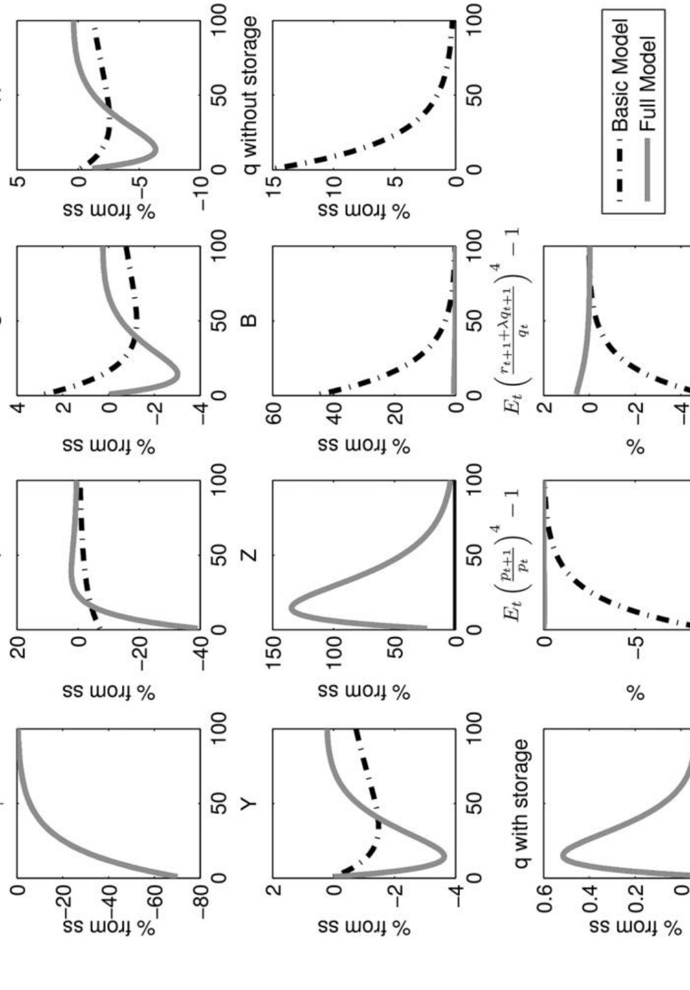

FIG. 3.—Impulse responses of economy with storage to liquidity shock; ss = steady state

How might the government, through its central bank, conduct open-market operations in response to the liquidity shock? A first-best allocation would not be affected by a liquidity shock. With this benchmark in mind, in our monetary economy the central bank can use open-market operations to offset the effects of the liquidity shock, by setting the feedback rule coefficient ψ_ϕ to be negative in (32). That is, the central bank can counteract the negative shock by purchasing equity with money to—at least partially—restore the liquidity of investing entrepreneurs. Figure 4 compares the impulse responses of the economy with this policy rule ($\psi_\phi = -0.1$; solid lines) and without ($\psi_\phi = 0$; dotted lines, taken from the solid lines in fig. 3).

The central bank's purchases of equity with money cause real balances to increase sharply, notwithstanding the relatively stable price of money. Storage increases less than in the economy without the policy intervention. Investment falls initially by almost 40 percent—almost as much as in the case of no policy, because at the time of the shock the investing entrepreneurs' portfolios are predetermined. However, in the subsequent periods, investing entrepreneurs (most of whom were savers previously) have a larger proportion of liquid assets thanks to the policy intervention, and investment recovers to a level of 20 percent below the steady state. Thus, capital stock and output do not fall as much as in the economy without intervention.

After the initial purchase of equity, government runs a surplus because equity yields a higher return. It uses this surplus to reduce the money supply by setting $\mu_t < 1$. Because this deflationary policy rewards money holders, the flight to liquidity is more pronounced: the equity price falls as a result.

In contrast, how might the central bank use open-market operations in response to a productivity shock? Once more taking a first-best allocation as a benchmark, the problem of our laissez-faire monetary economy appears to be that investment does not react enough to productivity shocks and consumption is not smooth enough. Here the central bank can provide liquidity procyclically to accommodate productivity shocks, by setting the feedback coefficient of ψ_a to be positive in (32). Figure 5 compares the impulse response functions of the laissez-faire monetary economy with an accommodating monetary policy ($\psi_a = 0.2$). As productivity rises by 1.43 percent, the central bank buys equity with money to provide an additional 3 percent liquidity. Entrepreneurs hold more money and less illiquid equity and thus invest more. Investment increases by 1.1 percent in the periods immediately following the shock, rather than increasing by 0.6 percent as in the economy without the intervention. But whereas investment and hence capital stock and output all increase more because of the policy, storage increases less.

The efficacy of these open-market operations relies on the purchase of an asset—here, equity—which is only partially resalable and hence earns a nontrivial liquidity premium. If the liquidity premium of short-term

