Return period of vegetation uprooting by flow

Giulio Calvani\textsuperscript{a,b,*}, Paolo Perona\textsuperscript{b}, Simone Zen\textsuperscript{b}, Valentina Bau\textsuperscript{b}, Luca Solari\textsuperscript{a}

\textsuperscript{a}Department of Civil and Environmental Engineering, School of Engineering, University of Florence, via S. Marta 2, 50139 Florence, Italy
\textsuperscript{b}School of Engineering, Institute for Infrastructure and Environment, The University of Edinburgh, The King’s Buildings, EH9 3FB Edinburgh, United Kingdom

Abstract

Fluvial environments are dynamic systems whose evolution and management are strongly affected by the resilience of riparian vegetation to uprooting by flow. Similarly to other natural phenomena, the interactions between flow, sediment and vegetation uprooting is governed by both the magnitude and duration of hydrological events. In this work, we analytically derive the link between probabilities of plant uprooting by flow and the return time of corresponding hydrologic erosion events. This physically-based analysis allows to define the key parameters involved in the plant uprooting dynamics, and to link the uprooting probability of riparian vegetation to plant biomechanical characteristics, hydrological regime and sediment parameters. For example, we show how the rooting depth changes the return time of critical hydrologic event uprooting plants with different probabilities. The model also shows

\begin{flushright}
\textsuperscript{*}Corresponding author at: Department of Civil and Environmental Engineering, School of Engineering, University of Florence, via S. Marta 2, 50139 Florence, Italy. Email address: giulio.calvani@unifi.it (Giulio Calvani)
\end{flushright}
the difference between magnitude driven and duration driven flow uprooting
events. The proposed approach is eventually validated against data from field
measurements and numerical simulations of pioneer woody species for two
flood events with different return period. Our approach demonstrates the
strong interrelations between the hydrological river regime and vegetation
properties and suggests that such interactions may be key for species recruit-
ment and consequent ecosystem shifts when hydrological regime is altered by
either human or climate changing scenarios.

Keywords: Peak Over Threshold, Poisson process, flow erosion, plant
uprooting, type II uprooting

1. Introduction

Fluvial environments are dynamic systems whose evolution is governed
by the interactions between vegetation dynamics, sediment processes and
flow regime. Riparian plants alter turbulence structures, flow velocity and
sediment transport (Nepf, 2012b). At the same time, the alternation of low
and high flow discharges drives the recruitment, growth and decay of ripar-
ian vegetation (Edmaier et al., 2011). Particularly during high stage events,
vegetation is subjected to drag force and plant removal occurs when root
anchoring force is reduced through bed erosion to equal the drag (named
uprooting Type II after (Edmaier et al., 2011)). Vegetation uprooting under flow and scour constraints (Type II) was investigated by Edmaier et al. (2015) in laboratory experiments with *Avena sativa* and by Bywater-Reyes et al. (2015) in field measurements. Calvani et al. (2019a) used flume experiments with *Avena sativa* and *Salix purpurea* and field measurements to test and validate a model able to predict the critical bed erosion depth for which uprooting occurs. All these studies agree that the amount of bed erosion leading to plant uprooting by flow is smaller than the initial rooting depth, thus supporting the critical rooting depth model (Edmaier et al., 2011; Calvani et al., 2019a). Perona and Crouzy (2018) hypothesized that for low plant size vs sediment size ratio, the critical rooting depth would correspond to a critical erosion depth. The latter is achieved by applying an erosion rate, which is the superposition of deterministic mean scouring (i.e., scouring happening over a characteristic longitudinal length scale) and random fluctuations mainly induced by turbulence and sediment transport mechanics.

Bed elevation changes, which include deposition and erosion, are regulated by the Exner equation, which states that time changing rate in bed elevation depends on the spatial variability of sediment fluxes. Specifically, in a
river reach, erosion takes place when downstream sediment outflow is larger than the sediment inflow coming from upstream. Under a 1D framework, the corresponding mathematical formulation is the 1D-Exner equation,

\[ \frac{\partial \eta(x, t)}{\partial t} = -\frac{1}{(1 - \lambda_s) B} \frac{\partial Q_s}{\partial x} \]  

(1)

where \( \eta(x, t) \) is the bed elevation, \( x \) is the coordinate along the streamwise direction of the main channel, \( t \) is time, \( \lambda_s \) is the sediment porosity, \( B \) is the channel width and \( Q_s(x, t) \) is the sediment discharge. At the time scale of a single flood event, the difference in sediment transport fluxes between two consecutive sections \( \left( \frac{\Delta Q_s}{\Delta x} = \frac{1}{\Delta x} \int_{x_1}^{x_2} dQ_s \right) \) is related to the bed shear stress acting at the bottom of the channel, which depends on the average flow velocity and, in turn, on the flow discharge. Therefore, the amount of erosion achieved during a flood event depends both on the magnitude and the duration of the event itself.

Flow discharge drives the uprooting process and, therefore, the hydrological time scale of flood events governs the recruitment of riparian vegetation species. Accordingly, riparian and aquatic species would have adapted their biomechanical properties in order to withstand the flow regime and increase survival chances during stress periods, due to either drought or flood events (Karrenberg et al., 2002; Gibling and Davies, 2012; Gurnell, 2014). As a
result, the link between vegetation dynamics and hydromorphological time scales may represent the key factor to understand the biological evolution of riparian species and predict their effects on ecosystem dynamics (Calvani et al., 2019b). Such link was seldom investigated in literature, mostly by focusing on short time horizon only (Corenblit et al., 2015), although the interactions among native and invasive alien species and river morphodynamics employ decades to evolve (Habersack, 2000; Solari et al., 2016). To this purpose, an analysis on the long term (return period) is therefore sought, as well as the definition of an hydrograph associated to such return period. This is particularly required when both the magnitude and the duration of the flow event play a fundamental role in flow-time related processes, such as flood risk modelling and management (e.g., Mignot et al., 2018; Tanaka et al., 2017), dam overtopping (e.g., Schmocker and Hager, 2009) and sediment transport (e.g., Powell et al., 2001), among others.

