News and the Cross-section of Expected Corporate Bond Returns

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Abstract

We study the cross-section of expected corporate bond returns using an inter-temporal CAPM (ICAPM) with three factors: innovations in future excess bond returns, future real interest rates and future expected inflation. Our test assets are a broad range of corporate bond market index portfolios. We find that two factors — innovations about future inflation and innovations about future real interest rates—explain the cross-section of expected corporate bond returns in our sample. Our model provides an alternative to the ad hoc risk factor models used, for example, in evaluating the performance of bond mutual funds.

JEL classification: G10, G12

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1 Introduction

We study the factors that explain the cross-section of expected corporate bond returns. Our model adapts the Campbell (1993) inter-temporal CAPM (ICAPM) to the case of an investor who invests only in the bond market.

There is, surprisingly, little research on the cross-section of expected bond returns in comparison to that on the cross-section of stock returns\textsuperscript{1}. This is striking given that, in 2005, according to the International Monetary Fund (2007), the capitalization of the US bond markets was US$24 trillion as compared to US$17 trillion for the US stock markets. The relative sizes of the corporate and government bond markets were US$18.1 trillion and US$5.9 trillion respectively. More importantly from an investor’s perspective, the most recent data (Investment Company Institute, 2007a) shows that, out of a total of US$18 trillion under management in US mutual funds in 2006, as much as US$2 trillion was invested in bond and money market funds compared to about US$10 trillion in equity funds. In terms of the number of funds, out of a total of about 8,100 mutual funds, 2,849 (35\%) were classified as bond and money market funds, 4,770 (58\%) as equity market funds and the remaining as hybrid funds (Investment Company Institute, 2007b).

Our main results are as follows. Using a return decomposition for a consol bond, we obtain a three-factor ICAPM in the spirit of Campbell (1993). We test this model using returns, over the period 1988–2006, on seven corporate bond index portfolios of different default categories. We find, using a standard Fama–MacBeth approach that our model cannot be rejected. Of the three factors in our model, innovations in future inflation rates (i.e. news about expected inflation) and future real rates are more important than innovations in expected excess bond returns in determining the cross-section of expected corporate bond returns. Our results are robust to a number of checks including the use of; alternative industry-based portfolios,\textsuperscript{1}

\textsuperscript{1}Selected examples include Chang and Huang (1990), Fama and French (1993) and Gebhardt, Hvidkjaer and Swaminathan (2005) among others.
different sub-samples of the data and an alternative GMM estimation technique.

The rest of the paper is organized as follows. Section 2 provides a brief outline of related research on the cross-section of expected corporate bond returns, while in Section 3, we describe the set-up of our model and the test methodology. In Section 4, we provide details of the data that we use and we discuss our empirical results in Section 5. Section 6 presents some robustness checks and Section 7 concludes the paper.

2 Related literature

As mentioned earlier, despite the large size of the US government and corporate bond markets relative to the equity markets and the substantial proportion of funds invested in bond-only mutual funds, there has been surprisingly little research on the factors that drive bond betas. In early work Chang and Huang (1990) find, using six portfolios based on Moody’s rating quality as a criteria, that excess returns on corporate bonds are driven by two unobservable factors. Fama and French (1993) find that a five-factor model that adds a term structure factor and a default premium factor to the now familiar Market, SMB and HML factors explains the cross-section of both stock and bond returns well. More recently, Gebhardt et al (2005) evaluate the factor loadings versus characteristics debate in the context of the cross-section of expected bond returns. They find that default betas and term betas are able to explain the cross-section of bond returns after controlling for characteristics such as duration and ratings. Their results imply that firm-specific information implicit in ratings and duration is not related to the cross-section of expected bond returns.

As pointed out earlier, there is a significant amount of investment in bond market mutual funds. The measurement of the performance of these funds using asset pricing models relies largely on ad hoc factor models. For example, Huij and Derwall (2008), who use a multifactor model with factors that include returns on the overall bond market, on low-grade debt, on a
mortgage-backed securities index, the aggregate stock market index and three more factors obtained by a principal components analysis of yield changes.

We also note here that the literature on the predictability of holding period returns on corporate bonds (in contrast to government bonds) is rather sparse. This is relevant in our context, because we need to identify state variables that have predictive power for excess corporate bond returns. We rely here on Baker et al (2003), who find that excess returns on corporate bonds are predicted by the real short rate and the term spread.

The model we use is based on the ICAPM derived in Campbell (1993). Campbell uses a log-linear approximation to an investor’s budget constraint to express unanticipated consumption as a function of current and future returns on wealth. In our adaptation of the Campbell (1993) model, we rely on a present value decomposition for the return on a consol bond, as in Engsted and Tanggaard (2001), which corresponds to the long-term investment horizon of our investor. We also assume that our investor invests only in the bond market. This may seem, at first blush, a restrictive assumption — but there are two points that make this assumption a reasonable one. Firstly, from an investor’s perspective, the most recent data (Investment Company Institute, 2007a) shows that, out of a total of US$18 trillion under management in US mutual funds in 2006, as much as US$2 trillion was invested in bond and money market funds, compared to about US$10 trillion in equity funds. In terms of the number of funds, out of a total of about 8,100 mutual funds, 2,849 (35%) were classified as bond and money market funds, 4,770 (58%) as equity market funds and the remaining as hybrid funds (Investment Company Institute, 2007b). This is because a large number of market participants such as pension funds and insurance companies, among others, have mandates that restrict the application of their funds to fixed-income securities. Secondly, as Ferson et al (2006) observe: ‘Ideally, one would like an SDF model or a set of factors to price both stocks and bonds. Empirically, however, this is challenging … However it is more common to find bond factors used for pricing bonds and stock factors for pricing stocks’.

\[\text{Using a consol-bond return decomposition rather than that for a coupon-bond with finite-maturity is not crucial to our results.}\]
Estimating the Campbell (1993) model requires the specification of the VAR, where the choice of the state variables is essentially an empirical issue. Campbell and Vuolteenaho (2004), for example, find that the success of their two-factor model relied critically on including the small-stock value spread as a state variable in their VAR estimation. Recently, Chen and Zhao (2008) also show that estimating innovations is sensitive to the specification of the VAR system. We find, in this paper, that our results are robust to an alternative vector of state variables. We also note that despite the critique about the specific choice of state variables, recent applications (see for example Brunnermeier and Julliard, 2007 among others) also use a similar VAR approach.

3 Model set-up and test methodology

We now provide brief details of our intertemporal CAPM and of the econometric methodology used in this paper\(^3\).

3.1 Bond return decomposition

In this paper, we use a return decomposition for a consol bond rather than that for zero coupon bond (see, for example, Campbell and Ammer, 1993) since our investor has a long horizon. We define the log one-period gross return from \(t\) to \(t+1\) on a consol bond as

\[
r_{b,t+1} = \log \left( \frac{C + P_{b,t+1}}{P_{b,t}} \right) = \log \left( C + \exp \left( p_{b,t+1} \right) \right) - p_{b,t}
\]

in which \(C\) denotes the coupon and \(P_{b,t}\) the price. It can then be shown (see Engsted and Tanggaard, 2001) that

\(^3\)Refer to the Appendix for further details on the derivations.
\[(E_{t+1} - E_t)(r_{b,t+1} - r_{f,t+1}) =
\]
\[- (E_{t+1} - E_t) \left\{ \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j}) + \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} + \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} \right\} \]  

(2)

in which \(\rho_b\) is the constant from the linearization and is a number slightly smaller than one. Using more compact notation, we define \(\tilde{x}_{b,t+1} = ((E_{t+1} - E_t) r_{b,t+1} - r_{f,t+1})\) as the innovation in the log excess one-period return, and the three terms on the right-hand-side of (2) as: \(\tilde{x}_{x,t+1}\), the innovation in the future log excess one-period return; \(\tilde{x}_{r,t+1}\), the innovation in the log excess one-period real return; \(\tilde{x}_{\pi,t+1}\), the innovation in the log excess one-period inflation. We can then rewrite equation (2) as

\[\tilde{x}_{b,t+1} = -\tilde{x}_{x,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{\pi,t+1}\]  

(3)

This expression is a dynamic accounting identity and holds by construction, having been obtained from the definition of the return on a consol bond. Unexpected excess bond returns must be due to ‘news’ (or changes in expectations) about either future excess bond returns, future inflation or future real interest rates, or combinations of these three. We note here that a similar decomposition can be derived based on the present value relation for a \(n\)-period coupon bond (see for example Campbell, Lo and MacKinlay, 1997). This analogous expression, using the definition of the return on a coupon bond, differs from (2) above only in that the summations run from 1 to \(n\) (where \(n\) is the time to maturity of the coupon bond) instead of from 1 to \(\infty\).\(^4\)

\(^4\)In empirical estimation this means summing the series from 1 to \(n\) (e.g. \(n=120\) if we use monthly data and assume a 10 year maturity bond) instead of an "infinite" sum to extract the news components from the VAR. We find (in results not reported here to conserve space) that our main empirical results remain unchanged even if we use the \(n\)-period coupon bond return decomposition.
3.2 Expected future bond returns and default risk

An issue that can be raised is that, if we are modelling the cross-section of expected corporate bond returns, we should provide for a factor that reflects default risk. It is possible to include in our decomposition a fourth factor specifically to model default risk. We could, for example, follow Perraudin and Taylor (2003) who consider a defaultable bond and obtain a return decomposition which has an additional factor that reflects the loss rate of default.