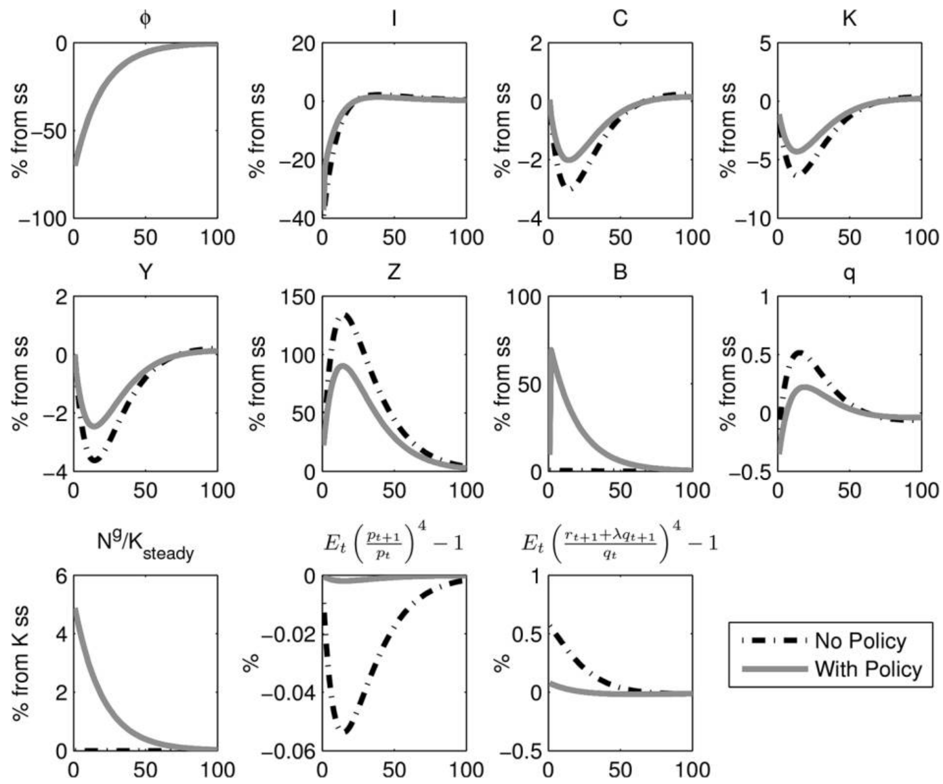

FIG. 4.—Impulse responses of full model to liquidity shock; ss = steady state

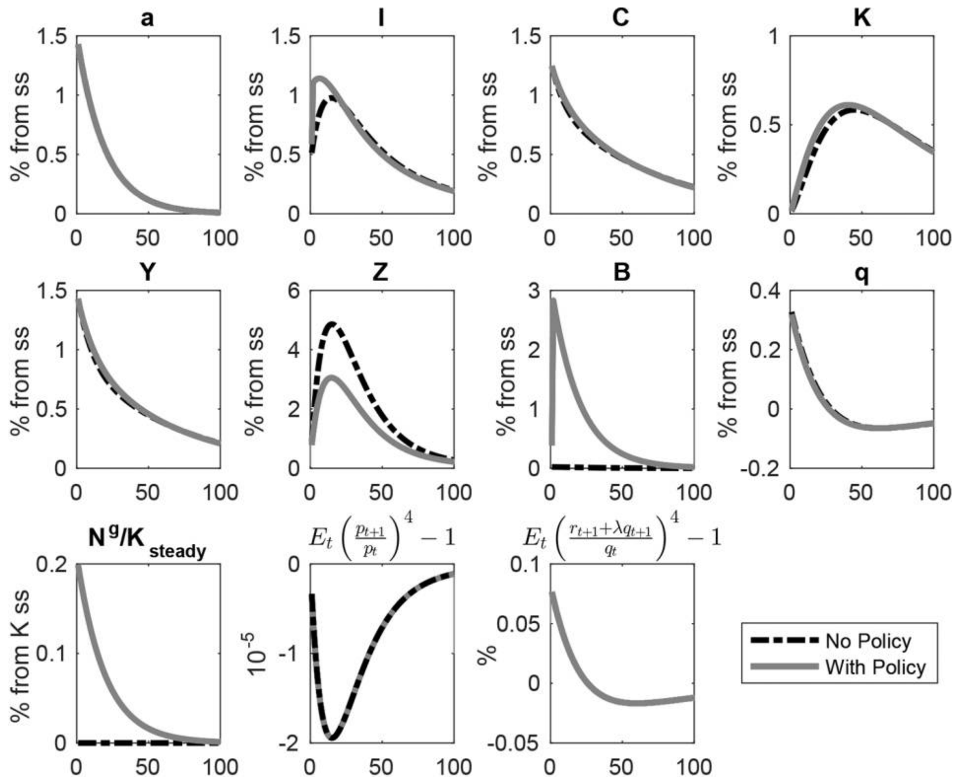

FIG. 5.—Impulse responses of full model to productivity shock; ss = steady state

government bonds is very low (as in Japan since the late 1990s and many other advanced economies since late 2008), then traditional open-market operations will only serve to change the composition of broad money and will have limited effects. The unorthodox policy of the Federal Reserve Bank during the recent financial crisis, such as the Term Securities Lending Facility, was an attempt to increase liquidity by supplying treasury bills against only partially resalable securities, such as mortgage-backed securities.

V. Related Literature and Final Remarks

We hope to have succeeded in constructing a model of money and liquidity in the tradition of Keynes (1936) and Tobin (1969). The two key equations of our model, (23) and (24)—which are generalized in (35)–(37)—have the flavor of the Keynesian system. We follow Tobin in placing emphasis on the spectrum of liquidity across different classes of asset. Also, Tobin's q theory finds echo in our model through the central role played by the equity price q: driving the feedback from asset markets to the rest of the economy. Our policy analysis—open-market operations change the liquidity mix of the private sector's asset holdings—parallels that in Metzler (1951). Perhaps, with its focus on liquidity, our framework harks back to an earlier tradition of interpreting Keynes and has less in common with the New Keynesian literature, with its emphasis on sticky prices, that has been dominant in the past few decades.

This paper is part of the recent literature on macroeconomics with financial frictions that includes Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Holmstrom and Tirole (1998); Bernanke, Gertler, and Gilchrist (1999); and more recently Brunnermeier and Sannikov (2014).[21] Naturally, the common thread of this literature has been some form of borrowing constraint, akin to our θ constraint. Our innovation here is to combine it with the ϕ constraint, the resalability constraint. We have shown that the presence of these two constraints opens up the possibility for fiat money to circulate, to lubricate the transfer of goods from savers to investors. There is a wedge between money and other assets that arises out of the assumed difference in their resalability.