In this work we link the uprooting probability \( P_T \) to the extreme value analysis of a flow discharge Compound Poisson Process (CPP) using the Peak Over Threshold (POT) methodology. POT is a common mathematical approach to evaluate the occurrence probability (i.e., return period) of rare extreme events and is widely used in many disciplines, such as meteorol-
ogy, geological, hydraulic and structural engineering and earth sciences (e.g., Leadbetter, 1991; Önöz and Bayazit, 2001; Novak, 2011; Castillo, 2012). We additionally provide a formulation for the statistically average hydrograph of a flow event associated to such threshold and its return period. We then apply the proposed formulation to the case study of vegetation removal by flow and bed erosion (Type II uprooting). We combine the POT of the CPP and the probabilistic model of plant removal to correlate the hydrological parameters to the return period of riparian vegetation uprooting probability. As last, we perform a sensitivity analysis on the parameters involved and test the proposed approach against field measurements data from Bywater-Reyes et al. (2015).

2. Methodology

2.1. The uprooting model

Consider figure 1, which represents the uprooting process investigated by Perona and Crouzy (2018). Scouring trajectories originate from the initial bed level ($\eta = 0$), reduce plant anchoring, until the critical erosion depth (i.e., $\eta = -L_e$) is achieved, then plant is uprooted. The different trajectories evolve according to the flow hydrograph $Q_\xi(t)$ and the stochasticity in the erosion
process, $g_t$. Such process results in a probability distribution function, $p_\tau(t)$, of the times leading to uprooting. According to Perona and Crouzy (2018), the probability distribution function of time to uprooting, $p_\tau(t)$, reads:

$$p_\tau(t) = \frac{L_e}{2\sqrt{\pi G^3(t)}} \left( \frac{g_t(t)}{2} \right) \exp \left[ -\frac{(L_e - V(t))^2}{4 G(t)} \right]$$

$$+ W(t) \exp \left[ \frac{L_e V(t)}{G(t)} \right]$$

where $L_e$ is the critical erosion depth for plant uprooting to occur, $g_t(t)$ describes the strength of uncorrelated Gaussian noise of the erosion process, $G(t) = \frac{1}{2} \int_0^t g_t(\tau) d\tau$, $V(t) = \int_0^t \dot{L}_d(\tau) d\tau$ and $W(t) = \sqrt{\pi G(t)} \text{Erfc} \left[ \frac{L_e + V(t)}{2 \sqrt{G(t)}} \right]$. With $\tau$ the dummy time variable of integration. Therein, the deterministic part of the root exposing rate due to bed erosion is $\dot{L}_d = \dot{\eta}_d(t) \frac{dL}{d\eta}$ where $dL/d\eta$ accounts for the root shape and architecture within the soil. We assume $dL/d\eta = 1$ under the simplifying hypothesis of root vertical development (Edmaier et al., 2015; Calvani et al., 2019a). This requires that the average hydrograph of an event must be defined in order to calculate the associated erosion rate, its total duration $\hat{T}_\xi$ (figure 1) and the correspondent uprooting probability $P_\tau(t = \hat{T}_\xi)$.

The quantity $g_t$ has the unit of a diffusivity (i.e., m$^2$s$^{-1}$) and models the stochasticity of turbulence and sediment transport mechanics. Since no
formulation are available in literature, we argue that a relationship for the quantity \( g_t \) can be sought in the formula of the eddy viscosity (Pope, 2001; Michael, 2015), as disturbances in sediment transport are directly related to fluid obstacle interactions and flow turbulence at the stem scale (Nepf, 2012a; Perona and Crouzy, 2018). Thus, the formula reads:

\[
g_t(t) = l_s \cdot u_*
\]

where \( l_s \) is the *sediment mixing length* (i.e., a length scale along the vertical direction \( y \)) and \( u_* \) is the shear velocity, that plays the role of a velocity scale along the longitudinal direction \( x \), similarly to the case of eddy viscosity \( \nu_t \).

We set the sediment mixing length \( l_s \) equal to the mobilized sediment layer thickness, which is in the order of magnitude of the \( D_{90} \). Accordingly, the equation for \( l_s \) reads

\[
l_s = k_g \cdot D_{90}
\]

where \( k_g \) is a multiplying coefficient equal to 2, according to Parker (1990).

For the sake of dimensional consistency in unit of measurement, a multiplying constant equal to \( 1 \text{s} \text{d}^{-1} \) has to be taken into account when considering the strength of the Wiener process (see Eq. (2.10) in Perona and Crouzy (2018)).

Finally, the relationship for the probability of Type II uprooting \( P_r(t) \)
reads (Perona and Crouzy, 2018):

\[ P_r(t) = \int_0^t p_r(\tau) d\tau \]  

(5)