In this paper, however, we do not specifically include a separate factor for default risk for the following reasons. The first is that we will get a new free parameter, i.e. the loss rate on default, for which we will have to use estimates that are outside of our data. This will bring in more parameter uncertainty and will move us away from our basic objective of understanding what drives the cross-section of expected corporate bond portfolio returns. In addition, increasing the number of free parameters and factors would bias our results in favour of finding a model with a better fit. Instead, we assume that the ‘news about future expected bond returns’ component of our three-way decomposition includes any news about the way in which ‘default risk’ will affect excess bond returns, since these future expected returns will capture and include investor’s expectations about the possibility of default in the corporate bond market and also incorporate expectations about default-related factors such as macroeconomic conditions. Further, it is likely that the ‘news about expected future bond returns’ factor in our three-way decomposition would be correlated with this fourth default risk factor (should we include it in our decomposition) and hence will complicate further the estimation of the factor betas and the market prices of risk in which we are interested. Second, our test assets are the Lehman Brothers corporate bond portfolios — these are investable indices that are tracked by hundreds of corporate bond funds. We can assume, for example, that our investor — who is not investing in individual corporate bonds, but in bond portfolios — can still invest in a matching or mimicking mutual fund, with the same credit risk characteristics, where he is not exposed to the default risk that he would be
were he to hold an individual bond. To sum up, we will assume that $\tilde{x}_{x,t+1}$ in (3) also captures investor’s expectations about the possibility of default in the corporate bond market.

### 3.3 Bond ICAPM

We follow Campbell (1993) and use the Epstein–Zin (1989) utility function, defined recursively, for an infinitely lived representative agent who invests only in the bond market. The Euler equation for asset $i$ has an associated pricing equation in simple returns given by

$$1 = E_t \left[ \left\{ \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right\}^\theta \left\{ \left( \frac{1}{R_{B,t+1}} \right) \right\}^{1-\theta} R_{i,t+1} \right]$$

in which $\theta = \frac{1-\gamma}{1-\psi}$, $\psi$ is the elasticity of intertemporal substitution, $\gamma$ is the coefficient of relative risk aversion, $\delta$ is a time discount factor, $C_t$ is consumption, $R_{B,t+1}$ is the return on the aggregate bond market and $R_{i,t+1}$ is the return on any asset $i$.

We now define the SDF as

$$M_{t+1} = \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left( \frac{1}{R_{B,t+1}} \right)^{1-\theta}.$$ After some algebra, the log of the SDF can be written as

$$m_{t+1} = E_t (m_{t+1}) - \frac{\theta}{\psi} (c_{t+1} - E_t (c_{t+1})) - (1 - \theta) (r_{B,t+1} - E_t (r_{B,t+1})).$$ We then substitute out consumption and use equation (3) to obtain

$$m_{t+1} = E_t (m_{t+1}) + \tilde{x}_{\pi,t+1} + \tilde{x}_{r,t+1} + \tilde{x}_{x,t+1}$$

Next, we define $f_{t+1} = \left( \tilde{x}_{\pi,t+1}, \tilde{x}_{r,t+1}, \tilde{x}_{x,t+1} \right)'$ and $b = (1,1,1)$, and use the standard result that if the log of the SDF, $m_{t+1}$, is a linear function of the $K$ risk factors then, the unconditional model in expected excess log return returns is

$$E (r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = b' \text{cov} (r_{t+1}; f_{t+1})$$

8
Note that (6) is a form of the expected return-beta form

\[ E (r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \lambda^T \beta_i \]  

(7)

in which \( r_{f,t+1} \) is the risk free-rate, \( \beta_i = [Var(f_{t+1})]^{-1} Cov(r_{i,t+1}, f_{t+1}) \) is a vector with the K factor betas for asset \( i \) and \( \lambda = -Var(f_{t+1})b \) is a vector of the market price of risk.

Now, we can rewrite the model in an expected return-beta representation, i.e.

\[ E (r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \lambda^T \beta_i = \lambda_\pi \beta_{i,\pi} + \lambda_r \beta_{i,r} + \lambda_x \beta_{i,x} \]  

(8)

where each beta is the beta of asset \( i \) with its corresponding news component, i.e.

\[ \beta_{i,\pi} = Var(\tilde{x}_{\pi,t+1})^{-1} Cov(r_{i,t+1}, \tilde{x}_{\pi,t+1}) \]  
\[ \beta_{i,r} = Var(\tilde{x}_{r,t+1})^{-1} Cov(r_{i,t+1}, \tilde{x}_{r,t+1}) \]  
\[ \beta_{i,x} = Var(\tilde{x}_{x,t+1})^{-1} Cov(r_{i,t+1}, \tilde{x}_{x,t+1}) \]  

and \( \lambda = (\lambda_x, \lambda_r, \lambda_\pi)^T \) denotes the vector of factor risk prices. Finally, we rewrite the left-hand side using simple expected returns to obtain our three-beta model for the bond market:

\[ E (r_{i,t+1} - r_{f,t+1}) = \lambda_\pi \beta_{i,\pi} + \lambda_r \beta_{i,r} + \lambda_x \beta_{i,x}. \]  

(9)

Equation (9) implies that, in the case of the bond market, the risk premium for an investor is independent of the long-term investor’s relative risk aversion \( \gamma \).

### 3.4 VAR estimation and extraction of news components

We can now use the VAR approach as in Campbell and Vuolteenaho (2004) to extract the components of equation (3) from the data. We specify our VAR using the state variables \( z_t = (x_{b,t}, r_t, sprd_t) \), in which \( x_{b,t} \), \( r_t \), and \( sprd_t \) are the excess return on the bond market, the real interest rate and the Baa–Aaa credit spread, respectively. We use these variables because the VAR needs to include the excess bond return and the real rate in order to compute their
corresponding news components. We include the credit spread because previous studies have found that this variable has significant predictive power for bond returns (see Section 2). We also assess the robustness of our results using an alternate plausible state variable, i.e. the dividend yield. Note that inflation is not included, because its news component will be calculated as a residual, as explained below.

We can write a first-order VAR (in companion form for higher lags if required) as

\[
z_{t+1} = Az_t + w_{t+1}
\]

in which \( A \) is the VAR parameter matrix and \( w_{t+1} \) is the vector of error terms. Using suitable unit vectors \( g_1 \) and \( g_2 \), the VAR estimate of \( A \) and its residuals, \( w_{t+1} \), each component is

\[
\begin{align*}
\tilde{x}_{b,t+1} &= g_1 w_{t+1} \\
\tilde{x}_{x,t+1} &= \rho g_1 A (I - \rho A)^{-1} w_{t+1} \\
\tilde{x}_{r,t+1} &= \rho g_2 A (I - \rho A)^{-1} w_{t+1} \\
\tilde{x}_{\pi,t+1} &= -\tilde{x}_{b,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{x,t+1}
\end{align*}
\]

Thus, we get the inflation news component as a residual, since we know the other components in this dynamic accounting identity. Therefore, we do not need to specify a specific data-generating process for the inflation. This mirrors the methodology followed by Campbell and Vuolteenaho (2004), who avoid specifying a process for the dividend yield in the case of stocks and obtain the cash-flow news component as a residual.

4 Data

We use monthly data obtained from Lehman Brothers, for the 1988–2006 period, on bond indices for the aggregate bond market and for different bond credit rating categories. We
note two points regarding the data. Firstly, we use holding period returns based on the ‘Total Return since Inception’ data so that the holding period return from $t$ to $t + 1$ reflects both capital gains as well as coupon payments. Many studies on bonds use other measures such as yields that are not useful in our context. We also note that these Lehman Brothers corporate bond indices, during our sample period 1988–2006\(^5\), consist of the most representative and liquid issues in each rating category that are followed by the traders who always post bid–ask prices. Sangvinatsos (2005) points out that Lehman Brothers corporate bond indices are used and replicated as benchmarks\(^6\) by a large proportion of bond portfolio managers, and that hence the computed returns represent returns that could actually be realized. The Lehman US Aggregate Index, which we use as a proxy for the US bond market, covers the dollar-denominated, investment-grade, fixed-rate taxable bond market, including Treasuries, government-related and corporate securities, MBS pass-through securities, asset-backed securities and commercial mortgage-based securities. We use as test assets the following seven indices from the Lehman Brothers fixed-income database: AAA; AA; A; BAA; BA; B; CA. In our tests for robustness we also use Citigroup corporate bond indices for 7 industry sectors over the period 1990-2006. The credit spread defined as Moody’s Baa–Aaa and the CPI data are both from the FRED database. We use the three-month T-bill rate from the CRSP and obtain the real rate as the difference between the T-bill rate and the growth rate in the CPI.

\(^5\)The earliest data available on corporate bond indices based on credit ratings (which is from Lehman Brothers) is from 1983 but is restricted to 4 categories only. Additional indices for high-yield or low-credit rated bonds were added in the late 1980s. This early data is, however, based on ‘backfilled’ data and matrix prices. After conducting a careful analysis, we find that 1988 is the earliest date from which we believe that reliable corporate bond index data for the seven credit rating categories is available.

\(^6\)Two examples from Morningstar are: SunAmerica High Yield Bond A normally invests at least 80% of its assets in below-investment-grade US and foreign junk bonds without regard to the maturities of such securities and the Fidelity US Bond Index Fund which has more than 70% in AAA US corporate bonds.
4.1 Test methodology

We use the standard Fama and MacBeth cross-sectional regressions to estimate our model given in equation (9).

In the first step of the method, for each test asset $i$, the betas are estimated with a time series regression of excess returns, $R_{it}^e$, onto a constant and the three factors:

$$R_{it}^e = \alpha_i + \beta_{i,x} \tilde{x}_{it,t+1} + \beta_{i,r} \tilde{x}_{rt,t+1} + \beta_{i,\pi} \tilde{x}_{\pi t,t+1} + \varepsilon_{it}$$  \hspace{1cm} (12)

We use, following much of the recent literature, estimates of betas over the full sample period. In the second step, for each period $t$, the risk premiums $\lambda_{t,x}, \lambda_{t,r}$, and $\lambda_{t,\pi}$ are estimated from a series of cross-sectional regressions of the excess returns on the estimated betas, i.e.

$$R_{it}^e = \hat{\beta}_{i,x} \hat{\lambda}_{t,x} + \hat{\beta}_{i,r} \hat{\lambda}_{t,r} + \hat{\beta}_{i,\pi} \hat{\lambda}_{t,\pi} + \alpha_{it} \quad i = 1, 2, \ldots, 7.$$  \hspace{1cm} (13)

Although the standard errors derived from the Fama–MacBeth technique correct for cross-sectional correlation in a panel, this technique assumes that the time series is not autocorrelated. Moreover, Fama–MacBeth standard errors do not correct for the fact that the betas are generated regressors. In response to the first issue, we follow Cochrane (2005) and report Fama–MacBeth standard errors corrected for autocorrelation. To account for the fact that betas are estimated regressors, we also report Shanken (1992) standard errors. But Shanken standard errors are to be preferred to those of Fama and MacBeth only in the case that the returns are conditionally homoskedastic, because the latter may be more precise when the returns are conditional heteroskedastic (see Jagannathan and Wang, 1996). Finally, we test the validity of our three-factor model by assessing whether the pricing errors are jointly zero using a $\chi^2$ test. We also report, as an informal criterion, plots of actual and

\footnote{We also estimated our model using GMM. Our results are qualitatively similar to those derived from the Fama–MacBeth procedure. We give details of the GMM procedure in the Appendix and report results for the period 1993-2006.}
predicted mean returns, which if the model fits perfectly should lie on the 45° line through the origin.