[21] Of these, Holmstrom and Tirole (1998) is perhaps the closest to the present paper. There, liquidity refers to the instrument used for transferring wealth across periods, in particular by firms arranging in advance to meet any future needs for additional finance (when they may hit a borrowing constraint). This liquidity is supplied up front by the firms themselves, possibly through intermediaries. That is, firms hold claims against each other. Firms can issue fully state-contingent claims so that they can mutually insure against idiosyncratic shocks to their future financing needs. Holmstrom and Tirole ask whether the private market supplies enough liquidity in aggregate and what role there may be for public intervention. Because full state contingency is allowed in their model, there is no need for private paper to circulate. Hence, even if there were some impediment to the resale of private paper (along the lines of our ϕ being less than one), it would not matter. See also Holmstrom and Tirole (2001, 2011). Surveys can be found in Gertler and Kiyotaki (2011); Brunnermeier, Eisenbach, and Sannikov (2013); and Gertler, Kiyotaki, and Prestipino (2016).

Wedges between assets can be generated in other ways. In limited-participation models, agents may have different access to asset markets.[22] Models with spatially separated markets—island models—assume that agents cannot visit all markets within the period, which limits trade across assets. Some models combine geographical separation with asynchronization, where agents have access to asset markets at different times.[23] If the assumption of competitive markets is dropped, as in matching models, assets can exhibit different degrees of resalability.[24] And there is a long tradition in the banking and finance literature that, implicitly or explicitly, has to do with the limited resalability of securities, dating back at least to Diamond and Dybvig (1983).[25]

Our model abstracts from private banks as separate agents who supply liquid paper. Instead, all our private assets are partially liquid to the same degree, and all our entrepreneurs serve as financial intermediaries by simultaneously providing funds for others' capital investment and raising funds for their own. That is to say, we have amalgamated the classical role of a banker (investing in financial assets) with the classical role of an entrepreneur (investing in productive assets). In the context of this abstraction, a fall in the resalability of private assets corresponds to a disruption of the financial system.

We should end by stressing that if, in particular, our model is to be used for proper policy analysis, then considerably more research is needed. While it might be argued that our θ-ϕ framework has the virtue of simplicity, the borrowing and resalability constraints as they stand are too stylized in nature, too reduced form. The borrowing constraint can be rationalized by invoking a moral-hazard argument—namely, to produce future output from new capital requires the specific skill of the investing entrepreneur, and he can renege on his promises. But the resalability constraint requires more modeling, not least because we need to understand where

There is a large empirical literature on financial frictions. One strand constructs simple indicators of financial frictions at the firm level using balance sheet or other types of firm-level data. The challenge here is to separate firms that cannot invest because of financial frictions from firms not wanting to invest because prospects do not look good; see, e.g., Farre-Mensa and Ljungqvist (2016). Another strand of the literature uses pseudonatural experiments: identify two groups of firms that are similar in their prospects and demand shocks but different in their financial frictions. This method has been applied to the recent financial crisis to identify the real effects (employment and/or investment) of credit shocks faced by firms; see, e.g., Almeida et al. (2012) and Chodorow-Reich (2014).

22 See, e.g., Allen and Gale (1994, 2007).

23 See, e.g., Townsend (1987), Townsend and Wallace (1987), Freeman (1996a, 1996b), and Green (1999).

24 Matching models that can be used for policy analysis include Shi (1997), Lagos and Wright (2005), Nosal and Rocheteau ([2011] 2017), and Lagos, Rocheteau, and Wright (2017).

25 For attempts to incorporate banking into standard business cycle models, see, e.g., Williamson (1987), Gertler and Karadi (2011), Gertler and Kiyotaki (2011), and Gertler et al. (2016).

the liquidity shocks, the shocks to ϕ, come from.[26] Can policies be devised that directly dampen these shocks (or even raise the average value of ϕ), rather than merely dealing with their effects?

To analyze the effects of open-market operations over the business cycle, we assumed that the government can commit to a policy. But can it? This question calls for further modeling, too, because if the government could commit to, say, a deflationary monetary policy that followed the Friedman rule (set the real return on money to equal agents' subjective discount rate), then it would in effect be using its taxation powers to substitute perfect public commitment for imperfect private commitment. In the long run, can the government be trusted more than the private sector? And to what extent do future tax liabilities crowd out a private agent's ability to issue credible promises to others?[27] These thorny issues warrant much careful thinking.

Appendix

A. *Proof of Claim 1*

We construct a steady-state equilibrium in which inequalities (5), (6), and (10) do not bind. From (7), we need $q = 1$. (If q were strictly larger than one, investing entrepreneurs would invest arbitrarily large amounts and [5] would bind. If q were strictly smaller than one, investing entrepreneurs would not invest at all and there would be no capital stock.) Given $q = 1$, it is immaterial to an entrepreneur whether he has an investment opportunity—so the missing insurance market does not matter—and the choice of consumption versus investment/saving of all the agents implies

$$1 = \beta(r + \lambda). \tag{A1}$$

Also, $p = 0$. (If p were strictly positive, then with [6] not binding we would have $1 = \beta(p_{t+1}/p_t)$, which is not consistent with a constant money supply.)

Equilibrium in the capital, labor, and goods markets implies

$$r = \gamma A\left(\frac{L}{K}\right)^{1-\gamma},$$

$$w = (1-\gamma)A\left(\frac{K}{L}\right)^{\gamma} = \omega L^{\nu},$$

$$AK^{\gamma}L^{1-\gamma} = C + C^{w} + (1-\lambda)K,$$

[26] Kiyotaki and Moore (2003) show how the resalability constraint can arise endogenously as a result of adverse selection and how securitization may mitigate the adverse selection. Other macroeconomic models of adverse selection in asset markets include Eisfeldt (2004), Moore (2010) Kurlat (2013), Guerrieri and Shimer (2014), and Chang (2018).

[27] A related question would be, If the government has a superior power to force private agents to pay, why not provide them with finance directly?

where C and C^w are aggregate consumption of entrepreneurs and workers. These are the conditions for a first-best allocation.