2.2. Peak Over Threshold analysis

We now approximate the flow discharge signal to a Compound Poisson Process. The Compound Poisson Process (CPP) is a common mathematical representation to describe the dynamics of stochastic systems where instantaneous perturbations cause sudden jumps in the state variable (Cox and Miller, 1965; Ridolfi et al., 2011). Forest fire spread (Daly and Porporato, 2006; Zen et al., 2018), avalanches induced by snowfall (Perona et al., 2007, 2012), groundwater recharge, soil moisture increase (Rodriguez-Iturbe et al., 1999; Botter et al., 2007), river flood events due to heavy rainfall (Todorovic, 1978; Önoz and Bayazit, 2001; Lague, 2010) and ecomorphodynamics (Crouzy and Perona, 2012; Bertagni et al., 2018) are only some of the natural processes that can be modelled using the CPP approach. In the following, we focus on flow discharges in a straight channel, characterized by constant width and bed slope. We assume flow discharge \( q(t) \) being driven by a deterministic drift (i.e., exponential decrease \( \text{Exp}[−t/\tau_P] \), with decay rate \( \tau_P \))
and instantaneous random positive jumps (with average frequency $\lambda_P$) representing the flood events (figure 2) (Botter et al., 2007). The average flow discharge $\mu_P$ of the CPP is $\mu_P = \gamma_P \cdot \lambda_P \cdot \tau_P$, where $\gamma_P$ is the mean values of the jumps. Accordingly, flow discharge can be modelled by a probabilistic distribution function, $p(q)$ (figure 2), of the form (Lague et al., 2005; Botter et al., 2007):

$$p(q) = \frac{1}{q \Gamma(\beta_P)} \text{Exp} \left[ -\frac{q}{\gamma_P} \right] \left( \frac{q}{\gamma_P} \right)^{\beta_P}$$

(6)

where $\Gamma[\beta_P]$ is the complete Gamma function (Abramowitz and Stegun, 1965) with $\beta_P = \lambda_P \tau_P$.

Next, we perform an extreme value analysis using the Peak Over Threshold (POT) approach developed by Todorovic (1970) and then applied to exponentially distributed peak events (CPP) by Zelenhasic (1970) and ¨On¨oz and Bayazit (2001), among others. Once a certain threshold $\xi$ is set, POT allows to evaluate the return period $T(\xi)$ of the flow discharge higher than such threshold. For the sake of brevity, only the main results are reported here below, whereas we address the reader to Calvani (2019) for the calculation steps. The return period $T(\xi)$ simply reads:

$$T(\xi) = \frac{1}{1 - P_\xi}$$

(7)

Therein, the probability of events higher than the threshold $\xi$, $P_\xi$ as given
by the POT analysis, is equal to:

$$ P_\xi = e^{-T} \lambda_P \, P^\xi_\xi $$

where $T$ is a temporal quantity set equal to 1d for the aim of the POT,

$\lambda_P = e^{-\phi} \frac{\phi}{\beta_P} \Gamma[\beta_P, \phi]$ is the average frequency of upcrossing the threshold $\xi$; $\phi$ is the ratio between the threshold $\xi$ and the mean value of pulses $\gamma_P$; $P^\xi_\xi$ is the probability of the signal $q(t)$ (figure 2) to be higher than the threshold $\xi$,

that is $P^\xi_\xi = \int_\xi^\infty p(q) dq = \frac{\Gamma[\beta_P, \phi]}{\Gamma[\beta_P]}$ (Ridolfi et al., 2011) where $\Gamma[\beta_P, \phi]$ is the upper incomplete Gamma function (Abramowitz and Stegun, 1965). It must be clear that the two frequencies, $\lambda_P$ and $\lambda_P'$, represent different quantities for the CPP. The first one, $\lambda_P$, is a property of the process and depends, in this case, on the hydrological regime of the river, only. On the contrary, the second one, $\lambda_P'$, depends on the threshold value, $\xi$. To clarify this point, one can compare the whole number of jumps in figure 2 (which depends on $\lambda_P$) to the number of jumps across above the threshold $\xi$ (which depends on $\lambda_P'$).

2.3. Reference mean event

For a given threshold $\xi$ and its return period $T(\xi)$ (Eq. (7)), we calculate the associated reference mean event, which represents a statistically averaged flow hydrograph following a jump (peak) above the threshold $\xi$ and lasts
until downcrossing the threshold $Q_{cr}$. As we focus on events able to uproot vegetation after riverbed erosion (i.e., Type II uprooting), we consider flow discharge above the threshold value for incipient motion of sediment $Q_{cr}$ only, which we assume equal to the one for the incipient erosion. Such value can be calculated as follows:

$$Q_{cr} = \tau_{cr}^* \left( \frac{\rho_s - \rho}{\rho} \right)^{5/3} D_{50}^{5/3} \frac{B}{n} S^{-7/6}$$

(9)

where $\tau_{cr}^*$ is the critical Shields parameter equal to either 0.03, according to Parker et al. (2007) for gravel bed rivers subjected to bedload transport, or $\tau_{SL}^*$ for sand-bed rivers with suspended load; $\rho_s$ and $\rho$ are sediment and water density, respectively; $D_{50}$ is the mean grain size; $B$ is the river width; $n$ is the Manning coefficient and $S$ is the bed slope. The critical Shields parameter for sand-bed rivers $\tau_{SL}^*$ can be calculated using Brownlie’s equation (Brownlie, 1981).

We address the reader to Calvani (2019) for the whole mathematical approach and report here the final equation of the reference mean event $Q_\xi(t)$ defined by a piecewise function:

$$Q_\xi(t) = \begin{cases} 
Q_0(\xi) e^{-t/\tau_1} & [0 \leq t \leq T_\xi^+] \\
\xi e^{-(t-T_\xi^+)/\tau_2} & [T_\xi^+ < t \leq \hat{T}_\xi] 
\end{cases}$$

(10)
Quantities $Q_0(\xi)$, $\tau_1$ and $\tau_2$ are calculated according to the properties of the Compound Poisson Process: particularly, the average time, $T^+_{\xi}$, and the average flow value, $\bar{Q}_{q>\xi}$, above the threshold $\xi$, and the average time, $T_{\xi\rightarrow Q_{cr}}$, from the threshold $\xi$ to the threshold $Q_{cr}$. The temporal quantities, $T^+_{\xi}$ and $T_{\xi\rightarrow Q_{cr}}$, are related to the concept of mean first passage time, that is the average time that a signal upcrosses or downcrosses a certain threshold value (Laio et al., 2001; Ridolfi et al., 2011). The total duration of the reference mean event is, therefore:

$$\hat{T}_{\xi} = T^+_{\xi} + T_{\xi\rightarrow Q_{cr}}$$

(11)