5 Empirical results

Table 1 provides some interesting summary statistics on our set of test assets. We note that unlike equity size portfolios, the average returns on bond portfolios are not monotonically related to the rating category: for example, the AA-rated portfolio has a higher return than the A and BAA-rated portfolios. The median returns also have a similar pattern. Further, the B and CA-rated portfolios returns are more than twice as volatile as those of AAA and other higher quality bond portfolios. Our summary statistics show that there is an interesting spread of average returns to explain: 0.71–1.51% per month, or about 1.10% per month spread in average returns.

The cross-correlations between the test assets are reported in Table 2. We note that the magnitude of the cross-correlations are related closely, as might be expected, to the rating categories: for example, the correlation between the AAA and the A portfolio is 0.96, but is only 0.06 in relation to the CA-rating category portfolio. On the other hand, the cross-correlation between the portfolios decreases in a monotonic way as we move from the AAA to the CA-rating category portfolios.

We report, in Table 3, some summary statistics on our three state variables: the excess return on the aggregate bond market index, the real rate and the credit spread over the sample period 1988–2006. Here, we find that the excess bond return is more than three times as volatile as the real interest rate and sixty times more volatile than the credit spread. The real interest rate and the spread appear, however, to be more persistent than the excess bond return. We also provide statistics on the cross-correlation between state variables in Table 4. The cross-correlations between the excess bond market return, the real rate and the credit spread are, in general, quite low.
5.1 VAR

We include variables in the VAR system that one might reasonably expect to capture predictable variation in bond returns. Our state variables are: the excess return on the aggregate bond market, the real rate and the credit spread. Table 5 reports the VAR coefficients based on equation-by-equation OLS estimates. We also obtained bootstrapped standard errors, but because these are qualitatively similar we do not report them to conserve space. Finally, we report the $R^2$ and $F$ statistics.

The first column of Table 5 shows that the real rate, $r_t$, and the spread, $sprd_t$, have some ability to predict excess bond returns. Excess bond returns display a small degree of persistence: the coefficient on the lagged excess bond return is 0.15 with a standard error of 0.067. A point to note is that compared to the low $R^2$ (typically 2–4%) seen in VARs with predictive variables for excess stock returns, the $R^2$ for the excess bond return regression is 5%. The remaining columns of Table 5 summarize the dynamics of the explanatory variables. These results show that the credit spread is highly persistent, with an autocorrelation coefficient of 0.95, while the excess bond market return and real rate display lower levels of persistence.

5.2 Fama–MacBeth risk premiums

We report, in Table 7, the results of the first stage of the Fama–MacBeth regressions, i.e. the time series estimates of the betas for our three factors; news about expected inflation, real rate and future bond returns. We find that all the betas are negative and they show some variation in size ranging from a high of (-0.53) to a low of (-2.72) for the riskiest CA-category bond index. Table 8 reports results for the second stage of the Fama–MacBeth regression. We present Fama–MacBeth estimates and measure the statistical significance of the risk premiums using $t$-statistics based on Fama–MacBeth standard errors, Shanken-
adjusted standard errors that account for measurement error in the first-pass beta estimates and Fama–MacBeth standard errors corrected for autocorrelation.

We find that the $\lambda$-coefficients for the news betas for expected future inflation and expected future real rates are statistically significant. For example, the Fama–MacBeth $t$-statistics are, respectively, -3.27 and 2.49, based on standard errors corrected for autocorrelation, and -2.03 and 2.34 with Shanken standard errors. The estimated risk price for inflation news beta is high and negative, at -0.60% per month, whereas that for real rate news is 0.18% per month. These estimates imply that a long-term investor who invests only in the bond market demands a higher premium to hold assets that covary with the negative market’s inflation news than that required to hold assets that covary with news about the market’s real discount rates. We now assess the fit of our model. Using the Fama–MacBeth chi-squared test, we find that the three-factor model cannot be rejected because the chi-squared statistic is 11.56, which is smaller than the 1% critical value for the $\chi^2_4$, i.e. 13.28. Our ICAPM is able to explain 63.18% of the cross-sectional variation in expected excess bond returns on the seven risk portfolios.

6 Robustness checks

We now briefly describe several additional tests that we have carried out to assess the robustness of our results.

6.1 Sensitivity to additional state variables

Our main VAR includes three variables: excess bond returns, real rate and credit spread. We re-estimate the VAR by adding the dividend yield on the CRSP VW index to the state vector. This is a plausible state variable because low-grade bonds are, in many ways, similar
to equity and the ‘conventional wisdom’ in the literature (see, for example Cochrane, 2005) is that dividend yields have predictive power for excess stock returns. Therefore, it seems natural and interesting to include this variable in our analysis. Descriptive statistics of this variable are reported in the last column of Table 3. We find that our main results are not materially altered when we add the dividend yield into the VAR (see Table 6). Dividend yield seems not to be useful as a predictor of excess bond returns, as its low OLS t-statistic shows (i.e. 0.82). We also estimated Fama–MacBeth cross-sectional regressions using the factors from this VAR. We find that our main results (not reported here in the interest of brevity) remain unchanged to the inclusion of dividend yield as a state variable.

6.2 Alternative Test Assets and Estimation Methodology

Lewellen, Nagel and Shanken (2008) suggest that, to improve empirical tests, it is advisable to expand the set of test portfolios using assets with a possibly different factor structure. For example, in the case of the equity market they suggest using industry-sorted portfolios in addition to the usual Fama French size and B/M portfolios. In this spirit, we add seven corporate bond industry indices to our original test assets. These indices are from Citigroup and include the following industrial sectors: manufacturing, service, transportation, utility, consumer, energy and other. These portfolios are available from Datastream from 1990, thus our sample period is now 1990-2006. We provide summary statistics of these portfolios in Table 10 and their cross-correlations are presented in Table 11.

The main results of the expanded portfolio of 14 test assets are given in Table 12. We find that the excess bond market news remains insignificant, whereas the real rate news is significant using either ordinary Fama–MacBeth standard errors, standard errors corrected for autocorrelation or Shanken-corrected standard errors. The inflation news component is significant with Fama–MacBeth standard errors corrected for autocorrelation, but loses its predictive ability when we use Shanken-corrected standard errors. More importantly, our
Fama–MacBeth chi-squared statistic, which tests whether all of the pricing errors are zero, cannot reject the null hypothesis. Here, the statistic is 18.65, which is smaller than the 1% critical value for the $\chi^2_{11}$, i.e. 24.72.

We re-estimate the model using a GMM methodology, where we treat the moments that generate the regressors $\beta$ at the same time as the moments that generate the regression coefficients $\lambda$ as outlined in the Appendix and again our main conclusions remain unchanged.

Following many empirical studies that present plots of the actual mean returns versus the model predictions, we focus on the economically interesting pricing errors themselves and not only on whether a test statistic is large or small by statistical standards. Figures 1 and 2 show that our three-beta model does reasonably well, in terms of the test portfolios lining up along the 45° line, in pricing the test assets.

### 6.3 Alternative Sub-samples

Our full sample period is from 1988-2006. In order to assess whether our results are robust, we also evaluate the performance of our model over sub-samples of the data. We therefore divide the full sample into two roughly equal sub-periods: from 01/1988 to 12/1996 and from 01/1997 to 09/2006. Our results are reported in Table 13. We find that our estimations are qualitatively similar to those obtained in our original calculations, i.e. the model specification tests for each sub-sample continue to indicate that there is insufficient evidence to reject the null that the pricing errors are zero.

### 7 Conclusion

Although the bond market constitutes a separate asset class with a larger market value than that of the entire equity market, there has been little attention paid to the covariance
risk of expected excess returns of bonds belonging to different risk classes. Some examples of this research include Chang and Huang (1990) and Gebhardt et al (2005). Previous research has used either stock market factor models augmented to include additional factors that affect bonds, or models with *ad hoc* factors (see for example, Elton et al, 2005) that seem important in the context of bond markets. For example, Huij and Derwall (2008) measure bond fund performance using a model that includes proxies for the overall bond market, low-grade debt, mortgage-backed securities and principal components-based factors extracted from yield changes in certain ranges of the bond maturity spectrum. In contrast, in this paper, we provide a motivation for our news factors based on a simple present value decomposition for consol bonds. Further, we operationalize this using a VAR framework, as in Campbell and Vuolteenaho (2004), to extract factors from variables that forecast bond returns. Clearly, a limitation of this approach are that it assumes that the econometrician knows enough about the investor’s information set using a specific set of state variables and that the parameters of the VAR represent changes in the investor’s environment. Despite this, however, our three-factor model, when taken to the data, is able to give a reasonable account of the cross-sectional variation in expected bond returns.

Our main results are as follows: we use a return decomposition for a consol bond, which, combined with Epstein-Zin preferences, leads to a three-factor ICAPM in the spirit of Campbell (1993). An interesting feature of our three-factor ICAPM for bonds is that it does not have the risk aversion coefficient as a free parameter and that the bond betas with the three factors are entirely data dependent. We test this model and find, using seven index portfolios of different default categories over the 1988–2006 sample period, that our model cannot be rejected. Of the three factors in our ICAPM, innovations in future inflation rates and future real rates are more important than news about future excess bond returns in determining the cross-section of expected corporate bond returns. Our robustness checks show that these results remain qualitatively similar to the use of an additional state variable, alternative test assets based on industry portfolios, different sub-samples and the use of an alternative
estimation methodology.