We need to verify that under condition 1, inequalities (5), (6), and (10) do not bind. With $q = 1$ and $p = 0$, inequality (5) for an investing entrepreneur becomes

$$rn_t - c_t = n_{t+1} - \lambda n_t \geq (1 - \theta)i_t - \phi\lambda n_t .$$

Aggregating this inequality across all the investing entrepreneurs and recalling that the arrival of an investment opportunity is independently and identically distributed across entrepreneurs and through time, we have

$$\pi(rN - C) \geq (1 - \theta)I - \phi\lambda\pi N,$$

where N is aggregate equity of entrepreneurs. Using the budget constraint of entrepreneurs $C = (r - 1 + \lambda)N$ and equity market equilibrium $K = N^w + N$ (where N^w is aggregate equity of workers), we can write this inequality as

$$\pi(1 - \lambda)N \geq (1 - \theta)(1 - \lambda)(N + N^w) - \phi\lambda\pi N. \tag{A2}$$

Given that condition 1 is a strict inequality, (A2) is satisfied as long as N^w is not too large. That is to say, there is a continuum of equilibria indexed by N^w as long as (A2) is satisfied. QED

B. Derivation of Consumption and Portfolio Equations

Let $V_t(m_t, n_t, z_t)$ be the value function of the entrepreneur who holds money, equity, and storage (m_t, n_t, z_t) at the beginning of the period t before meeting an opportunity to invest with probability π. The Bellman equation can be written as

$$\begin{aligned} V_t(m_t, n_t, z_t) = \pi \times & \max_{\substack{c_t^i,\, i_t,\, m_{t+1}^i,\, n_{t+1}^i,\, z_{t+1}^i \\ \text{s.t.}(5,6,7)}} \{\ln c_t^i + \beta E_t[V_{t+1}(m_{t+1}^i, n_{t+1}^i, z_{t+1}^i)]\} \\ & + (1 - \pi) \times \max_{\substack{c_t^s,\, m_{t+1}^s,\, n_{t+1}^s,\, z_{t+1}^s \\ \text{s.t.}(5,6,7),\, i_t = 0}} \{\ln c_t^s + \beta E_t[V_{t+1}(m_{t+1}^s, n_{t+1}^s, z_{t+1}^s)]\}. \end{aligned}$$

Solving the flow-of-funds conditions for c_t^s and c_t^i, we find that the Bellman equation is

$$\begin{aligned} V_t(m_t, n_t, z_t) = \pi \times \max_{m_{t+1}^i, n_{t+1}^i, z_{t+1}^i} & \{\ln([r_t + \phi_t\lambda q_t + (1 - \phi_t)\lambda q_t^R]n_t + p_t m_t + z_t - q_t^R n_{t+1}^i \\ & - p_t m_{t+1}^i - \sigma_t z_{t+1}^i) + \beta E_t[V_{t+1}(m_{t+1}^i, n_{t+1}^i, z_{t+1}^i)]\} \\ + (1 - \pi) \times \max_{m_{t+1}^s, n_{t+1}^s, z_{t+1}^s} & \{\ln((r_t + \lambda q_t)n_t + p_t m_t + z_t - q_t n_{t+1}^s - p_t m_{t+1}^s \\ & - \sigma_t z_{t+1}^s) + \beta E_t[V_{t+1}(m_{t+1}^s, n_{t+1}^s, z_{t+1}^s)]\}. \end{aligned}$$

Let R_{mt+1} and R_{zt+1} be the rates of return on money and storage from date t to date $t + 1$, and let $R_{t+1}^{hh'}$ be the implied rate of returns on equity for the entrepreneur when his type is h ($h = i$ for investing and $h = s$ for saving or noninvesting) at date t and h' at date $t + 1$; that is,

$$R_{mt+1} = \frac{p_{t+1}}{p_t}, \qquad R_{zt+1} = \frac{1}{\sigma_t},$$
$$R^{ss}_{t+1} = \frac{r_{t+1} + \lambda q_{t+1}}{q_t}, R^{si}_{t+1} = \frac{r_{t+1} + \lambda\phi_{t+1}q_{t+1} + \lambda(1-\phi_{t+1})q^R_{t+1}}{q_t},$$
$$R^{is}_{t+1} = \frac{r_{t+1} + \lambda q_{t+1}}{q^R_t}, R^{ii}_{t+1} = \frac{r_{t+1} + \lambda\phi_{t+1}q_{t+1} + \lambda(1-\phi_{t+1})q^R_{t+1}}{q^R_t}.$$

Then the first-order conditions that we need to confirm are

$$1 = E_t\left[\pi\frac{\beta c^i_t}{c^{ii}_{t+1}}R^{ii}_{t+1} + (1-\pi)\frac{\beta c^i_t}{c^{is}_{t+1}}R^{is}_{t+1}\right], \tag{A3}$$

$$1 > E_t\left[\pi\frac{\beta c^i_t}{c^{ii}_{t+1}}R_{mt+1} + (1-\pi)\frac{\beta c^i_t}{c^{is}_{t+1}}R_{mt+1}\right], \tag{A4a}$$

$$1 > E_t\left[\pi\frac{\beta c^i_t}{c^{ii}_{t+1}}R_{zt+1} + (1-\pi)\frac{\beta c^i_t}{c^{is}_{t+1}}R_{zt+1}\right], \tag{A4b}$$

$$1 = E_t\left[\pi\frac{\beta c^s_t}{c^{si}_{t+1}}R^{si}_{t+1} + (1-\pi)\frac{\beta c^s_t}{c^{ss}_{t+1}}R^{ss}_{t+1}\right], \tag{A5}$$

$$1 = E_t\left[\pi\frac{\beta c^s_t}{c^{si}_{t+1}}R_{mt+1} + (1-\pi)\frac{\beta c^s_t}{c^{ss}_{t+1}}R_{mt+1}\right], \tag{A6}$$

$$1 = E_t\left[\pi\frac{\beta c^s_t}{c^{si}_{t+1}}R_{zt+1} + (1-\pi)\frac{\beta c^s_t}{c^{ss}_{t+1}}R_{zt+1}\right], \tag{A7}$$

where c^h_t is date t consumption of the entrepreneur of type h and $c^{hh'}_{t+1}$ is date $t+1$ consumption of the entrepreneur when his type is h at date t and h' at date $t+1$.