To this regard, we must point out that flow volume conservation is exactly satisfied for the first part of the reference mean event only (i.e., from $Q_0$ to $\xi$), as this is imposed using the conditions for $T^+_{\xi}$ and $\bar{Q}_{q>\xi}$. The second exponential decay (i.e., from $\xi$ to $Q_{cr}$) is calculated using the exact formulation for the average time $T_{\xi\rightarrow Q_{cr}}$. This may lead to error in the flow volume conservation, and the outcomes of this assumption will be explored in Section 3.1.
2.4. Erosion rate

In order to account for bed elevation changes and scouring events promoting Type II uprooting during high flow events, we couple the time-varying flow discharge to the 1D Exner (Eq. (1)) and sediment transport relationships. For the sediment transport, we consider both the cases of bed and suspended load. Specifically, we assume a Meyer-Peter-Müller type formula (Wong and Parker, 2006) for bedload and the van Rijn’s model (van Rijn, 1984) for the suspended load. For the resultant relationships to be as simple as possible, we neglect the effects of the time derivative in the momentum equation at the time scale of the process. As a result, bed shear stress $\tau_b$ and water depth $Y$ can be calculated from flow discharge only, by knowing channel geometry and involving the Manning relation for normal flow. Additionally, for the channel geometry, we assume a wide rectangular cross-section with constant width and bed slope. By combining the aforementioned formulas and assuming negligible upstream sediment discharge (Perona and Crouzy, 2018), we obtain a relationship for the net (deterministic) erosion rate $\dot{\eta}_d$ where the typical structure of sediment transport formula above critical threshold and exponent 3/5 coming from Manning relation can be rec-
ognized. The relation reads:

\[ \dot{\eta}_d(t) = \psi_1 \psi_2 \left( q^3(t) - Q_{cr}^3 \right)^b \cdot \left( q_{m}(t) \cdot I(q(t), D_{50}) \right)^{a_{ST}} \]  \hspace{1cm} (12)

where \( \psi_1 \) is a coefficient depending on physical parameters, river size and sediment properties, \( \psi_2 \) is a coefficient depending on the main type of sediment load, \( b \) is the exponent in the sediment transport formula (e.g., \( 3/2 \) in the case of van Rijn’s and MPM’s models), \( I(q(t), D_{50}) \) is a quantity given by the Einstein’s integrals (Einstein, 1950) and depending on mean grain size \( D_{50} \) and flow discharge in the case of suspended load only, and \( a_{ST} \) is a parameter equal to either 0 for bedload or 1 for suspended load. The relation for the parameter \( \psi_1 \) reads:

\[ \psi_1 = \sqrt[3]{g} D_{50}^{1-b} \left( \frac{\rho_s - \rho}{\rho} \right)^{-b} \left( \frac{n}{B} \right)^{\frac{3b}{2}} S^{\frac{7b}{10}} \]  \hspace{1cm} (13)

where \( g \) is the acceleration due to gravity, \( \lambda_s \) is the sediment porosity, \( \Delta x \) is the length scale along the streamwise direction where the spatial derivative of sediment transport (right-hand side term in Eq. (1)) can be approximated by the finite difference. Following the approximation suggested by Perona and Crouzy (2018), \( \Delta x \) is the spatial scale, where net (parallel) bed erosion takes place. The coefficient \( \psi_2 \) depends on the main type of sediment transport,
according to the following relation:

\[
\psi_2 = \begin{cases} 
\alpha_{BL} \ D_{50}^{1/2} \left( \frac{\rho_s - \rho}{\rho} \right)^{1/2} & a_{ST} = 0 \\
\alpha_{SL} \ \left( \frac{n}{B} \right)^{3/10} S^{7/9} R_{ep}^{2/3} \ \tau_{SL}^b & a_{ST} = 1 
\end{cases}
\]  
(14)

Therein, \( \alpha_{BL} \) is the coefficient in the bedload formula (e.g., 3.97 in Wong and Parker (2006)), \( \alpha_{SL} = 0.174 \) is the coefficient in van Rijn’s formula for suspended load (van Rijn, 1984) and \( R_{ep} \) is the particle Reynolds number. It is worth to mention that, in the case of bedload (\( a_{ST} = 0 \) in Eq. (14)), when \( b = 1.5 \) (e.g., Meyer-Peter and Müller, 1948; Wong and Parker, 2006), the mean grain size \( D_{50} \) cancels out in the product \( \psi_1 \cdot \psi_2 \) in Eq. (12). As a result, the erosion rate \( \dot{\eta}_d(t) \) depends on the mean grain size, \( D_{50} \), by the critical flow for incipient motion of sediment, \( Q_{cr} \) (Eq. (9)), only.

3. Results

3.1. Reference mean event

A graphical explanation of the reference mean event \( Q_\xi(t) \) (Eq. (10)) is reported in figure 3a), with the associated erosion rates due to bedload and suspended load and the critical thresholds, \( Q_{cr} \), for incipient sediment transport (Eq. (9)). Figure 3b) shows the comparison between a reference
mean event (blue line) and some Compound Poisson Process events (thin black lines) between the two thresholds $\xi$ and $Q_{cr}$.