There are a number of ways in which this study could be extended. Firstly, one obvious concern is that our results are sample-specific, especially in relation to the choice of state variables. In ongoing work, we are investigating techniques for estimation that may allow us to be more agnostic about this choice. Secondly, it would be useful to see how the model performs in the analysis of the performance of bond market mutual funds relative to models that use ad hoc factor representations. Finally, extensions to the model that allow for heteroskedasticity (see for example Guo, 2006 and De Goeij and Marquering, 2006) may also be fruitful avenues for future work.
### Table 1: Descriptive statistics—Lehman Brothers corporate bond portfolios (Intermediate Maturity) for different credit rating categories—Sample 01/1988–09/2006—Percentage holding period returns

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BAA</th>
<th>BA</th>
<th>B</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.7103</td>
<td>0.7315</td>
<td>0.7278</td>
<td>0.7253</td>
<td>1.0207</td>
<td>1.0360</td>
<td>1.5073</td>
</tr>
<tr>
<td>Median</td>
<td>0.6746</td>
<td>0.7187</td>
<td>0.8455</td>
<td>0.7345</td>
<td>1.1986</td>
<td>1.1834</td>
<td>1.2683</td>
</tr>
<tr>
<td>Std Dev.</td>
<td>1.2002</td>
<td>1.2755</td>
<td>1.2714</td>
<td>1.3123</td>
<td>2.1720</td>
<td>3.2881</td>
<td>8.9148</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1344</td>
<td>-0.1812</td>
<td>-0.1371</td>
<td>-0.1176</td>
<td>-0.9159</td>
<td>-0.1478</td>
<td>2.3770</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.0622</td>
<td>0.3725</td>
<td>0.2786</td>
<td>0.1464</td>
<td>5.1188</td>
<td>6.7152</td>
<td>23.9827</td>
</tr>
</tbody>
</table>

### Table 2: Pairwise correlation matrix—Lehman Brothers corporate bond portfolios (Intermediate maturity) for different rating categories—Sample 01/1988–09/2006—Percentage holding period bond returns.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BAA</th>
<th>BA</th>
<th>B</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>1</td>
<td>0.9871</td>
<td>0.9642</td>
<td>0.8872</td>
<td>0.3508</td>
<td>0.1740</td>
<td>0.0603</td>
</tr>
<tr>
<td>AA</td>
<td>0.9871</td>
<td>1</td>
<td>0.9802</td>
<td>0.9180</td>
<td>0.4003</td>
<td>0.2234</td>
<td>0.1106</td>
</tr>
<tr>
<td>A</td>
<td>0.9642</td>
<td>0.9802</td>
<td>1</td>
<td>0.9532</td>
<td>0.4796</td>
<td>0.3086</td>
<td>0.1705</td>
</tr>
<tr>
<td>BAA</td>
<td>0.8872</td>
<td>0.9180</td>
<td>0.9532</td>
<td>1</td>
<td>0.6206</td>
<td>0.4438</td>
<td>0.2678</td>
</tr>
<tr>
<td>BA</td>
<td>0.3508</td>
<td>0.4003</td>
<td>0.4796</td>
<td>0.6206</td>
<td>1</td>
<td>0.8662</td>
<td>0.6622</td>
</tr>
<tr>
<td>B</td>
<td>0.1740</td>
<td>0.2234</td>
<td>0.3086</td>
<td>0.4438</td>
<td>0.8662</td>
<td>1</td>
<td>0.7597</td>
</tr>
<tr>
<td>CA</td>
<td>0.0603</td>
<td>0.1106</td>
<td>0.1705</td>
<td>0.2678</td>
<td>0.6622</td>
<td>0.7597</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 3: State variables—Descriptive statistics—Sample 10/1987–09/2006—bondmkt is the excess aggregate bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and growth rate in the CPI; credit spread is the difference between Moody’s seasoned Baa and Aaa corporate bond yields; dividend yield is the difference between vwret and vwrex from CRSP. The credit spread data and the CPI data is from the FRED database. ACF refers to the autocorrelation at lag 1.

<table>
<thead>
<tr>
<th></th>
<th>bondmkt</th>
<th>real rate</th>
<th>credit spread</th>
<th>dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.3165</td>
<td>0.1167</td>
<td>.0714</td>
<td>0.2110</td>
</tr>
<tr>
<td>Median</td>
<td>0.3691</td>
<td>0.1348</td>
<td>.0692</td>
<td>0.1846</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.1706</td>
<td>1.1255</td>
<td>0.1175</td>
<td>0.6252</td>
</tr>
<tr>
<td>Minimum</td>
<td>-3.0170</td>
<td>-0.9135</td>
<td>0.0458</td>
<td>0.0891</td>
</tr>
<tr>
<td>Std Dev.</td>
<td>1.0917</td>
<td>0.2849</td>
<td>0.0177</td>
<td>0.0919</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1547</td>
<td>-0.3407</td>
<td>0.7189</td>
<td>1.4619</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.0086</td>
<td>1.0344</td>
<td>-0.2376</td>
<td>2.4450</td>
</tr>
<tr>
<td>ACF</td>
<td>.1700</td>
<td>.3810</td>
<td>.9480</td>
<td>.3170</td>
</tr>
</tbody>
</table>
Table 4: State variables — Pairwise correlations — Sample 10/1987–09/2006 — bondmkt is the excess aggregate bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and the growth rate in the CPI; credit spread is the difference between Moody’s seasoned Baa and Aaa corporate bond yields; dividend yield is the difference between vwret and vwretx from CRSP. The credit spread data and the CPI data is from the FRED database.

<table>
<thead>
<tr>
<th></th>
<th>bondmkt</th>
<th>real rate</th>
<th>credit spread</th>
<th>dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>bondmkt</td>
<td>1</td>
<td>0.0884</td>
<td>0.0686</td>
<td>0.0295</td>
</tr>
<tr>
<td>real rate</td>
<td>0.0884</td>
<td>1</td>
<td>-0.0676</td>
<td>0.1844</td>
</tr>
<tr>
<td>credit spread</td>
<td>0.0686</td>
<td>-0.0676</td>
<td>1</td>
<td>0.2254</td>
</tr>
<tr>
<td>dividend</td>
<td>0.0295</td>
<td>0.1844</td>
<td>0.2254</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5: VAR — Sample 10/1987–09/2006 — All variables have been demeaned and a constant term has been included — bondmkt is the excess bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and the growth rate in the CPI; credit premium is the difference between Moody’s seasoned Baa and Aaa corporate bond yields. The credit premium data and the CPI data is from the FRED database. Figures correspond to OLS estimates, standard errors are inside parenthesis and t-statistics are in brackets.

<table>
<thead>
<tr>
<th></th>
<th>bondmkt</th>
<th>real rate</th>
<th>credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>bondmkt (-1)</td>
<td>0.1550</td>
<td>0.0071</td>
<td>0.0001</td>
</tr>
<tr>
<td>real rate (-1)</td>
<td>(0.0661)</td>
<td>(0.0159)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>credit spread (-1)</td>
<td>[2.3440]</td>
<td>[0.4434]</td>
<td>[0.0633]</td>
</tr>
<tr>
<td></td>
<td>0.4809</td>
<td>0.3708</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td>0.2578</td>
<td>0.0622</td>
<td>0.0013</td>
</tr>
<tr>
<td></td>
<td>[1.8652]</td>
<td>[5.966]</td>
<td>[0.3694]</td>
</tr>
<tr>
<td></td>
<td>4.2982</td>
<td>-1.7339</td>
<td>0.9484</td>
</tr>
<tr>
<td></td>
<td>(4.0746)</td>
<td>(0.9823)</td>
<td>(0.0201)</td>
</tr>
<tr>
<td></td>
<td>[1.0548]</td>
<td>[-1.7650]</td>
<td>[47.1041]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>bondmkt</th>
<th>real rate</th>
<th>credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.0476</td>
<td>0.1577</td>
<td>0.9102</td>
</tr>
<tr>
<td>F-statistic</td>
<td>3.6828</td>
<td>13.7886</td>
<td>747.0744</td>
</tr>
<tr>
<td></td>
<td>bondmkt</td>
<td>real rate</td>
<td>credit spread</td>
</tr>
<tr>
<td>----------------</td>
<td>---------</td>
<td>-----------</td>
<td>---------------</td>
</tr>
<tr>
<td>bondmkt (-1)</td>
<td>0.1552</td>
<td>0.0071</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.0662)</td>
<td>(0.0160)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td></td>
<td>[2.3433]</td>
<td>[0.4457]</td>
<td>[.0625]</td>
</tr>
<tr>
<td></td>
<td>0.4509</td>
<td>0.3640</td>
<td>0.0005</td>
</tr>
<tr>
<td>real rate (-1)</td>
<td>0.2646</td>
<td>0.0638</td>
<td>0.0013</td>
</tr>
<tr>
<td></td>
<td>[1.7038]</td>
<td>[5.7053]</td>
<td>[0.38055]</td>
</tr>
<tr>
<td></td>
<td>3.7603</td>
<td>−3.8559</td>
<td>0.9489</td>
</tr>
<tr>
<td>credit spread (-1)</td>
<td>4.2107</td>
<td>1.0152</td>
<td>0.0208</td>
</tr>
<tr>
<td></td>
<td>[0.8930]</td>
<td>[−1.8281]</td>
<td>[45.5790]</td>
</tr>
<tr>
<td></td>
<td>0.4274</td>
<td>.0969</td>
<td>−0.0004</td>
</tr>
<tr>
<td>dividend (-1)</td>
<td>.8225</td>
<td>0.1983</td>
<td>0.0041</td>
</tr>
<tr>
<td></td>
<td>[0.5196]</td>
<td>[0.4887]</td>
<td>[−0.0954]</td>
</tr>
</tbody>
</table>

|                | $R^2$   | 0.0488    | 0.1586        | 0.9102   | 0.1707   |
|                | F-statistic | 2.8204 | 10.3565 | 557.7959 | 11.3222 |