We guess that

$$\begin{aligned}
c^i_t &= (1-\beta)\{[r_t + \phi_t\lambda q_t + (1-\phi_t)\lambda q^R_t]n_t + p_t m_t + z_t\},\\
c^s_t &= (1-\beta)\{(r_t + \lambda q_t)n_t + p_t m_t + z_t\},\\
n^i_{t+1} &= \beta\{[r_t + \phi_t\lambda q_t + (1-\phi_t)\lambda q^R_t]n_t + p_t m_t + z_t\}/q^R_t,\\
m^i_{t+1} &= 0, z^i_{t+1} = 0,\\
n^s_{t+1} &= \beta f_{nt}[(r_t + \lambda q_t)n_t + p_t m_t + z_t]/q_t,\\
m^s_{t+1} &= \beta f_{mt}[(r_t + \lambda q_t)n_t + p_t m_t + z_t]/p_t,\\
z^s_{t+1} &= \beta f_{zt}[(r_t + \lambda q_t)n_t + p_t m_t + z_t]/\sigma_t,\\
c^{ii}_{t+1} &= (1-\beta)[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q^R_t]n^i_{t+1},\\
c^{is}_{t+1} &= (1-\beta)(r_{t+1} + \lambda q_{t+1})n^i_{t+1},\\
c^{si}_{t+1} &= (1-\beta)\{[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1-\phi_{t+1})\lambda q^R_t]n^s_{t+1} + p_{t+1}m^s_{t+1} + z^s_{t+1}\},\\
c^{ss}_{t+1} &= (1-\beta)\{(r_{t+1} + \lambda q_{t+1})n^s_{t+1} + p_{t+1}m^s_{t+1} + z^s_{t+1}\},
\end{aligned}$$

where $f_{mt} \equiv p_t m^s_{t+1}/(p_t m^s_{t+1} + q_t n^s_{t+1} + \sigma_t z^s_{t+1})$, $f_{nt} \equiv q_t n^s_{t+1}/(p_t m^s_{t+1} + q_t n^s_{t+1} + \sigma_t z^s_{t+1})$, and $f_{zt} \equiv \sigma_t z^s_{t+1}/(p_t m^s_{t+1} + q_t n^s_{t+1} + \sigma_t z^s_{t+1})$ are the date t shares of money, equity, and storage in the portfolio of the noninvesting entrepreneur. Under this guess, we learn that

$$\frac{c^{ii}_{t+1}}{c^i_t} = \beta R^{ii}_{t+1} \text{ and } \frac{c^{is}_{t+1}}{c^i_t} = \beta R^{is}_{t+1},$$

and thus (A3) is satisfied. We can write (A4a) and (A4b) as

$$1 > E_t\left[\pi \frac{R_{mt+1}}{R^{ii}_{t+1}} + (1-\pi)\frac{R_{mt+1}}{R^{is}_{t+1}}\right], \tag{A8a}$$

$$1 > E_t\left[\pi \frac{R_{zt+1}}{R^{ii}_{t+1}} + (1-\pi)\frac{R_{zt+1}}{R^{is}_{t+1}}\right]. \tag{A8b}$$

We also learn that

$$\frac{c^{si}_{t+1}}{c^s_t} = \beta(f_{mt}R_{mt+1} + f_{nt}R^{si}_{t+1} + f_{zt}R_{zt+1}),$$

$$\frac{c^{ss}_{t+1}}{c^s_t} = \beta(f_{mt}R_{mt+1} + f_{nt}R^{ss}_{t+1} + f_{zt}R_{zt+1}).$$

Thus, (A5)–(A7) become

$$1 = E_t\left[\frac{\pi R^{si}_{t+1}}{f_{mt}R_{mt+1} + f_{nt}R^{si}_{t+1} + f_{zt}R_{zt+1}} + \frac{(1-\pi)R^{ss}_{t+1}}{f_{mt}R_{mt+1} + f_{nt}R^{ss}_{t+1} + f_{zt}R_{zt+1}}\right], \tag{A9}$$

$$1 = E_t\left[\frac{\pi R_{mt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{si}_{t+1} + f_{zt}R_{zt+1}} + \frac{(1-\pi)R_{mt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{ss}_{t+1} + f_{zt}R_{zt+1}}\right], \tag{A10}$$

$$1 = E_t\left[\frac{\pi R_{zt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{si}_{t+1} + f_{zt}R_{zt+1}} + \frac{(1-\pi)R_{zt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{ss}_{t+1} + f_{zt}R_{zt+1}}\right]. \tag{A11}$$

We are seeking values of (f_{mt}, f_{nt}, f_{zt}) that solve these equations, but we have only two degrees of freedom because $f_{mt} + f_{nt} + f_{zt} = 1$. However, f_{nt} times the right-hand side of (A9) plus f_{mt} times the right-hand side of (A10) plus f_{zt} times the right-hand side of (A11) equals 1, so one of (A9)–(A11) is not independent. Subtracting (A10) from (A9) and rearranging, we get

$$\pi E_t\left[\frac{R_{mt+1} - R^{si}_{nt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{si}_{t+1} + f_{zt}R_{zt+1}}\right] = (1-\pi)E_t\left[\frac{R^{ss}_{t+1} - R_{mt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{ss}_{t+1} + f_{zt}R_{zt+1}}\right], \tag{A12a}$$

$$\pi E_t\left[\frac{R_{zt+1} - R^{si}_{nt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{si}_{t+1} + f_{zt}R_{zt+1}}\right] = (1-\pi)E_t\left[\frac{R^{ss}_{t+1} - R_{zt+1}}{f_{mt}R_{mt+1} + f_{nt}R^{ss}_{t+1} + f_{zt}R_{zt+1}}\right]. \tag{A12b}$$

These are equivalent to (36) and (37). Because $q_t > 1 > q^R_t$ in our equilibrium, we always have

$$R^{ii}_{t+1} > R^{si}_{t+1} \text{ and } R^{is}_{t+1} > R^{ss}_{t+1}.$$

In the neighborhood of the steady-state equilibrium, we have

$$R^{ii}_{t+1} > R^{ss}_{t+1} > R_{mt+1} \text{ and } R^{ss}_{t+1} > R_{zt+1}.$$

Thus, comparing (A10) and (A11) with (A8), we learn that inequalities (A8) hold in the neighborhood of the steady state.