Due to assumptions made in the calculations of the reference event (Eq. (10)), particularly the second exponential decay from $\xi$ to $Q_{cr}$ (Section 2.3), we compared the flow volume of the reference mean event (i.e., $\int_0^{T_Q} Q(t)dt$) to the average flow volume of some events taken from a CPP, according to various combinations of the parameters $\lambda_P$ and $\tau_P$, and with respect to different values of the threshold $\xi$. The comparison was carried out for two ideal rivers characterized by different hydro-morphological parameters. The first one, here named the Small River, has a cross section width, $B$, equal to 50m, bed slope, $S$, equal to 0.005, and grain size distribution characterized by $D_{50}$ equal to 0.1m and $D_{90}$ equal to 0.15m. The corresponding hydrology is characterized by a mean flow discharge, $\mu_P$, equal to $15m^3s^{-1}$, average frequency of events, $\lambda_P$, equal to $0.1d^{-1}$, and exponential decay rate, $\tau_P$, equal to $1.5d$. The second river, here named the Large River, has a cross section width, $B$, equal to 100m, bed slope, $S$, equal to 0.002, and grain size distribution characterized by $D_{50}$ equal to 0.04m and $D_{90}$ equal to 0.1m. The corresponding hydrology is characterized by a mean flow discharge, $\mu_P$, equal to $400m^3s^{-1}$, average frequency of events, $\lambda_P$, equal to $0.1d^{-1}$, and
exponential decay rate, $\tau_P$, equal to 1.5d. For both the rivers, the $D_{90}$ was used to calculate the Manning coefficient $n$, according to the empirical relation $n = D_{90}^{1/6}/26$. The results of the comparison are shown in figure 4.

The comparison shows that, as expected, the formulation of the reference mean event (Eq. (10)) does not capture exactly the average flow volume during the exponential decay from the upper threshold $\xi$ to the lower threshold $Q_{cr}$. Nevertheless, the agreement seems satisfactorily as the relative error is overall less than 5%, with a maximum of 15% for some very particular combinations of the parameters $\lambda_P$ and $\tau_P$ (e.g., $\lambda_P=0.02d^{-1}$ and $\tau_P=7d$) which are uncommon in natural rivers. It is worth to note that the flow volume calculated using the reference mean event overestimates the numerical data for most of the $\lambda_P-\tau_P$ combinations in the Small River. On the contrary, it has the tendency to underestimate the numerical data in the Large River. As a result, the formulation of the reference mean event yields to predicting errors in the uprooting probability. To this regard, we compared the average uprooting probability of fifty events taken from a CPP with two different values of the higher threshold $\xi$ for the Small River and the Large River (figure 3c,d). For the Small River, for the first value of the threshold, $\xi=125m^3s^{-1}$, the average uprooting probability of the CPP events was $P_r=0.24$, whereas...
the uprooting probability calculated using the corresponding reference mean event was $P(t = \hat{T}_\xi)$=0.32. For the second threshold value, $\xi$=180m$^3$s$^{-1}$ (figure 3c), the uprooting probability from the CPP was $P=0.24$, whereas $P(t = \hat{T}_\xi)$=0.56. For the Large River, the first threshold value was set to $\xi$=550m$^3$s$^{-1}$ and the uprooting probability of the CPP events was $P=0.53$, with the corresponding $P(t = \hat{T}_\xi)$ of the reference mean event equal to 0.54 (figure 3d). For the second value of the threshold, $\xi$=750m$^3$s$^{-1}$, the uprooting probabilities were $P=0.58$ and $P(t = \hat{T}_\xi)$=0.55. As a consequence of the approximation of the flow volume (figure 4), the approach leads to slight underestimations of the uprooting probability in the case of the Large River and overestimations in the case of the Small River. Therefore, we are confident that the case of $\xi$=550m$^3$s$^{-1}$ in the Large River with slight overestimation of the uprooting probability depends on the particular randomly chosen events that are mainly lying below the reference mean event (see figure 3d).

3.2. Resilience to vegetation uprooting

We performed the calculations of $P(t = \hat{T}_\xi)$ for both the rivers presented in the previous section, in the case of bedload transport (i.e., $\alpha_{ST}$=0 in Eq. (14)). For the sake of simplicity, we did not consider the case of suspended load (i.e., $\alpha_{ST}$=1 in Eq. (14)), even when the Shields number would be
large enough to support its occurrence at high value of the flow discharge.

The length scale, $\Delta x$ in Eq. (13), was set equal to $6 \cdot B$, which is roughly the length scale of potential river bars (Leopold and Wolman, 1957). Due to the highly non-linear relationships involved in the calculation of the uprooting probability $P_r(t = \hat{T}_\xi)$, we performed a graphical analysis on the effects of varying parameter values, one at a time. In particular, we considered the effects of the critical erosion for uprooting, $L_e$ and the coefficient $\alpha_{BL}$ in bed load sediment transport formula, by accounting for constant values of the hydrological parameters, $\mu_P$, $\lambda_P$ and $\tau_P$. Additionally, we kept constant the fluctuations of the sediment transport rate ($g_t = 0.05m^2d^{-1}$), regardless of Eq. (3), to highlight the changes induced by varying the tested parameter.

Figure 5 shows the trend of the uprooting probability function, $P_r(t = \hat{T}_\xi)$, versus the corresponding return period $T_\xi$ at varying the parameters, for the Small River and the Large River, respectively.

For both rivers, the critical erosion depth $L_e$ plays an important role in the probability of uprooting. In case of the Small River, figure 5a) shows that an increment of 0.25m in $L_e$ (e.g., from 0.5m to 0.75m) raises survival chances ($= 1 - P_r$) by more than 30% for a yearly flow event. For the Large River (figure 5c), the same consideration implies an increment of 20% in the
survival chances. According to Calvani et al. (2019a), plants do not need
to grow root as deep as that amount, as soil strength increases with depth.
Furthermore, the same gain in $L_e$ can be achieved by reducing the frontal
area subjected to drag, either by increasing flexibility (i.e., reconfiguration)
or by physically losing leaves. The latter mechanism appears to be a possible
strategy for riparian plants in the temperate zone to adapt their deciduous
period to autumn and winter seasons, not only to save energy, but also to
withstand the larger and more frequent peak events.