Table 6: VAR — Sample 10/1987–09/2006 — All variables have been demeaned and a constant term has been included — bondmkt is the excess aggregate bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and the growth rate in the CPI; credit spread is the difference between Moody’s seasoned Baa and Aaa corporate bond yields; and the dividend yield is the difference between vwretd and vwretx from CRSP. The credit spread data and the CPI data is from the FRED database. Figures correspond to OLS estimates, standard errors are inside parenthesis and t-statistics are in brackets.
Table 7: Time series—Sample 01/1988–09/2006 —Bond Market, Real Rate and Inflation News
Factors—The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium —bondmkt is the excess aggregate bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and the growth rate in the CPI; credit spread is the difference between Moody’s seasoned Baa and Aaa corporate bond yields. The credit spread data and the CPI data is from the FRED database. Inflation news were obtained as a residual. The corporate bond portfolios are bond market index portfolios of different default categories from Lehman Brothers.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BAA</th>
<th>BA</th>
<th>B</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>bondmkt news</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>-1.0333</td>
<td>-1.0853</td>
<td>-1.0929</td>
<td>-1.1836</td>
<td>-0.5313</td>
<td>-0.6318</td>
<td>-2.7219</td>
</tr>
<tr>
<td>OLS t-stat</td>
<td>-13.4720</td>
<td>-13.3988</td>
<td>-12.1164</td>
<td>-8.8922</td>
<td>-1.2078</td>
<td>-0.9295</td>
<td>-1.4386</td>
</tr>
<tr>
<td>inflation news</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>-1.0546</td>
<td>-1.1225</td>
<td>-1.071</td>
<td>-1.0813</td>
<td>-0.5700</td>
<td>-0.4203</td>
<td>-0.6284</td>
</tr>
<tr>
<td>GMM t-stat</td>
<td>-34.0194</td>
<td>-36.2097</td>
<td>-29.8329</td>
<td>-22.6213</td>
<td>-4.3914</td>
<td>-2.0984</td>
<td>-1.5146</td>
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<tr>
<td>real rate news</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>-1.0424</td>
<td>-1.1046</td>
<td>-0.8774</td>
<td>-0.5098</td>
<td>1.3080</td>
<td>3.2037</td>
<td>5.9428</td>
</tr>
<tr>
<td>OLS t-stat</td>
<td>-10.2096</td>
<td>-10.2468</td>
<td>-7.3056</td>
<td>-2.8786</td>
<td>2.2325</td>
<td>3.5392</td>
<td>2.3585</td>
</tr>
</tbody>
</table>

Table 8: Fama MacBeth Cross-Sectional Regressions—The news components were obtained from the residuals and the companion matrix of the VAR in Table 5. The corporate bond portfolios are bond market index portfolios of different default categories from Lehman Brothers.

<table>
<thead>
<tr>
<th></th>
<th>bondmkt news</th>
<th>inflation news</th>
<th>real rate news</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.1181</td>
<td>-0.5994</td>
<td>0.1797</td>
</tr>
<tr>
<td>Fama–MacBeth t-stat</td>
<td>0.5602</td>
<td>-2.5323</td>
<td>2.9507</td>
</tr>
<tr>
<td>Fama–MacBeth t-stat</td>
<td>0.6122</td>
<td>-3.2736</td>
<td>2.4889</td>
</tr>
<tr>
<td>corrected for autocorrelation</td>
<td>0.4383</td>
<td>-2.0332</td>
<td>2.3428</td>
</tr>
<tr>
<td>Shanken-corrected t-stat</td>
<td>11.5648</td>
<td>63.18%</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Fama MacBeth Cross-Sectional Regressions—The news components were obtained from the residuals and the companion matrix of the VAR in Table 5. The corporate bond portfolios are bond market index portfolios of different default categories from Lehman Brothers.
Table 9: Fama MacBeth Cross-Sectional Regressions—Sample 01/1988–09/2006—Excess bond returns—Intermediate Maturity (less than ten years)—The news components were obtained from the residuals and the companion matrix of the VAR in Table 6. The corporate bond portfolios are bond market index portfolios of different default categories from Lehman Brothers.

<table>
<thead>
<tr>
<th></th>
<th>bondmkt news</th>
<th>inflation news</th>
<th>real rate news</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.1218</td>
<td>-0.5939</td>
<td>0.1724</td>
</tr>
<tr>
<td>Fama–MacBeth t-stat</td>
<td>0.5729</td>
<td>-2.5732</td>
<td>2.9171</td>
</tr>
<tr>
<td>Fama–MacBeth t-stat</td>
<td>0.6262</td>
<td>-3.3142</td>
<td>2.3584</td>
</tr>
<tr>
<td>corrected for</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>autocorrelation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shanken-corrected t-stat</td>
<td>0.4533</td>
<td>-2.0912</td>
<td>2.3424</td>
</tr>
<tr>
<td>Fama–MacBeth chi-squared statistic</td>
<td>11.3823</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>63.77</td>
<td></td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>Manufacturing</th>
<th>Transport</th>
<th>Consumer</th>
<th>Energy</th>
<th>Service</th>
<th>Other</th>
<th>Utility</th>
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</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.6263</td>
<td>0.6945</td>
<td>0.6766</td>
<td>0.6477</td>
<td>0.6665</td>
<td>0.6585</td>
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<tr>
<td>Median</td>
<td>0.6220</td>
<td>0.7746</td>
<td>0.7817</td>
<td>0.7210</td>
<td>0.7385</td>
<td>0.6316</td>
<td>0.6404</td>
</tr>
<tr>
<td>Maximum</td>
<td>5.1549</td>
<td>5.5990</td>
<td>5.1849</td>
<td>5.3689</td>
<td>5.3889</td>
<td>10.1852</td>
<td>5.7564</td>
</tr>
<tr>
<td>Std Dev.</td>
<td>1.3966</td>
<td>1.6322</td>
<td>1.5108</td>
<td>1.5394</td>
<td>1.4836</td>
<td>1.8005</td>
<td>1.6048</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.0844</td>
<td>-0.3357</td>
<td>-0.2960</td>
<td>-0.3649</td>
<td>-0.1644</td>
<td>0.3223</td>
<td>-0.1861</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.3349</td>
<td>0.6883</td>
<td>0.6592</td>
<td>0.8725</td>
<td>0.6616</td>
<td>4.3219</td>
<td>0.6226</td>
</tr>
</tbody>
</table>
Table 11: Pairwise correlation matrix — Sample 01/1990–09/2006 — Corporate Bond Indices based on Industry: Citigroup — Manufacturing includes: aerospace/defence, automotive manufacturers, building products, chemicals, conglomerate, electronics, information/data technology, machinery, metals/mining, paper/forest products, textiles/apparel/shoes, vehicle parts, manufacturing–other — Service includes: cable/media, gaming/lodging/leisure, healthcare supply, pharmaceuticals, publishing, restaurants, food/drugs, retail stores–other, service–other — Transportation includes: airlines, railroads, transportation–other — Consumer includes: beverage/bottling, consumer products, food processors, tobacco — Utility includes: electric, power, gas-local distribution, telecommunications, utility–other — Energy includes: gas-pipelines, oil and gas, oilfield machinery and services.

Table 12: Fama MacBeth Cross-sectional Regressions—The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium — bondmkt is the excess aggregate bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and the growth rate in the CPI; credit spread is the difference between Moody’s seasoned Baa and Aaa corporate bond yields. The spread premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual. Our test assets are seven industry corporate bond portfolios obtained from Citigroup and seven corporate bond market index portfolios of different default categories from Lehman Brothers.
<table>
<thead>
<tr>
<th></th>
<th>bondmkt news</th>
<th>inflation news</th>
<th>real rate news</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimate</strong></td>
<td>0.1505</td>
<td>-0.5670</td>
<td>0.0403</td>
</tr>
<tr>
<td>Fama–MacBeth t-stat</td>
<td>0.7769</td>
<td>-2.0536</td>
<td>1.2476</td>
</tr>
<tr>
<td>Fama-MacBeth t-stat</td>
<td>0.9044</td>
<td>-2.5244</td>
<td>1.6652</td>
</tr>
<tr>
<td>corrected for autocorrelation</td>
<td>0.6083</td>
<td>-1.6925</td>
<td>0.9710</td>
</tr>
<tr>
<td>Shanken-corrected t-stat</td>
<td>9.5008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fama-MacBeth chi-squared statistic</td>
<td>9.4877</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PANEL B: Sub-sample Period 01/1997-09/2006**

<table>
<thead>
<tr>
<th></th>
<th>bondmkt news</th>
<th>inflation news</th>
<th>real rate news</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimate</strong></td>
<td>-0.0343</td>
<td>-0.3979</td>
<td>0.1999</td>
</tr>
<tr>
<td>Fama–MacBeth t-stat</td>
<td>-0.1346</td>
<td>-1.5198</td>
<td>1.1751</td>
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<tr>
<td>Fama-MacBeth t-stat</td>
<td>-0.1218</td>
<td>-1.3936</td>
<td>1.0684</td>
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<td>corrected for autocorrelation</td>
<td>0.6083</td>
<td>-1.6982</td>
<td>1.2181</td>
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<tr>
<td>Shanken-corrected t-stat</td>
<td>9.4877</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fama-MacBeth chi-squared statistic</td>
<td>9.4877</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Different Sample Periods: Cross-Section—The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium — bondmkt is the excess bond market return measured as the Lehman Brothers monthly US aggregate bond return in excess of the three months Treasury bill; real rate is the monthly real short-term interest rate, i.e. the difference between the risk-free rate and the growth rate in the CPI; credit premium is the difference between Moody’s seasoned Baa and Aaa corporate bond yields. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual. The corporate bond portfolios are bond market index portfolios of different default categories from Lehman Brothers.

A P P E N D I X

This Appendix provides details of the bond return decomposition, the factor model and the VAR methodology used in the paper. It collects in one place, and draws heavily on, previous work by Campbell (1993, 1996), Campbell and Ammer (1993), Campbell and Vuolteenaho (2004), Perraudin and Taylor (2003) and Shiller and Beltratti (1992).