C. *Proof of Claim 2*

From (26)–(28), the steady-state value of (r, b, q) solves

$$\beta r - (1-\beta)b = \left[1 - \lambda + \pi\lambda(1-\beta)\frac{1-\phi}{1-\theta}\right] + (1-\beta)\left(1 - \pi\frac{1-\phi}{1-\theta}\right)\lambda q, \tag{A13}$$

$$\pi\beta r + \pi\beta b = \left[1 - \lambda + \pi\lambda(1-\beta)\frac{1-\phi}{1-\theta}\right](1-\theta q) - \pi\beta\lambda\phi q, \tag{A14}$$

$$(1-\pi)\frac{r-(1-\lambda)q}{r+\lambda q+(b/\chi)} = \pi\lambda\frac{[(1-\phi)/(1-\theta)](q-1) - [r-(1-\lambda)q]}{r+\lambda q+(b/\chi) - \lambda[(1-\phi)/(1-\theta)](q-1)}. \tag{A15}$$

From (A13) and (A14), we have

$$\begin{pmatrix} 1 & -(1-\beta) \\ \pi & \pi\beta \end{pmatrix}\begin{pmatrix} \beta r \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\kappa + \begin{pmatrix} \lambda(1-\beta)(1-\pi\eta) \\ -\left[\hat{\theta} + \lambda\pi\beta(1-\eta)\right] \end{pmatrix} q,$$

where $\eta = (1-\phi)/(1-\theta)$, $\kappa = 1 - \lambda + \lambda\pi(1-\beta)\eta$, and $\hat{\theta} = (1 - \lambda + \lambda\pi\eta)\theta$. Thus,

$$\pi\beta r = (1 - \beta + \pi\beta)\kappa + (1-\beta)\left[\lambda\pi\beta(1-\pi)\eta - \hat{\theta}\right]q, \tag{A16}$$

$$\pi b = (1-\pi)\kappa - \left[\lambda\pi - \lambda\pi(\beta + \pi - \pi\beta)\eta + \hat{\theta}\right]q. \tag{A17}$$

Because $\lambda\pi - \lambda\pi(\beta + \pi - \pi\beta)\eta + \hat{\theta} = \lambda\pi(1-\pi)(1-\beta)\eta + \lambda\pi\phi + \theta(1-\lambda) > 0$, we have

$$b > 0 \text{ iff } q < \frac{(1-\pi)\kappa}{\lambda\pi - \lambda\pi(\beta+\pi-\pi\beta)\eta + \hat{\theta}} \equiv \hat{q}. \tag{A18}$$

Then there is a monetary equilibrium with financing constraint only if $\hat{q} > 1$, or

$$(1-\lambda)\theta + \lambda\pi\phi < (1-\lambda)(1-\pi),$$

that is, condition 1 for a first-best allocation is violated.

Equation (A15) can be written as

$$[r - (1-\lambda)q]\left[r - (1-\lambda)q + q + \frac{b}{\chi}\right] = \lambda\eta(q-1)\left[r - (1-\lambda)q + \pi\left(q + \frac{b}{\chi}\right)\right]. \tag{A19}$$

Together with (A16) and (A17), we have the condition for steady-state q

$$\begin{aligned} 0 &= \Psi(q) \\ &\equiv \lambda\pi\beta\eta(q-1)\,(\{(1-\beta+\pi\beta)\kappa + q[(1-\beta)(\lambda\pi\beta\eta-\hat{\theta}) - \pi\beta\kappa]\} \\ &\qquad \times [\lambda(1-\pi\eta)+\hat{\theta}] + \pi\beta(1-\pi)[\kappa + q(\lambda\pi\beta\eta-\hat{\theta})]) \\ &\quad - \{(1-\beta+\pi\beta)\kappa + q[(1-\beta)(\lambda\pi\beta\eta-\hat{\theta}) - \pi\beta\kappa]\} \\ &\quad \times (\{(1-\beta+\pi\beta)\kappa + q[(1-\beta)(\lambda\pi\beta\eta-\hat{\theta}) - \pi\beta\kappa]\} \\ &\qquad \times [\lambda(1-\pi\eta)+\hat{\theta}] + \beta(1-\pi)[\kappa + q(\lambda\pi\beta\eta-\hat{\theta})]). \end{aligned} \tag{A20}$$

Then we learn that

$$\Psi(1) = -(1-\beta)(\kappa+\lambda\pi\beta\eta-\hat{\theta})^2\{(1-\beta)[\lambda(1-\pi\eta)(1-\theta)+\theta] + \beta(1-\pi)\} < 0.$$

Therefore, the necessary and sufficient condition for the existence of monetary equilibrium with $q \in (1, \hat{q})$ is

$$0 < \Psi(\hat{q}). \tag{A21}$$

Using (A18), inequality (A21) becomes

$$\begin{aligned} 0 &< \beta\lambda\eta(1-\pi-\chi)[\chi-\beta(1-\pi)+\pi\beta(1-\pi)] - [\chi-\beta(1-\pi)]\chi\kappa \\ &= \pi\beta^2\lambda\eta(1-\pi)(1-\pi-\chi) \\ &\quad - [\chi-\beta(1-\pi)](1-\lambda+\lambda\pi\eta)[\chi-\beta\lambda(1-\pi)(1-\phi)]. \end{aligned}$$

Multiplying both sides by $1-\theta$, we get the condition

$$\begin{aligned} 0 &< \pi\beta^2\lambda(1-\pi)(1-\phi)[(1-\lambda)(1-\pi)-(1-\lambda)\theta-\pi\lambda\phi] \\ &\quad + [(\beta-\lambda)(1-\pi)-(1-\lambda)\theta-\pi\lambda\phi][(1-\lambda)(1-\theta)+\lambda\pi(1-\phi)] \\ &\quad \times [\lambda(1-\beta)(1-\pi)+(1-\lambda)\theta+\lambda(\pi+\beta-\pi\beta)\phi]. \end{aligned}$$