For the effects of the coefficient of the bedload transport formula, we
considered the original value proposed by Wong and Parker (2006) and four
other values, differing by $\pm 25\%$ and $\pm 50\%$. For the Large River, figure 5d)
shows that increasing the coefficient $\alpha_{BL}$ by 25% raises the uprooting prob-
ability by roughly 5% in the whole range of the tested return periods. A
similar behaviour in the function $P_\tau$ can be observed when the coefficient
$\alpha_{BL}$ decreases by 25% ($\alpha_{BL}=2.978$). In this case we observed a decrease in
the uprooting probability by 5%, only. As a result, the parameter $\alpha_{BL}$ in
the range of tested values does not seem to significantly affect the uproot-
ing probability. Different results were obtained for the Small River, where
the variation imposed in the bedload transport coefficient, $\alpha_{BL}$, affect the
uprooting probability by more than 10% for a yearly flow event (figure 5b).

Particularly, for the case of doubling the bedload transport coefficient, the uprooting probability increases by 25%.

Additionally, we investigated the effects of varying the hydrological parameters, specifically the average jump value $\gamma_P$, the average frequency of jumps $\lambda_P$, and the exponential decay rate $\tau_P$, and the grain size distribution, with particular focus on the mean grain size $D_{50}$. The results of the analysis are reported in figures 6 and 7, for the Small River and the Large River, respectively.

Both rivers show similar trends of the uprooting probability, while varying the same parameter. Similarly to the case with constant mean flow discharge $\mu_P$ (figure 5), the influence of the investigated parameters is more evident in the Small River, when compared to the Large River. Such result is particularly clear in figures 6 and 7 when comparing panels c), varying the mean value of jump, $\gamma_P$, and panels d), at varying the mean grain size $D_{50}$.

Consider now, the case of figure 8, where the uprooting probabilities of two different cross sections in the same river are shown. Hydro-morphological parameters are representative of the Thur River (CH), at the two measuring stations of Jonschwil, Mühlau (upstream) and Andelfingen (downstream).
Data are reported within the figure. The uprooting probability $P_r(t = \hat{T}_\xi)$, for the same critical erosion length ($L_e=0.75\text{m}$) and erosion process noise ($g_t=0.05\text{m}^2\text{d}^{-1}$), shows the existence of a return period for which the two curves intersect. Such return period corresponds to equal uprooting probability in both the cross sections, thus supporting the idea of selecting the survival of equal vegetation species along the whole river reach. We explain this trend by considering the different reference mean events and the associated $p_r$ obtained for the two different cross sections. For low return periods (e.g., $T(\xi) \approx 0.3\text{y}$), the uprooting probability is higher in the downstream cross section (DS). The main reason is the longer duration of the reference mean event for the DS cross section, if compared to that in the upstream (US) one (i.e., $\hat{T}_\xi^{DS} > \hat{T}_\xi^{US}$). On the contrary, for higher return periods (e.g., $T(\xi) \approx 10\text{y}$), the uprooting probability is higher in the US cross section. Although the condition $\hat{T}_\xi^{DS} > \hat{T}_\xi^{US}$ still applies, the probability distribution function, $p_r$, in case of the US cross section (see bottom-left inset panel in figure 8) shows a very remarkable peak, leading to a higher integral value, $P_r$. We refer to this dualism as duration driven and magnitude driven uprooting events, respectively.
3.3. Real case application

We applied the proposed methodology and the uprooting model to the case study of the Santa Maria River (Arizona, USA). This river was investigated by Bywater-Reyes et al. (2015) and plants on a bar along it were mechanically uprooted under different conditions of scouring. As a result, data of flow discharge to fit the CPP and measurements of root resistance and plant geometry are both available.

Figure 9a) shows the reference mean event and its associated erosion rate \( \dot{\eta} \) driven by suspended load \( (a_{ST}=1 \text{ in Eq. (12)}) \) for the 10y return period peak event. We calculated the uprooting probability according to different critical erosion length \( L_e \) and compared the results for the two flow discharges investigated by Bywater-Reyes et al. (2015) \( (Q_2=80\text{m}^3\text{s}^{-1}; Q_{10}=460\text{m}^3\text{s}^{-1}) \) and the plants uprooted under 0.30m scouring condition. For the measured plants, we calculated the minimum, median and maximum of uprooting probability of according to the corresponding velocities as output numerical simulation carried out by Bywater-Reyes et al. (2015) for the two investigated return periods. Figure 9b) shows the uprooting probability \( P_r(t = \dot{T}_\xi) \) versus the threshold \( \xi \) for different values of the unknown variable \( L_e \) for the Santa Maria River. The critical erosion length \( L_e = 0.33\text{m} \) used in figure 9b) was
calculated according to the model proposed by Calvani et al. (2019a) for the mechanically uprooted plants for which measurements of intact root (i.e., main root length) were available. Uprooting probability for measured plants are shown as boxplots. As a final result of our analysis, we found a very good agreement between measured and modelled uprooting probability for both the flow discharges. Therefore, this supports the validity of our analysis and the robustness of our approach.

4. Discussion

For the sake of clarity, we have considered the reference mean event $Q_\xi(t)$ starting when a jump in the Compound Poisson Process up-crosses the threshold $\xi$. This is replicated in the reference mean event by the initial jump from the critical value $Q_{cr}$ to the flow discharge $Q_0(\xi)$. This assumption is often legitimated by the generally shorter duration of the raising limb compared to that of the falling limb in a flow hydrograph. However, such hypothesis can not be satisfied in case of high correlated signals, for instance when the temporal scale $\tau_P$ governing the exponential decrease (deterministic drift in the CPP) is larger than the average interval between shots (i.e., $\lambda_P^{-1}$).