A. Bond Decomposition

There are two versions of the variance decomposition for bonds in the literature. The first uses a zero coupon bond (see Campbell and Ammer, 1993) and the second, a consol bond (see Shiller and Beltratti, 1992, and Engsted and Tanggaard, 2001).
Consol Bond

Campbell (1993) uses a log-linear approximation to the return on a real consol bond that pays one unit of consumption good each period and with no maturity date. Here, we follow Shiller and Beltratti (1992) and Engsted and Tanggaard (2001), and use a log-linear version of the present value of a nominal consol bond or a perpetuity. We denote the coupon by \( C \) and the price \( P_{bt} \), then the log one period gross return from \( t \) to \( t + 1 \) is given by

\[
r_{bt+1} = \ln \left( \frac{C + P_{bt+1}}{P_{bt}} \right) = \ln (C + \exp(p_{bt+1})) - p_{bt}
\]  

(A1)

We now take a first-order Taylor expansion around the mean of \( \ln (C + \exp(p_{bt+1})) \), i.e.

\[
\ln (C + \exp(p_{bt+1})) \\
\approx \ln (C + \exp(E_t(p_{bt+1}))) - E_t(p_{bt+1}) \frac{\exp(E_t(p_{bt+1}))}{C + \exp(E_t(p_{bt+1}))}
\]

to get

\[
r_{bt+1} = \ln (C + \exp(E_t(p_{bt+1}))) - E_t(p_{bt+1}) \frac{\exp(E_t(p_{bt+1}))}{C + \exp(E_t(p_{bt+1}))} \\
\kappa_b \\
+ \frac{\exp(E_t(p_{bt+1}))}{C + \exp(E_t(p_{bt+1}))} p_{bt+1} - p_{bt}
\]

We simplify notation by rewriting the above equation as

\[
r_{bt+1} = \kappa_b + \rho_b p_{bt+1} - p_{bt}
\]

in which \( \kappa_b = \ln (C + \exp(E_t(p_{bt+1}))) - E_t(p_{bt+1}) \frac{\exp(E_t(p_{bt+1}))}{C + \exp(E_t(p_{bt+1}))} \) is a constant arising from the linearization. The term \( \rho_b \), given by

\[
\rho_b \equiv \frac{\exp(E_t(p_{bt+1}))}{C + \exp(E_t(p_{bt+1}))} \approx \frac{E(P_{bt+1})}{C + E(P_{bt+1})} \approx \frac{E(P_{bt+1})}{C + E(P_{bt+1})} = \frac{1}{E(R_{bt+1})}
\]

is approximately equal to \( R_{bt+1} \equiv \frac{C + P_{bt+1}}{P_{bt}} \)

Now, we note that

\[
r_{bt} = \kappa_b + \rho_b - p_{bt+1} - p_{bt}
\]  

(A2)
is a difference equation in the log bond price $p_{bt}$. We can then solve equation (A2) forward, impose the usual transversality condition and take conditional expectations at time $t$ to get

$$p_{bt} = -E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j}$$  \hspace{1cm} (A3)$$

Substituting equation (A3) back into

$$r_{b,t+1} = \kappa_b + \rho_b p_{b,t+1} - p_{b,t} p_{bt}$$

leads to

$$r_{b,t+1} = \kappa_b + \rho_b \left( -E_{t+1} \sum_{j=0}^{\infty} \rho_b^j r_{b,t+2+j} \right) - \left( -E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \right)$$

$$r_{b,t+1} = \kappa_b - \rho_b \left( E_{t+1} \sum_{j=0}^{\infty} \rho_b^j r_{b,t+2+j} \right) + \left( E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \right)$$

$$r_{b,t+1} = \kappa_b - \rho_b \left( E_{t+1} \sum_{j=0}^{\infty} \rho_b^j r_{b,t+2+j} \right) + \left( E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \right)$$

\[ \therefore \ (E_{t+1} - E_t) r_{b,t+1} = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_b^j r_{b,t+1+j} \]  \hspace{1cm} (A4)

If we assume that $\rho_b = \rho$ — in other words, that the linearization constant for bonds is approximately equal to the linearization coefficient for the intertemporal budget constraint— then we get

$$ (E_{t+1} - E_t) r_{b,t+1} = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_b^j r_{b,t+1+j} \]  \hspace{1cm} (A4)$$

To obtain excess returns, we add and subtract the risk-free rate and use the fact that \((E_{t+1} - E_t) r_{f,t} = 0\) to get the decomposition for innovations in the excess bond returns:

$$ (E_{t+1} - E_t) (r_{b,t+1} - r_{f,t+1}) = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_b^j (r_{b,t+1} - r_{f,t+1+j})$$  \hspace{1cm} (A5)$$

$$- (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_b^j r_{f,t+1+j}$$
Next, we can write the nominal risk-free rate as

\[ r_{f,t+1} = r_{r,t+1} + \pi_{t+1} \]

in which \( r_{r,t+1} \) and \( \pi_{t+1} \) are, respectively, the real interest rate and inflation rate. Then the last term in equation (A12) can be written as

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{rt+1+j} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} \quad (A6)
\]

Thus we can write

\[
(E_{t+1} - E_t) (r_{b,t+1} - r_{f,t+1}) = -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1+j} - r_{f,t+1+j})
\]

\[
- (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} \quad (A7)
\]

\[
= -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1})
\]

\[
- (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j}
\]

Now, for ease of exposition, we use the notation in Campbell and Ammer (1993) for ‘innovations’ and define \( \tilde{x}_{b,t+1} = (E_{t+1} - E_t) (r_{b,t+1} - r_{f,t+1}) \) as the innovation in the log excess one-period return on a consol bond from \( t \) to \( t+1 \), \( \tilde{x}_{x,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1+j} - r_{f,t+1+j}) \) as the innovation in the future log excess one-period return on a consol bond held from \( t \) to \( t+1 \), \( \tilde{x}_{r,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} \) as the innovation in the log excess one-period real return, and \( \tilde{x}_{\pi,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} \) as the innovation in the log excess one-period inflation.

Substituting in the above expression, we get

\[
\tilde{x}_{b,t+1} = -\tilde{x}_{x,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{x,t+1} \quad (A8)
\]
Bond Return Decomposition with Default Risk

As mentioned above, this version is based on that of Perraudin and Taylor (1993). Consider a default-free pure discount bond with price $P_t$ and maturity of $T$ periods. The gross return of this bond, $R_{t+1}$, is implicitly defined by

$$P_t = R_{t+1}^{-1} P_{t+1}$$  \hfill (A9)

If $P_t^*$ is the price of a defaultable pure discount bond (conditional on no default until date $t$), then its gross return, $R_t^*$ can be defined as

$$P_t^* = R_{t-1}^{*^{-1}} (P_{t+1}^* - 1_{t+1} (P_{t+1}^* - \gamma 100))$$

in which $1_{t+1}$ is a dummy or indicator variable that is equal to one in the event of default up to, and including, date $t$, and is zero otherwise, and $\gamma$ is the recovery rate in the event of default. Perraudin and Taylor (1993) define the loss rate on default as $\alpha_{t+1} = \frac{(P_{t+1}^* - \gamma 100)}{P_{t+1}^*}$. Then we can write the defaultable bond price as

$$P_t^* = R_{t-1}^{*^{-1}} (P_{t+1}^* - 1_{t+1} \alpha_{t+1} P_{t+1}^*)$$  \hfill (A10)

Next, dividing equation (A10) by equation (A9) we obtain:

$$\frac{P_t^*}{P_t} = R_{t+1} R_{t-1}^{*^{-1}} \left( \frac{P_{t+1}^* - 1_{t+1} \alpha_{t+1} P_{t+1}^*}{P_{t+1}^*} \right)$$

Defining $A_{t+1} = 1 - 1_{t+1} \alpha_{t+1}$ and taking logs, we get

$$p_t^* - p_t = r_{t+1}^* - r_{t+1} + \log \frac{P_{t+1}^* A_{t+1}}{P_{t+1}^*}$$

$$= r_{t+1}^* - r_{t+1} + p_{t+1}^* - p_{t+1} + a_{t+1}$$

This can be iterated forward and, with some more algebra, we can write

$$p_t^* - p_t = E_t \left[ \sum_{i=1}^{T} (\alpha_{t+i} - r_{t+i}^e) \right]$$
Hence the log price difference is the expectation of future recovery and future excess returns. While this expression is for a coupon bond with a time to maturity $T$, it shows that we could incorporate the notion of default risk (in this case, defined as the loss rate on default) into the return decomposition for a bond. Our bond decomposition in Section 3.1 could therefore be augmented, in principle, to include this extra factor. As indicated in the paper, however, there are costs to this and we let the innovations in expected future bond returns capture the risk from default.

### B. Expression for log SDF

We follow Campbell (1993, 1996) and use the Epstein–Zin (1989) utility function, defined recursively, for an infinitely lived representative agent as

$$U_t = \left[ (1 - \delta) C_t^{1 - \gamma} + \delta \left( E_t U_{t+1}^{1 - \gamma} \right)^{\frac{1}{\delta}} \right]^{\frac{\theta}{1 - \gamma}}$$  \hfill (A11)

in which $\theta = \frac{1 - \gamma}{1 - \psi}$, $\psi$ is the elasticity of intertemporal substitution, $\gamma$ is the coefficient of relative risk aversion, $\delta$ is a time discount factor and $C_t$ is consumption. We assume that our investor can invest only in the bond market or in bond market mutual funds, i.e. the aggregate bond market portfolio. The Euler equation for asset $i$, following Epstein and Zin (1989, 1991), has an associated pricing equation in simple returns given by

$$1 = E_t \left\{ \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right\}^\theta \left\{ \left( \frac{1}{R_{B,t+1}} \right) \right\}^{1 - \theta} R_{i,t+1}$$ \hfill (A12)

in which $R_{B,t+1}$ is the return on the aggregate bond market index. The corresponding SDF is

$$M_{t+1} = \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left( \frac{1}{R_{B,t+1}} \right)^{1 - \theta}$$ \hfill (A13)

and the log of the SDF is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{B,t+1}$$ \hfill (A14)