This is assumption 2 in the text. Therefore, under assumption 2, we have a competitive equilibrium in which fiat money has a positive value (claim 2*a*) and $q_t >$ 1 (claim 2*b*). Claim 2*c* directly follows from inequality (A4a) or (A8a), which we proved above given that $q_t > 1$ in the neighborhood of the steady state. QED

D. *Proof of Claim 3*

Claim 3a.—Under assumption 2, we have $(1-\lambda)\theta + \pi\lambda\phi < (1-\lambda)(1-\pi)$. Thus, we have

$$\begin{aligned} \frac{\partial \text{ RHS of (26)}}{\partial q} &= \lambda(1-\beta)\left(1-\pi+\pi\phi-\pi\frac{1-\phi}{1-\theta}\theta\right) \\ &> \lambda(1-\beta)\frac{\pi\phi}{(1-\lambda)(1-\theta)} > 0. \end{aligned}$$

Given $q > 1$ and $b \geq 0$, we have from (26)

$$r = \text{RHS of } (26) > \text{RHS of } (26)|_{q=1,b=0}$$
$$= 1 - \lambda + (1 - \beta)(r + \lambda),$$

or

$$r + \lambda > \frac{1}{\beta}.$$

QED

Claim 3d.—Suppose that claim $3d$ is not true. Then

$$\frac{r + \phi\lambda q + (1 - \phi)\lambda q^R}{q} \geq 1. \tag{A22}$$

Because $q^R < 1 < q$, this implies that

$$\frac{r}{q} + \lambda > 1.$$

But then the left-hand side of (28) is strictly positive, whereas the right-hand side is nonpositive by (A22). This is a contradiction. QED

Claim 3c.—From (28), claim $3d$ implies that $(r/q) + \lambda > 1$. QED

Claim 3b.—Using $q^R < 1 < q$ in (28), we have

$$(1 - \pi)[r - (1 - \lambda)q] > \pi[q - r - \lambda\phi q - \lambda(1 - \phi)q^R],$$

or

$$r - (1 - \lambda)q > \pi\lambda\eta(q - 1) > 0.$$

Hence, given $b > 0$, from (A19) it follows that

$$\frac{\Delta(\Delta + 1)}{\Delta + \pi} < \lambda\eta\left(1 - \frac{1}{q}\right),$$

where $\Delta \equiv (r/q) + \lambda - 1 > 0$ by claim $3c$. But from (A16),

$$\frac{1}{q} = \frac{1}{(1 - \beta + \pi\beta)\kappa}\left\{\pi\beta(\Delta + 1 - \lambda) - (1 - \beta)\left[\lambda\pi\beta(1 - \pi)\eta - \hat{\theta}\right]\right\}.$$

Substituting this into the above inequality, we get

$$\frac{\Delta(\Delta + 1)}{\lambda\eta(\Delta + \pi)} + \frac{\pi\beta\Delta}{(1 - \beta + \pi\beta)\kappa}$$
$$< 1 - \frac{1}{(1 - \beta + \pi\beta)\kappa}\left\{\pi\beta(1 - \lambda) - (1 - \beta)\left[\lambda\pi\beta(1 - \pi)\eta - \hat{\theta}\right]\right\}.$$

The left-hand side of this inequality is increasing in Δ.

Suppose that claim $3b$ is not true; that is, $\Delta \geq (1 - \beta)/\beta$. Then

$$\frac{1 - \beta}{\lambda\eta\beta(1 - \beta + \pi\beta)} + \frac{\pi(1 - \beta)}{(1 - \beta + \pi\beta)\kappa}$$
$$< 1 - \frac{1}{(1 - \beta + \pi\beta)\kappa}\left\{\pi\beta(1 - \lambda) - (1 - \beta)\left[\lambda\pi\beta(1 - \pi)\eta - \hat{\theta}\right]\right\}.$$

Multiplying this inequality through by $\lambda\eta\beta(1-\beta+\pi\beta)\kappa$, we have

$$(1-\beta)[1-\lambda+\lambda\pi(1-\beta)\eta]+\lambda\eta\beta\pi(1-\beta)$$
$$<\lambda\eta\beta(1-\beta+\pi\beta)[1-\lambda+\lambda\pi(1-\beta)\eta]-\lambda\eta\beta^2\pi(1-\lambda)$$
$$+\lambda^2\eta^2\beta^2\pi(1-\pi)(1-\beta)-\lambda\eta\beta(1-\beta)(1-\lambda+\lambda\pi\eta)\theta.$$

Canceling the two terms that do not have factor $1-\beta$ and dividing by $1-\beta$, we get

$$(1-\lambda+\lambda\pi\eta)[1-\lambda\eta\beta(1-\theta)]<0$$

or

$$(1-\lambda+\lambda\pi\eta)[1-\lambda\beta(1-\phi)]<0.$$

This is a contradiction. QED

References

Ajello, Andrea. 2016. "Financial Intermediation, Investment Dynamics, and Business Cycle Fluctuation." *A.E.R.* 106 (August): 2256–2303.

Allen, Franklin, and Douglas Gale. 1994. "Limited Market Participation and Volatility of Asset Prices." *A.E.R.* 84 (September): 933–55.

———. 2007. *Understanding Financial Crises.* Oxford: Oxford Univ. Press.

Almeida, Heitor, Murillo Campello, Bruno Laranjeira, and Scott Weisbenner. 2012. "Corporate Debt Maturity and the Real Effects of the 2007 Credit Crisis." *Critical Finance Rev.* 1 (1): 3–58.

Bernanke, Ben, and Mark Gertler. 1989. "Agency Costs, Net Worth and Business Fluctuations." *A.E.R.* 79 (March): 14–31.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, vol. 1, edited by John Taylor and Michael Woodford, 1341–93. Amsterdam: Elsevier.