In this case, a more appropriate formulation for the raising limb of the refer-
ence mean event must be provided. This is object of ongoing investigations. Nevertheless, hydrological regimes with such characteristics are uncommon so we are confident that the proposed formulation and methodology can be satisfactorily applied to most rivers (e.g., figure 9).

Additionally, in this section, we focused on Eq. (3) and the associated time-varying $g_t$. We investigated the effects of different values of $k_g$ (Eq. (4)), representing the variability of the mobilized sediment layer thickness. We compared the resulting uprooting probabilities with constant and time-varying $g_t$. For the sake of the analysis, we consider the constant $g_t$ as the integral average of the time-varying one over the entire duration $\hat{T}_\xi$ of the reference mean event $Q_\xi(t)$ for a given return period $T(\xi)$.

Figure 10 shows the comparison among uprooting probabilities with constant and time-varying $g_t$ according to different values of $k_g$. Time-varying $g_t$ plays a significant role in modifying the resultant $P_\tau(t = \hat{T}_\xi)$ only for either very high or very low values of $k_g$ (e.g., $k_g = 0.2$ or $k_g = 20$). For more reasonable values (e.g., $k_g = 2$ (Parker, 1990)), the uprooting probabilities are very similar and, therefore, the average value defines the entire trend.

This behaviour is clearly explained by the corresponding probability distribution functions, $p_\tau$, plotted in the inset panels of figure 10 for two different
return periods, $T(\xi)$. Moreover, for values of $k_g$ equal to 4, time-varying $g_t$ increases the uprooting probability for low return periods (e.g., $T(\xi) < 11y$), whereas $P_r(t = \hat{T}_\xi)$ is almost equal for slightly higher recurrence intervals (e.g., $10y < T(\xi) < 50y$). For higher return period ($T(\xi) > 50y$), the uprooting probability with the time-varying $g_t$ is lower than the correspondent with constant $g_t$. For even higher values ($k_g = 20$), the uprooting probability with time-varying $g_t$ is always larger, for the tested range of return period and hydrological parameters. It is interesting to highlight that for $k_g$ lower than 4, the uprooting probability function behaves in the opposite way. We didn’t investigate on the threshold value of $k_g$ that switches between the two different trends.

5. Conclusions

In this work, we linked the uprooting probability given by the stochastic model of Perona and Crouzy (2018) to the return period of flood events, calculated using the Peak Over Threshold method on a Compound Poisson Process. We proposed a simple approach to calculate a reference mean event for a given return period and its application to the stochastic model for the uprooting probability.
Our analysis has been carried out for one single event and returns the probability of uprooting associated to characteristic flood/erosion events of assigned return period. However, riparian vegetation may withstand many more erosion events during its life. This suggests that the interval between consecutive peak events and the ability for riparian species to recover and grow in this interval play a fundamental role in the evolution of water driven patterns (Bertagni et al., 2018), both from the biological and the morphological point of view (Edmaier et al., 2015; Perona and Crouzy, 2018; Calvani et al., 2019b). For this reason, the role of the intertime between consecutive flood events and their cumulative effects should be further investigated.

Our results suggest that the critical erosion depth $L_e$ and average frequency of peak events $\lambda_P$ are the key parameters to define the uprooting probability of riparian vegetation in a given river basin. Yet, this study confirms that long time scale interactions between river hydro-morphology and riparian vegetation are fundamental to shape the riverine environment (Bywater-Reyes et al., 2015). For a given hydrological regime, the mechanisms at the base of such interactions may be key to select species according to their ability to survive in water-driven environments. For instance, invasive riparian plants can take advantage of these interactions, leading to
colonization of new fluvial landforms and suppression of local species, due to alteration in the hydrological regime by either human impacts (Tealdi et al., 2011; Coletti et al., 2017) or climate change (Serrat-Capdevila et al., 2007; House et al., 2016).

Acknowledgments

This work was performed while the author was visiting the Chair of Environmental Engineering at the School of engineering of the University of Edinburgh, which is therefore deeply acknowledged. We thank Editors and two anonymous Reviewers for comments and suggestions that greatly improved the manuscript.

References


on the caliber of bed load sediment yield in ephemeral gravel bed rivers.

Water Resources Research 37, 1463–1474.

environmental sciences. Cambridge University Press.


Probabilistic modelling of water balance at a point: the role of climate,
soil and vegetation, in: Proceedings of the Royal Society of London A:
3789–3805.

Schmocker, L., Hager, W.H., 2009. Modelling dike breaching due to overtop-

Serrat-Capdevila, A., Valdés, J.B., Pérez, J.G., Baird, K., Mata, L.J., Mad-
dock III, T., 2007. Modeling climate change impacts—and uncertainty—on
the hydrology of a riparian system: The san pedro basin (arizona/sonora).


Zelenhasic, E.F., 1970. Theoretical probability distributions for flood peaks. Hydrology papers (Colorado State University); no. 42.