Adding and subtracting both $\frac{\theta}{\psi} E_t (\Delta c_{t+1}) \Delta c_{t+1}$ and $(1 - \theta) E_t (r_{b,t+1})$ from the above
equality leads to

\[ m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - \frac{\theta}{\psi} E_t (\Delta c_{t+1}) + \frac{\theta}{\psi} E_t (\Delta c_{t+1}) \]

\[ - (1 - \theta) r_{B,t+1} - (1 - \theta) E_t (r_{B,t+1}) + (1 - \theta) E_t (r_{B,t+1}) \]

Regrouping terms leads to

\[ m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} E_t (\Delta c_{t+1}) - (1 - \theta) E_t (r_{B,t+1}) \]

\[ = E_t (m_{t+1}) \]

\[ - \frac{\theta}{\psi} (\Delta c_{t+1} - E_t (\Delta c_{t+1})) - (1 - \theta) (r_{B,t+1} - E_t (r_{B,t+1})) \]

The above expression can be written as

\[ m_{t+1} = E_t (m_{t+1}) - \frac{\theta}{\psi} (\Delta c_{t+1} - E_t (\Delta c_{t+1})) - (1 - \theta) (r_{B,t+1} - E_t (r_{B,t+1})) \quad (A15) \]

We know that

\[ \Delta c_{t+1} - E_t (\Delta c_{t+1}) = \log \left( \frac{c_{t+1}}{c_t} \right) - E_t \left( \log \left( \frac{c_{t+1}}{c_t} \right) \right) \]

\[ = c_{t+1} - c_t - E_t c_{t+1} - E_t c_t \]

\[ = c_{t+1} - E_t c_{t+1} - c_t - E_t c_t \]

\[ \Rightarrow \Delta c_{t+1} - E_t (\Delta c_{t+1}) = c_{t+1} - E_t c_{t+1} \]

Hence, we can write the log SDF as

\[ m_{t+1} = E_t (m_{t+1}) - \frac{\theta}{\psi} (c_{t+1} - E_t (c_{t+1})) - (1 - \theta) (r_{B,t+1} - E_t (r_{B,t+1})) \quad (A16) \]

Next, we use the following result from Campbell (1993) (equation 21, p. 494)

\[ c_{t+1} - E_t c_{t+1} = r_{b,t+1} - E_t (r_{b,t+1}) + (1 - \psi) (E_t c_{t+1} - E_t (r_{B,t+1})) \sum_{j=1}^{\infty} \rho^j r_{B,t+1+j} \quad (A17) \]
to substitute out consumption from equation (A16). The log SDF is then

\[
m_{t+1} = E_t (m_{t+1}) - \frac{\theta}{\psi} \left( r_{B,t+1} - E_t (r_{B,t+1}) + (1 - \psi) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{B,t+1+j} \right) \\
- (1 - \theta) (r_{B,t+1} - E_t (r_{B,t+1})) \\
\therefore \theta = \frac{1 - \gamma}{1 - \frac{r}{\psi}}, \therefore \frac{\theta}{\psi} = \frac{1 - \gamma}{(1 - \gamma)} \\
\therefore m_{t+1} = E_t (m_{t+1}) - \left( \frac{\theta}{\psi} + (1 - \theta) \right) (r_{B,t+1} - E_t (r_{B,t+1})) \\
- (1 - \psi) \left( \frac{1 - \gamma}{\psi - 1} \right) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{B,t+1+j} \\
\therefore \theta = \frac{1 - \gamma}{1 - \frac{r}{\psi}}, \therefore \frac{\theta}{\psi} = \frac{1 - \gamma}{(1 - \gamma)} \\
\therefore m_{t+1} = E_t (m_{t+1}) - \gamma (r_{B,t+1} - E_t (r_{B,t+1})) \\
+ (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{B,t+1+j}
\]

Adding and subtracting \( r_f \) from the right-hand side gives

\[
m_{t+1} = E_t (m_{t+1}) - \gamma (r_{B,t+1} - E_t (r_{B,t+1})) + (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{B,t+1+j} + r_f - r_f \\
\therefore m_{t+1} = E_t (m_{t+1}) - \gamma (E_{t+1} - E_t) (r_{B,t+1} - r_{f,t+1}) - \gamma (E_{t+1} - E_t) r_{f,t+1} \\
+ (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{B,t+1+j} - r_{f,t+1+j}) + (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} \\
\therefore (E_{t+1} - E_t) r_{f,t+1} = 0 \\
m_{t+1} = E_t (m_{t+1}) - \gamma (E_{t+1} - E_t) (r_{B,t+1} - r_{f,t+1}) \\
+ (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{B,t+1+j} - r_{f,t+1+j}) + (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}
\]

We substitute the following expression

\[
(E_{t+1} - E_t) (r_{B,t+1} - r_f) = -\tilde{x}_{x,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{x,t+1}
\]
into $m_{t+1}$, i.e.

$$m_{t+1} = E_t (m_{t+1}) - \gamma (E_{t+1} - E_t) (r_{B,t+1} - r_{f,t+1}) + (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{B,t+1+j} - r_{f,t+1+j})$$

$$+ (1 - \gamma) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}$$

$$\therefore m_{t+1} = E_t (m_{t+1}) - \gamma \left(-\tilde{x}_{\pi,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{x,t+1}\right) + (1 - \gamma) \left(\tilde{x}_{x,t+1}\right)$$

$$+ (1 - \gamma) \left(\tilde{x}_{\pi,t+1} + \tilde{x}_{r,t+1}\right)$$

$$\therefore m_{t+1} = E_t (m_{t+1}) + \tilde{x}_{\pi,t+1} + \tilde{x}_{r,t+1} + \tilde{x}_{x,t+1}$$

Notice here that, unlike in the case of the stock market, the bond market decomposition does not have any free parameter, i.e. $\gamma$ (see, for example, Campbell and Vuolteenaho, 2004).

C. The Expected Return Beta Model with Bond Market News Components

Next, we use a standard result from Cochrane (2005). Given

$$E_t (M_{t+1}R_{t+1}) = 1$$

and assuming that the log of the SDF $m_{t+1}$ is a linear function of the $K$ risk factors $f_{t+1}$

$$m_{t+1} = a + b' f_{t+1} \quad \text{(A18)}$$

The unconditional model in expected return form for returns on a bond or bond portfolio in logs is then

$$E (r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = b' \text{cov} (r_{t+1}; f_{t+1}) \quad \text{(A19)}$$

Equation (A19) can be written in a return-beta form:

$$E (r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \lambda' \beta_i \quad \text{(A20)}$$
in which \( \beta_i = [\text{Var}(f_{t+1})]^{-1} \text{Cov}(r_{t+1}, f_{t+1}) = \) vector with the \( K \) betas for asset \( i \),
\( \lambda = -\text{Var}(f_{t+1}) \) and \( b = \) vector of factor risk prices.

Equation (A20) can also be written in vector notation as follows:

\[
E(r_{i,t+1} - r_{f,t+1}1_N) + \frac{1}{2} \text{diag}(\text{Var}(r_{i,t+1})) = \beta \lambda \tag{A21}
\]

in which \( \beta = \text{Cov}(r_{t+1}, f_{t+1}) [\text{Var}(f_{t+1})]^{-1} = N \times K \) factor beta matrix with row \( i \) of factor loadings for asset \( i \) and \( 1 \) is a \( N \)-dimension vector of ones.

Now, we define

\[
f_{t+1} = \left( \tilde{x}_{\pi,t+1}, \tilde{x}_{r,t+1}, \tilde{x}_{x,t+1} \right)'
\]

\[
b = (1, 1, 1)
\]

Because

\[
m_{t+1} = E_t(m_{t+1}) + \tilde{x}_{\pi,t+1} + \tilde{x}_{r,t+1} + \tilde{x}_{x,t+1} \tag{A22}
\]

we get

\[
E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = -\sigma_{i,x} - \sigma_{i,x} - \sigma_{i,\pi} \tag{A23}
\]

in which \( \sigma_{i,x} = \text{Cov}(r_{i,t}, \tilde{x}_{x,t+1}) = \) covariance of asset return with bond excess return news, \( \sigma_{i,r} = \text{Cov}(r_{i,t}, \tilde{x}_{r,t+1}) = \) covariance of asset return with real interest rate news, \( \sigma_{i,\pi} = \text{Cov}(r_{i,t}, \tilde{x}_{\pi,t+1}) = \) covariance of asset return with inflation news.

We can write the equation above in terms of factor beta’s risk prices as

\[
E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = -\sigma_{x,x}^2 \beta_{i,x} - \sigma_{r,r}^2 \beta_{i,r} - \sigma_{\pi,\pi}^2 \beta_{i,\pi} \tag{A24}
\]

in which \( \sigma_{x,x}^2, \sigma_{r,r}^2, \) and \( \sigma_{\pi,\pi}^2 \) are the variances of \( \tilde{x}_{x,t+1}, \tilde{x}_{r,t+1} \) and \( \tilde{x}_{\pi,t+1} \). The risk prices for betas can be derived by defining \( \lambda = (\lambda_x, \lambda_r, \lambda_\pi)^T = \sigma_f b \), in which \( \sigma_f \) is a diagonal matrix with the factor variances along its main diagonal. In addition, we can rewrite the model in an expected return-beta representation, i.e.

\[
E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \lambda^T \beta_i = \lambda_x \beta_{i,x} + \lambda_r \beta_{i,r} + \lambda_\pi \beta_{i,\pi} \tag{A25}
\]
in which \( \lambda = (\lambda_x, \lambda_r, \lambda_\pi)^T = -Var(f_{t+1})b \) denotes the vector of factor risk prices and 
\( \beta_i = Var(f_{t+1})^{-1} Cov(r_{i,t+1}, f_{t+1}) \) represents the \((3 \times 1)\) vector of multiple regression betas 
for asset \( i \). The \( \lambda_s \) represent the risk prices of multiple regression beta risk for each of the factors. Finally, we take unconditional expectations and rewrite the left-hand side in simple 
expected return form, to obtain our three-beta model for the bond market

\[
E(r_{i,t+1} - r_{f,t+1}) = \lambda_x \beta_{i,x} + \lambda_r \beta_{i,r} + \lambda_\pi \beta_{i,\pi}
\]

(A26)

**VAR Estimation and Extraction of News Components**

We can now use the VAR approach of Campbell and Shiller (1988) to extract the components 
of equation (A15) from the data. We specify a VAR with excess bond returns, the real interest 
rate and other variables that help to forecast returns and real rates. Suppose we use the 
following VAR in which the vector \( z_t \) is specified as follows:

\[
z_t = (x_{b,t}, r_t, sprd_t, dy_t)
\]

(A27)

Here, \( x_{b,t}, r_t, sprd_t \) and \( dy_t \) are the excess return on the bond market, the real interest 
rate, the Baa–Aaa credit yield spread and the dividend yield on the CRSP VW index.