Brunnermeier, Markus, Thomas Eisenbach, and Yuliy Sannikov. 2013. "Macroeconomics with Financial Frictions: A Survey." In *Advances in Economics and Econometrics*, vol. 2, edited by Daron Acemoglu, Manuel Arellano, and Eddie Dekel, 3–94. New York: Cambridge Univ. Press.

Brunnermeier, Markus, and Lasse Pedersen. 2009. "Market Liquidity and Funding Liquidity." *Rev. Financial Studies* 22 (June): 2201–38.

Brunnermeier, Markus, and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *A.E.R.* 104 (February): 379–421.

Chang, Briana. 2018. "Adverse Selection and Liquidity Distortion." *Rev. Econ. Studies* 85 (January): 275–306.

Chodorow-Reich, Gabriel. 2014. "The Employment Effects of Credit Market Disruptions: Firm-Level Evidence from the 2008–09 Financial Crisis." *Q.J.E.* 129 (February): 1–59.

Cui, Wei, and Soren Radde. 2016. "Search-Based Endogenous Liquidity and the Macroeconomy." ECB Working Paper no. 1917, European Central Bank, Frankfurt.

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki. 2017. "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities." *A.E.R.* 107 (March): 824–57.

Diamond, Douglas, and Philip Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *J.P.E.* 91 (June): 401–19.

Eisfeldt, Andrea. 2004. "Endogenous Liquidity in Asset Markets." *J. Finance* 59 (February): 1–30.

Farre-Mensa, Joan, and Alexander Ljungqvist. 2016. "Do Measures of Financing Constraints Measure Financing Constraints?" *Rev. Financial Studies* 29 (February): 271–308.

Freeman, Scott. 1996a. "Clearinghouse Banks and Banknote Over-issue." *J. Monetary Econ.* 38 (August): 101–15.

———. 1996b. "Payment System, Liquidity, and Rediscounting." *A.E.R.* 86 (December): 1126–38.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *J. Monetary Econ.* 58 (January): 17–34.

Gertler, Mark, and Nobuhiro Kiyotaki. 2011. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*, vol. 3A, edited by Benjamin Friedman and Michael Woodford, 547–99. Amsterdam: Elsevier.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino. 2016. "Wholesale Banking and Bank Runs in Business Cycle Analysis." In *Handbook of Monetary Economics*, vol. 2B, edited by John Taylor and Harald Uhlig, 1345–1425. Amsterdam: Elsevier.

Green, Edward. 1999. "Money and Debt in the Structure of Payments." *Fed. Reserve Bank Minneapolis Q. Rev.* 23 (Spring): 13–29.

Guerrieri, Veronica, and Robert Shimer. 2014. "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality." *A.E.R.* 104 (July): 1875–1908.

Hart, Oliver, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital." *Q.J.E.* 109 (November): 841–79.

Hicks, John. 1937. "Mr. Keynes and the 'Classics': A Suggested Interpretation." *Econometrica* 5 (April): 147–59.

Holmstrom, Bengt, and Jean Tirole. 1998. "Private and Public Supply of Liquidity." *J.P.E.* 106 (February): 1–40.

———. 2001. "LAPM: A Liquidity-Based Asset Pricing Model." *J. Finance* 56 (October): 1837–67.

———. 2011. *Inside and Outside Liquidity*. Cambridge, MA: MIT Press.

Keynes, John Maynard. 1936. *The General Theory of Employment, Interest, and Money*. London: Macmillan.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *J.P.E.* 105 (April): 211–48.

———. 2001. "Liquidity, Business Cycles and Monetary Policy." Clarendon Lectures 2, London School Econ.

———. 2002. "Evil Is the Root of All Money." *A.E.R.* 92 (May): 62–66.

———. 2003. "Inside Money and Liquidity." Working paper, London School Econ.

———. 2005a. "Financial Deepening." *J. European Econ. Assoc.* 3 (May): 701–13.

———. 2005b. "Liquidity and Asset Prices." *Internat. Econ. Rev.* 46 (May): 317–49.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. "The Aggregate Demand for Treasury Debt." *J.P.E.* 120 (April): 233–67.

Kurlat, Pablo. 2013. "Lemons Markets and the Transmission of Aggregate Shocks." *A.E.R.* 103 (June): 1463–89.

Lagos, Ricardo, Guillaume Rocheteau, and Randall Wright. 2017. "Liquidity: A New Monetarist Perspective." *J. Econ. Literature* 55 (June): 371–440.

Lagos, Ricardo, and Randall Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *J.P.E.* 113 (June): 463–84.

Metzler, Lloyd A. 1951. "Wealth, Saving, and the Rate of Interest." *J.P.E.* 59 (April): 93–116.

Moore, John. 2010. "Contagious Illiquidity." Technical handout of presidential address at World Congress of Econ. Soc., Shanghai.

Nosal, Ed, and Guillaume Rocheteau. (2011) 2017. *Money, Payments, and Liquidity*. 2nd ed. Cambridge, MA: MIT Press.

Santos, Manuel, and Michael Woodford. 1997. "Rational Asset Pricing Bubbles." *Econometrica* 65 (January): 19–57.

Shi, Shouyong. 1997. "A Divisible Search Model of Fiat Money." *Econometrica* 65 (January): 75–102.

———. 2015. "Liquidity, Assets and Business Cycles." *J. Monetary Econ.* 70 (March): 116–32.

Tobin, James. 1969. "A General Equilibrium Approach to Monetary Theory." *J. Money, Credit, and Banking* 1 (February): 15–29.

Townsend, Robert. 1987. "Asset Return Anomalies in a Monetary Economy." *J. Econ. Theory* 41 (April): 219–47.

Townsend, Robert, and Neil Wallace. 1987. "A Model of Circulating Private Debt: An Example with a Coordination Problem." In *Contractural Arrangements for Intertemporal Trade*, edited by Edward Prescott and Neil Wallace. Minneapolis: Univ. Minnesota Press.

Williamson, Stephen. 1987. "Financial Intermediation, Business Failures, and Real Business Cycles." *J.P.E.* 95 (December): 1196–1216.