Figure 1: Illustration of the approach described by Eq. (2). The erosion rate evolves driven by flow hydrograph $Q_\xi(t)$, lasting $\hat{T}_\xi$, and erosion noise, $g_t$, so that different scouring trajectories result to a probabilistic distribution function, $p_\tau$, of the times to uprooting. Vegetation is removed when total erosion reaches the critical erosion depth, $L_e$. 
Figure 2: A sample realization of a Compound Poisson Process of flow discharge $q(t)$ (continuous blue line). Dashed blue line is the threshold $\xi$ for extreme value analysis. Dashed red line is the critical threshold $Q_{cr}$ for bed erosion. On the left the probabilistic distribution function $p(q)$ of flow discharge (continuous red line).
Figure 3: The reference mean event $Q_\xi(t)$ is the statistically averaged hydrograph associated to jumps above the threshold $\xi$. a) The reference mean event (continuous blue line) and its associated erosion rate, both in case of bedload (continuous dark-yellow line) and suspended load (dashed dark-yellow line) (see section 2.4). b) A comparison between the calculated hydrograph and some events above the threshold $\xi$ extracted from a Compound Poisson Process. c) The reference mean event for the Small River with $\xi=180\text{m}^3\text{s}^{-1}$ and some events taken from the CPP above such threshold. d) The reference mean event for the Large River with $\xi=550\text{m}^3\text{s}^{-1}$ and comparison to some events taken from the CPP.
Figure 4: The flow volume comparison between the reference mean event (analytical results) and some events taken from the CPP (numerical data) for the two ideal rivers involved in the analysis. Values are in m$^3$. a) The comparison for the Small River. b) The comparison for the Large River. Inset panels show the agreement for different combinations of the parameters $\lambda_P$ and $\tau_P$, according to different values of the threshold $\xi$. 

42
Figure 5: The uprooting probability, $P_r(t)$, in the Small River (panels a) and b)) and the Large River (panels c) and d)), at the end of the reference mean event ($t = \hat{T}_\xi$), according to different values of the parameters involved in Eq. (5). Values of the parameters are shown and, when not explicitly written, units are: [m] for $L_e$, [d$^{-1}$] for $\lambda_P$, and [d] for $\tau_P$. 

a) and c) $P_r(t = \hat{T}_\xi)$ versus return period $T(\xi)$ varying the critical length of erosion $L_e$, for the Small River and the Large River, respectively; b) and d) $P_r(t = \hat{T}_\xi)$ versus return period $T(\xi)$ varying the coefficient $\alpha_{BL}$ in the bedload transport formula, for the Small River and the Large River, respectively.
Figure 6: The uprooting probability, $P_{\tau}(t)$, in the Small River, at the end of the reference mean event ($t = \hat{T}_\xi$), according to different values of the parameters involved in Eq. (5).

Noise in erosion process $g_t$ is set to 0.05 m$^2$ d$^{-1}$, values of the other constant parameters are shown. When not explicitly written, units are: [m] for $L_e$, [d$^{-1}$] for $\lambda_{P}$, and [d] for $\tau_{P}$. a) $P_{\tau}(t = \hat{T}_\xi)$ versus return period $T(\xi)$ varying the mean frequency of jumps $\lambda_{P}$; b) $P_{\tau}(t = \hat{T}_\xi)$ versus return period $T(\xi)$ varying the exponential decay rate $\tau_{P}$; c) $P_{\tau}(t = \hat{T}_\xi)$ versus return period $T(\xi)$ varying the mean jump value $\gamma_{P}$; d) $P_{\tau}(t = \hat{T}_\xi)$ versus return period $T(\xi)$ varying the mean grain size $D_{50}$.
Figure 7: The uprooting probability, $P_\tau(t)$, in the Large River, at the end of the reference mean event ($t = \hat{\tau}_\xi$), according to different values of the parameters involved in Eq. (5).

Noise in erosion process $g_t$ is set to 0.05m$^2$ d$^{-1}$, values of the other constant parameters are shown. When not explicitly written, units are: [m] for $L_c$, [d$^{-1}$] for $\lambda_P$, and [d] for $\tau_P$. a) $P_\tau(t = \hat{\tau}_\xi)$ versus return period $T(\xi)$ varying the mean frequency of jumps $\lambda_P$; b) $P_\tau(t = \hat{\tau}_\xi)$ versus return period $T(\xi)$ varying the exponential decay rate $\tau_P$; c) $P_\tau(t = \hat{\tau}_\xi)$ versus return period $T(\xi)$ varying the mean jump value $\gamma_P$; d) $P_\tau(t = \hat{\tau}_\xi)$ versus return period $T(\xi)$ varying the mean grain size $D_{50}$. 

45
Figure 8: The uprooting probability, $P_\tau(t)$, at varying cross section. Hydro-morphological parameters are reported in the inset table. Blue line is for the upstream cross section (US), red line for the downstream one (DS). Inset panels show the probability distributions functions, $p_\tau$, for short (e.g., $T(\xi) \approx 0.3$y) and long (e.g., $T(\xi) \approx 10$y) return periods.
Figure 9: The uprooting probability $P_\tau(t)$ in the Santa Maria River, Arizona (USA) and comparison to the data calculated by Bywater-Reyes et al. (2015). a) The reference mean event $Q_\xi(t)$ for 10y return period and its associated erosion rate $\eta_{SL}(t)$ due to suspended load. b) Comparison of $P_\tau(t = \hat{T}_\xi)$ with different $L_c$. Boxplots are the probability of uprooting calculated with measured data by Bywater-Reyes et al. (2015) for 2 and 10y return periods.
Figure 10: Graphical comparison of uprooting probability $P_r(t = \hat{T}_\xi)$ versus return period $T(\xi)$ for different values of the time-varying noise $g_t(t)$ (Eq. (3)) and its integral mean over the duration $\hat{T}_\xi$ for different values of the coefficient $k_g$ of the sediment mixing length $l_s$ (Eq. (4)) in the Large River. Continuous lines are for the uprooting probability with constant $g_t$, dashed lines are for the uprooting probability with time-varying $g_t$. In the inset panels the probability distribution functions, $p_r$, corresponding to the reference mean event of two different return period (i.e., $T(\xi) = 1y$ and $T(\xi) = 20y$) for different values of the coefficient $k_g$. 

48