We need a few results before we can get compact expressions for the ‘news’ components 
from the VAR. We know that we can write a first-order VAR (in companion form for higher 
lags if required) as

\[
z_t = Az_{t-1} + w_t
\]

(A28)

in which \( A \) is the VAR parameter matrix and \( w_t \) is the vector of error terms. The 
difference equation in (A28) can be solved by recursive substitution as in Hamilton (1994),
e.g.

\[
\begin{align*}
  z_t &= Az_{t-1} + w_t \\ 
  z_{t+1} &= Az_t + w_{t+1} \\
  z_{t+1+1} &= Az_{t+1} + w_{t+2} \\
  & \vdots \\
  z_{t+1+1} &= A(Az_t + w_{t+1}) + w_{t+2} \\
  z_{t+1+1} &= A^2z_t + Aw_{t+1} + w_{t+2} \\
  \therefore z_{t+1+j} &= A^{j+1}z_t + \text{(terms in } w) \\
  \therefore E_t (z_{t+1+j}) &= A^{j+1}E_t (z_t) \\
  \therefore E_t (z_{t+1+j}) &= A^{j+1}z_t
\end{align*}
\]

Now we need expressions for terms of the type

\[
(E_{t+1} - E_t) (z_{t+1+j})
\]

So we can now expand the above expression

\[
\begin{align*}
  (E_{t+1} - E_t) (z_{t+1+j}) \\
  &= E_{t+1} (z_{t+1+j}) - E_t (z_{t+1+j}) \\
  \text{using } E_t (z_{t+1+j}) &= A^{j+1}z_t \text{ and} \\
  E_{t+1} (z_{t+1+j})&= A^j z_{t+1+j} \\
  \therefore (E_{t+1} - E_t) (z_{t+1+j}) &= A^j z_{t+1+j} - A^{j+1}z_t \\
  \therefore (E_{t+1} - E_t) (z_{t+1+j}) &= A^j (z_{t+1} - Az_t) \\
  \text{But, } z_{t+1} &= Az_t + w_{t+1} \\
  \therefore (E_{t+1} - E_t) (z_{t+1+j}) &= A^j w_{t+1}
\end{align*}
\]
Now we need to extract from the VAR expressions for the following news components:

\[
(E_{t+1} - E_t) (r_{b,t+1} - r_{f,t+1}) = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j}) \\
- (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} \\
- (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j}
\]

Because our state vector includes the excess bond market returns as its first element and real rates as its second element, we can define row selection vectors \( g_1 = (1, 0, 0, 0) \) and \( g_2 = (0, 1, 0, 0) \) that will pick out the first and second components that we need from the VAR. For example:

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j}) = g_1' A^j w_{t+1}^{\prime} \text{ (A29)}
\]

Similarly:

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} = g_2' \rho A (I - \rho A)^{-1} w_{t+1}^{\prime} \text{ (A30)}
\]

We can obtain the inflation news components, \((E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j}\), as a residual, because we know the other three components (note that \((E_{t+1} - E_t) (r_{b,t+1} - r_{f,t+1}) = g_1 w_{t+1}\) in the dynamic accounting identity)

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} = - g_1 w_{t+1} - g_1' \rho A (I - \rho A)^{-1} w_{t+1} - g_2' \rho A (I - \rho A)^{-1} w_{t+1} \text{ (A31)}
\]

In the case of bonds, we can avoid specifying the process for inflation as a state variable as long as we use the excess bond returns and the real rate in the VAR estimation. We can
then obtain the inflation component of the decomposition as the residual term, using the identity in equation (A8).

D. GMM estimation

We re-estimate our model using a GMM procedure as in Cochrane (2005). Using this methodology avoids the problem of generated regressors that arises in the two-step Fama-McBeth procedure.

In our case, the GMM system has a set of moments given by:

$$
g_T(\Theta) = \frac{1}{T} \begin{bmatrix}
\sum_{t=1}^{T} (R_t^e - \alpha - \beta f_t) \\
\sum_{t=1}^{T} (R_t^e - \alpha - \beta f_t) \otimes f_t \\
\sum_{t=1}^{T} (R_t^e - \beta \lambda)
\end{bmatrix} = \begin{bmatrix}
0_{(N \times 1)} \\
0_{(3N \times 1)} \\
0_{(N \times 1)}
\end{bmatrix}
$$

where $R_t^e (N \times 1)$ is the vector of excess returns for the $N$ test assets; $\alpha (N \times 1)$ is the vector of constants for the time series regressions; $\beta (N \times 3)$ is the matrix of three factor loadings; $f_t (3 \times 1)$ is the news’ vector used to price assets; $\lambda (3 \times 1)$ is the vector of beta risk prices; $\otimes$ denotes the Kronecker product and $0$ denotes conformable vectors of zeros.

The vector of parameters that we want to estimate using this GMM system is

$$
\Theta' = \begin{bmatrix}
\alpha' & \beta^* & \lambda'
\end{bmatrix}
$$

in which $\beta^* \equiv vec(\beta')'$ and $vec$ is the operator that enables us to stack the factor loadings for each of the $N$ assets into a column vector.

The matrix

$$
a = \begin{bmatrix}
I_N \otimes I_{K+1} & 0_{(N(K+1) \times N)} \\
0_{(K \times N(K+1))} & \beta'
\end{bmatrix}
$$
chooses which moments are set to zero in the first-order condition, \( a_g T (\hat{\Theta}) = 0 \), where \( I_N \) denotes an identity matrix of order \( N \). The matrix

\[
d = \frac{\partial g_T(\Theta)}{\partial \theta} = \begin{bmatrix}
-I_N & -I_N \otimes \left( \frac{1}{T} \sum_{t=1}^{T} f_t \right) & 0_{(N \times 3)} \\
-I_N \otimes \left( \frac{1}{T} \sum_{t=1}^{T} f_t \right) & -I_N \otimes \left( \frac{1}{T} \sum_{t=1}^{T} f_t f_t' \right) & 0_{(NK \times 3)} \\
0_{(N \times N)} & -I_N \otimes \lambda & -\beta \\
\end{bmatrix}
\]

is the sensitivity of the moment conditions to the parameters. The variance-covariance matrix of the moments, \( S \), has the following form

\[
S = \sum_{j=-\infty}^{\infty} E \begin{bmatrix}
(R_t^f - \alpha - \beta f_t) & (R_{t-j}^f - \alpha - \beta f_{t-j}) \\
(R_t^f - \alpha - \beta f_t) \otimes f_t & (R_{t-j}^f - \alpha - \beta f_{t-j}) \otimes f_{t-j} \\
(R_t^f - \beta \lambda) & (R_{t-j}^f - \beta \lambda) \\
\end{bmatrix}.
\]

\[
= \sum_{j=-\infty}^{\infty} E \begin{bmatrix}
\epsilon_t & \epsilon_{t-j} \\
\epsilon_t \otimes f_t & \epsilon_{t-j} \otimes f_{t-j} \\
\beta (f_t - E(f_t)) + \epsilon_t & \beta (f_{t-j} - E(f_t)) + \epsilon_{t-j} \\
\end{bmatrix},
\]

where \( \epsilon_t = R_t^f - \alpha - \beta f_t \), represents the vector of the time-series residuals. Note that in the last equality, we directly impose the null that the asset pricing relation is true, i.e. \( E(R_{t-j}^f) = \beta \lambda \).

\( S \) is estimated by the Newey-West procedure. By using the GMM formula for the variance-covariance matrix of the parameter estimates we can define

\[
Var \left( \hat{\Theta} \right) = \frac{1}{T} (ad)^{-1} a \hat{S} a' (ad)^{-1} \text{ and }
\]

\[
Var \left( g_T (\hat{\Theta}) \right) = \frac{1}{T} \left( I_{N(K+2)} - d (ad)^{-1} a \right) \hat{S} \left( I_{N(K+2)} - d (ad)^{-1} a \right)'.
\]

Using the last \( N \) elements of the diagonal we obtain the variance of the cross-sectional pricing errors (\( \hat{\alpha} \)) which can be used to conduct the tests that the pricing errors are jointly equal to zero

\[
\hat{\alpha}' \text{var} (\hat{\alpha})^{-1} \hat{\alpha} \sim \chi^2_{(N-3)}.
\]
We now present 3 sets of cross-sectional regression results based on the GMM procedure outlined above, using the sample period 1993/01-2006/09.

(1) In Panel A of Table 14, we present the results corresponding to a model with 3 state variables in the VAR (excess aggregate bond market return, real rate and the credit spread) and 7 Lehman Brothers index portfolios of different default categories.

(2) In Panel B of Table 14, we present the results corresponding to a model with 4 state variables in the VAR (excess aggregate bond market return, real rate, credit spread and dividend yield) and 7 Lehman Brothers index portfolios of different default categories.

(3) In Panel C of Table 14, we present the results corresponding to a model with 3 state variables in the VAR (excess aggregate bond market return, real rate, credit spread and dividend yield) and 14 assets in the cross-section (the 7 Lehman Brothers indices and 7 Citigroup industry portfolios).
### PANEL A

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<th>bondmkt news</th>
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<th>real rate news</th>
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<td>Estimate</td>
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Table 14: 1993-2006: Cross-Section— Panel A: The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium. The corporate bond portfolios are bond market index portfolios of different default categories from Lehman Brothers. Panel B: the dividend yield was added to the state vector defined in Panel A and the corporate bond portfolios are the Lehman Brothers indices defined in Panel A. Panel C: uses the same state vector as in Panel B and adds to the Lehman Brothers portfolios, 7 Citigroup industry indices.
References


Investment Company Institute, 2007a. A review of trends and activity in the investment company industry, 47th edn, ICI.


